Effect of Magnetic Field Curvature on Penetration of the Magnetic Field into the Plasma

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The penetration of a magnetic field into a cylindrical plasma, in the time scale that is much longer than electron cyclotron period, is studied. A linear wave analysis is shown that the magnetic field penetrates rapidly into the plasma in radii smaller than the ion skin depth. Due to the axial symmetry, the problem reduces to a two-dimensional problem. The magnetic field evolution is numerically calculated. The ion density is also calculated. It is shown that during the penetration of the magnetic field, a gap appears between cathode and plasma. At the early times, at the plasma boundary, electrons move radially, and coupling of the electron velocity and the electric field induces the magnetic field. Electrons then gain a drift due to the field curvature that results in fast penetration of the magnetic field into the plasma.

Keywords: magnetic field, nonlinear penetration, fast penetration, skin depth, cylindrical plasma

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1. Introduction

Magnetic field penetration into plasma is one of the most important issues studied in plasma physics. It has direct application in laboratory plasmas and in space plasma [1–4]. When the penetration is due to linear effects, it usually occurs in the skin depth: in collisional plasma, skin depth is $(\eta t/\mu_0)^{1/2}$, where η is the resistivity, while in collisionless plasma skin depth is c/ω_{pe}, where $\omega_{\mathrm{pe}} = (ne^2/m\varepsilon_0)^{1/2}$, n is the plasma density and m is the electron mass. On the other hand, nonlinear effects result in deeper penetration of the magnetic field into the plasma, provided that plasma is inhomogeneous [4–8]. Moreover, the penetration occurs on a time scale much faster than the ion cyclotron period. In such short time scales, the ion motion can be neglected relative to the electron motion [4]. For scale lengths $L = (\partial \ln n/\partial y)^{-1}$ that are smaller than the ion skin depth c/ω_{pi}, the magnetic field penetrates quickly into the plasma [7]. The velocity of the penetration v, perpendicular to the density gradient, is $v = (B/\mu_0 e)(\partial Ln(n)/\partial n)$, where B is the magnetic field amplitude, e is electron charge ($v \gg v_{\mathrm{A}}$, v_{A} is the Alfven's velocity). This fast penetration is induced by density gradient and in this case the magnetic field is convected with the electron fluid with the characteristic velocity v. A study of the fast penetration of the magnetic field into the plasma by taking into account of electron inertia has been examined by Zaburdaev in [9]. Moreover, fast penetration of the magnetic field into an initially homogeneous plasma was studied in [10]; penetration is induced by a density gradient along the current lines that is formed by the magnetic pressure. The ion dynamics in a two-ion species plasma under penetration of the magnetic field was investigated in [11]. Furthermore, it was shown that the behavior of the plasma is different from that of a single-ion species plasma.

In this paper we calculated the evolution of an azimuthal magnetic field into a cylindrical plasma. Due to the axial symmetry, the problem reduces to a two-dimensional problem. We suppose that the initial density of the plasma is uniform so that the magnetic field penetration is due to the field curvature and not due to the density gradient. A linear wave analysis is shown that the magnetic field penetrates rapidly into the plasma in radii smaller than the ion skin depth. Also, the ion density is calculated; during the penetration of the magnetic field, a gap appears between cathode and plasma. We will present a mechanism for the evolution of the magnetic field in the plasma. At the early times, electrons move radially and the space-dependent electric field induces the magnetic field. Fast penetration of the magnetic field into the plasma then has been attributed to drift of electrons due to field curvature.

The paper is organized as follows: in Sec. 2 the model is introduced. In Sec. 3 numerical solutions and discussion are included. Conclusions of the paper are summarized in Sec. 4.

2. The Model

We study the magnetic field evolution in plasma by assuming that time scale is much longer than electron cyclotron period, so that we can ignore the electron inertia in the momentum equation

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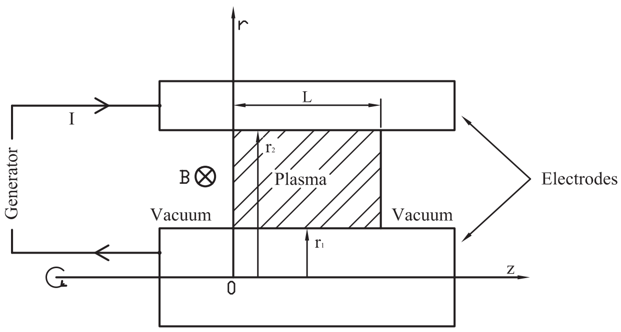

Fig. 1 A hollow cylindrical plasma fills the gap between two concentric cylindrical conductors and closes the circuit for a current which flows in one conductor and return in the other conductor. The cathode is at the inner conductor.

$$\frac{\partial n}{\partial t} + \nabla \cdot n\vec{V}_{\mathrm{i}} = 0, \tag{1}$$

$$M_{\mathrm{i}} n\left(\frac{\partial \vec{V}_{\mathrm{i}}}{\partial t} + \vec{V}_{\mathrm{i}} \cdot \nabla \vec{V}_{\mathrm{i}}\right) = \vec{J} \times \vec{B} - \nabla nT, \tag{2}$$

$$\vec{E} + \vec{V}_{\mathrm{i}} \times \vec{B} - \frac{1}{ne}\vec{J} \times \vec{B} = \eta \vec{J}, \tag{3}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}, \tag{4}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{5}$$

where n, $\vec{V}_{\mathrm{i}}$, and M_{i} are the density, velocity and mass of ion, respectively; $\vec{E}$ and $\vec{B}$ are the electric and magnetic field, respectively. $\vec{J}$ is the current density and T is the plasma temperature. The Eq. (3) is the generalized Ohm's law. We will not use directly it; instead its curl will be used to provide the induction equation (combining (3), (5)). Consequently, the term of the pressure gradient ∇P_{e} (P_{e} is the electron pressure) in Eq. (3) is dropped ($\nabla \times \nabla P_{\mathrm{e}}$). In Eq. (3) the diffusion term (the term in the right hand side) is negligible because we consider the fast magnetic field penetration into the plasma on time scales shorter than an ion cyclotron period ($t < \Omega_{\mathrm{i}}^{-1}$). In such short time scales, the diffusion term can not predict appreciably the magnetic field penetration into the plasma [3,9]. Therefore, the plasma is not tangibly heated and we can assume that the plasma temperature is constant. However, a nonzero resistivity is physically necessary to allow the magnetic field penetration [5,12]. In next section, that we will solve Eqs. (1)-(5), the parameter η (Spitzer resistivity) is 10^{-4} Ω·m [13,14]. In Eq. (4), i.e., Ampere's law, we neglected the displacement current and therefore quasineutrality is kept. We will now describe the linear wave analysis of Eqs. (1)-(5) in cylindrical geometry (Fig. 1): the magnetic field is uniform in the θ direction, $B = B_\theta$, the temperature T is constant, the initial plasma density is uniform. We assume that all linearized dependent variables have the wave-like dependency $\exp i(k_z z - \omega t)$. Also we assume $k_z R \gg 1$, where R is the magnetic field curvature radius. By performing some straightforward algebra calculations, the dispersion equation can be found as follows

$$\omega^2 - \omega k_z V_R - k_z^2(V_{\mathrm{A}}^2 + C_{\mathrm{s}}^2) = 0, \tag{6}$$

where $V_{\mathrm{A}} = B_0/(M_{\mathrm{i}} n\mu_0)^{1/2}$ is the Alfven velocity, $V_R = V_{\mathrm{A}}(c/\omega_{\mathrm{pi}} R)$, and $C_{\mathrm{s}} = (2T/M_{\mathrm{i}})^{1/2}$ is ion-acoustic velocity. In Eq. (6), V_R arises from the magnetic field curvature. The dispersion relation (6) can be written as

$$\omega = \frac{k_z V_{\mathrm{A}}}{2}\left\{\frac{c}{\omega_{\mathrm{pi}} R} \pm \left[\left(\frac{c}{\omega_{\mathrm{pi}} R}\right)^2 + 4(1+\beta)\right]^{1/2}\right\}, \tag{7}$$

where, β is $C_{\mathrm{s}}^2/V_{\mathrm{A}}^2$. If we assume that the pressure is zero, the Alfven mode is restored in Cartesian geometry ($R \to \infty$): $\omega = k_z V_{\mathrm{A}}$. On the other hand, for a cylindrical geometry with $c/\omega_{\mathrm{pi}} R \gg 4(1+\beta)$, a mode with dispersion relation

$$\omega = k_z\left(\frac{c}{\omega_{\mathrm{pi}} R} V_{\mathrm{A}}\right) = k_z V_R, \tag{8}$$

appears. Since we have assumed $c/\omega_{\mathrm{pi}} R \gg 4(1+\beta)$, first, this mode moves much faster than the Alfven mode. Second, the mode is excited when the radius is smaller than skin depth. Third, it can lead to fast penetration of the magnetic field into the plasma.

The linear wave analysis above predicts only fast penetration of the magnetic field into the plasma for radii of smaller than the ion skin depth. In the next section, the nonlinear terms of Eqs. (1)-(5) are not neglected and we solve Eqs. (1)-(5), simultaneously.

3. Numerical Solutions and Discussion

The geometry of the problem has been represented in Fig. 1. A hollow cylindrical plasma fills the gap between two concentric cylindrical conductors and closes the circuit for a current which flows in one conductor and return in the other conductor. According to the legitimate hypothesis of axisymmetry ($\partial/\partial\theta = 0$) the magnetic field is restricted to the azimuthal direction ($B = B_\theta$) since current flows only in the (r, z) plane [4]. To investigate of the quantity effect of the curvature of the magnetic field, we solve Eqs. (1)-(5), simultaneously. As mentioned earlier, the initial density is uniform. The cylindrical plasma is located at $r_1 < r < r_2$ and $0 < z < L$. We now transform Eqs. (1)-(5) to the dimensionless form. The free parameters of our numerical method are as follows: the maximum value of the magnetic field in the calculation region, B_0 (B_0 is negative so that the radial current is inward), the initial ion density, n_0, the radius of the inner cylinder, r_1, the Alfven's velocity, V_{A}, and the time of $t_0 = L/V_{\mathrm{A}}$. The dimensionless plasma parameters are, therefore $V_{\mathrm{ir}} = V_{\mathrm{ir}}/V_{\mathrm{A}}$, $V_{\mathrm{iz}} = V_{\mathrm{iz}}/V_{\mathrm{A}}$, $r = r/r_1$, $z = z/L$, $B = B/B_0$, $t = t/t_0$, and $n = n/n_0$. Finally, the dimensionless equations are as follows:

$$\frac{\partial n}{\partial t} = -\frac{\partial}{r\partial r}(nrV_{\mathrm{ir}}) - \frac{\partial}{\partial z}(nV_{\mathrm{iz}}), \tag{9}$$

$$\frac{\partial V_{\rm ir}}{\partial t} = -V_{\rm ir}\frac{\partial V_{\rm ir}}{\partial r} - V_{\rm iz}\frac{\partial V_{\rm ir}}{\partial z} - B\frac{\partial}{r\partial r}(rB) - \frac{2\mu_0 n_0 T}{B_0^2}\frac{\partial n}{\partial r}, \tag{10}$$

$$\frac{\partial V_{\rm iz}}{\partial t} = -V_{\rm ir}\frac{\partial V_{\rm iz}}{\partial r} - V_{\rm iz}\frac{\partial V_{\rm iz}}{\partial z} - B\frac{\partial B}{\partial z} - \frac{2\mu_0 n_0 T}{B_0^2}\frac{\partial n}{\partial z}, \tag{11}$$

$$\frac{\partial B}{\partial t} = -\left[\frac{\partial}{\partial r}(BV_{\rm ir}) + \frac{\partial}{\partial z}(BV_{\rm iz})\right] + \frac{B_0}{\mu_0 n e r_1 V_A}\left[\frac{\partial}{\partial r}\left(B\frac{\partial B}{\partial z}\right) - \frac{\partial}{\partial z}\left((\frac{B}{r}\frac{\partial (Br)}{\partial r}\right)\right] + \frac{\eta}{\mu_0 V_{\rm A} r}\left[\frac{\partial}{\partial r}(r\frac{\partial B}{\partial r}) + \frac{\partial}{\partial z}(r\frac{\partial B}{\partial z})\right]. \tag{12}$$

The boundary and initial conditions for Eqs. (9)-(12) for the magnetic field, the ion velocity components, and ion density are:

$$B(r, z = 0, t) = B_0\frac{r_1}{r}, \quad n(r, z, t = 0) = n_0 \tag{13}$$

$$V_{\rm ir}(r = r_1, z) = V_{\rm ir}(r = r_2, z) = 0,$$
$$V_{\rm iz}(r, z = 0) = V_{\rm iz}(r, z = L) = 0 \tag{14}$$

The electrodes are located in $r = r_1, r_2$. The ions can flow to the electrodes but they cannot penetrate into the electrodes (the electrodes are rigid). So the boundaries $r = r_1, r_2$ are nonpenetrative and the plasma is surrounded by vacuum at $z = 0, L$. Moreover the magnetic field, $B = B_0 r_1/r$, in $z = 0$. The electric field parallel to the conductors is zero

$$E_z|_{r=r_{1,2}} = \frac{1}{ne}\vec{J}\times\vec{B}\Big|_z + \eta J_z - \vec{V}_{\rm i}\times\vec{B}\Big|_z = 0 \tag{15}$$

where the Eq. (3) is used.

We solve Eqs. (9)-(12) by numerical scheme described in [15]. First, we use a uniform rectangular grid in the (r, z) plane with $\Delta r = \Delta z = 0.01$. Second, we substitute a Crank-Nicolson implicit formulation for the space and time derivatives. Third, we obtain a system of nonlinear equations linearized by a Taylor series expansion of the nonlinear terms. Finally the set of linear equations is solved.

Figure 2(a)-(b) shows the distribution of the magnetic field at instants, $t = 0.01t_0$, and $t = 0.1t_0$, respectively. One can see that penetration depth is larger at smaller radii i.e., the penetration occurs in the plasma near of the inner cylinder (cathode). As mentioned earlier (section 2), when $c/\omega_{\rm pi}R \gg 1$, the magnetic field penetrates rapidly into the plasma. The dimensionless parameter in the Eq. (12), i.e., the coefficient of the third term is as follows

$$\frac{B_0}{\mu_0 n e r_1 V_{\rm A}} = \frac{V_{\rm r1}}{V_{\rm A}} = \frac{c}{\omega_{\rm pi}R} \tag{16}$$

where $V_{r1} = B_0/\mu_0 n e r_1$. Consequently, the condition $c/\omega_{\rm pi}R \gg 1$ is the same as $V_R/V_{\rm A} \gg 1$. As radius R decreases, the inequality $c/\omega_{\rm pi}R \gg 1$, or $V_R/V_{\rm A} \gg 1$ is more easily satisfied.

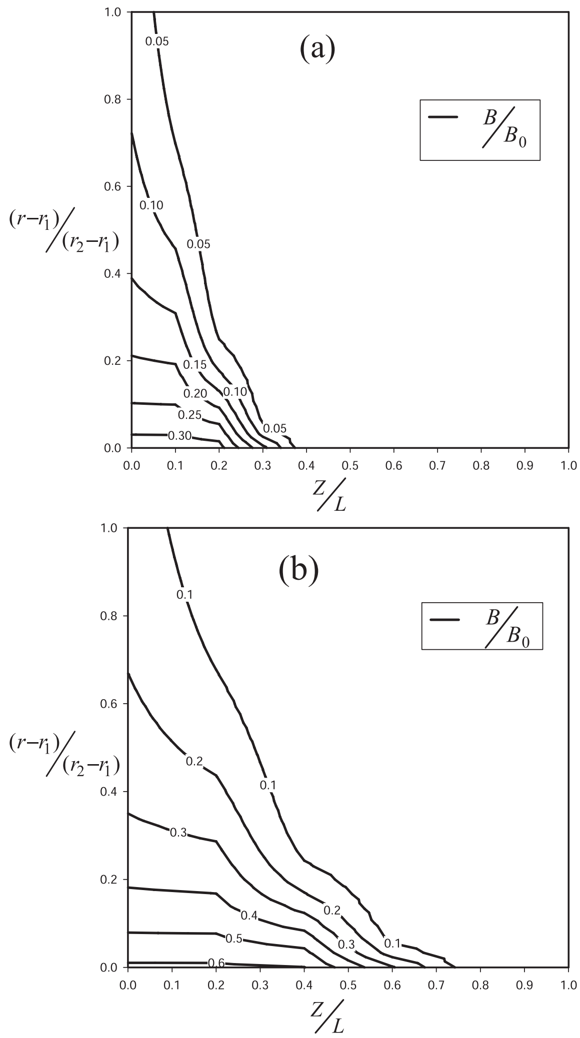

Fig. 2 Mapping of the constant magnetic field contours for $c/\omega_{\rm pi}r_1 = 2$ at time $t = 0.01t_0$, (b) the same as (a), but in $t = 0.1t_0$. The magnetic field is normalized to the maximum value of the magnetic field B_0. The magnetic field propagates rapidly along the cathode due to coaxial geometry. Here, $r_1$is the cathode radius, $r_2$is the anode radius and $(r - r_1)/(r_2 - r_1)$ shows distance from cathode.

The Eq. (4), Ampere's law, has the following form:

$$\vec{J} = -e_r\frac{\partial (Br)}{\mu_0 r\partial z} + e_z\frac{\partial (Br)}{\mu_0 r\partial r}, \tag{17}$$

This equation shows that the current lines coincide with the contour lines Br = const.

In addition, in a one-dimensional problem, the Hall term, in Eq. (12), can be written as [4]

$$\nabla\times\left(\frac{\vec{J}\times\vec{B}}{ne}\right) = \frac{\vec{B}}{\mu_0 n e r}\frac{\partial B}{\partial z}, \tag{18}$$

Thus, it is clear that, when radius r is large the magnetic field penetration is diffusive and the curvature effect is negligible.

The distribution of the ion density is shown in Fig. 3. It can be seen that the ion density is reduced near the

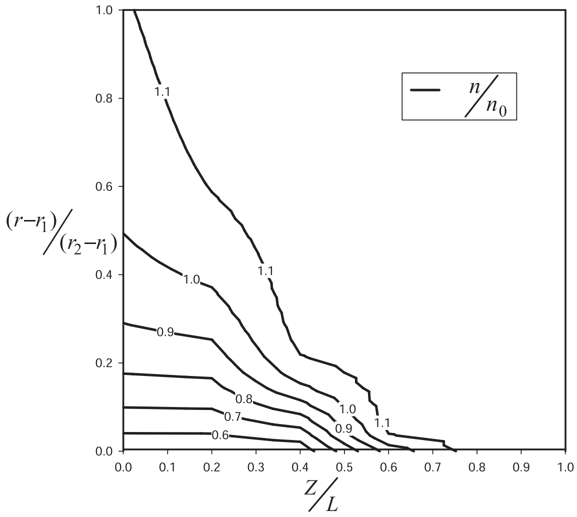

Fig. 3 Mapping of the constant ion density contours at time $t = 0.1t_0$. The plasma density is normalized to initial density. Here, r_1 is the cathode radius, r_2 is the anode radius and $(r - r_1)/(r_2 - r_1)$shows distance from cathode.

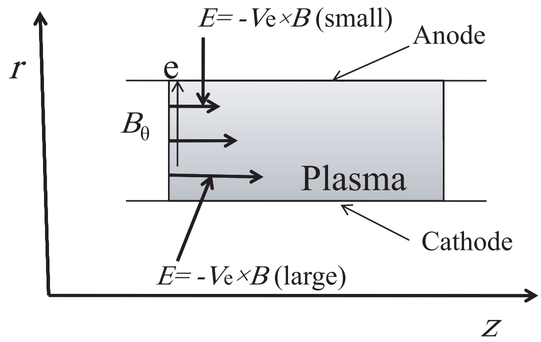

Fig. 4 Penetration of the magnetic field into the plasma, when the magnetic field arrives at the plasma boundary. The electric field in the region near the cathode is larger than the anode because the magnetic field at the plasma boundary is proportional to r^{-1}.

cathode, i.e., a gap appears between the cathode and the plasma. This is reason that when the magnetic field penetrates into the plasma, the force $J_z \times B$ pushes radially the plasma.

The time of the magnetic field propagation through the plasma is smaller than $t_0 = L/V_A$ (Fig. 2). Thus, the propagation speed of the magnetic field or current velocity J/ne is much larger than the characteristic hydrodynamic plasma velocity V_A. As a result, during the magnetic field penetration, distributions of the magnetic field are governed by the dynamics of electrons. We now discuss the mechanism that governs the evolution of the magnetic field, based on the electrons motion. By combining Eqs. (3),(5), and continuity equation of electron, and considering $V_i - V_e = J/ne$ and performing some algebra calculations in cylindrical geometry, we find

$$\frac{\partial}{\partial t}(\frac{B_\theta}{n}) = -\frac{V_{er}}{n}\frac{\partial B_\theta}{\partial r}. \tag{19}$$

This relation shows that if the density is uniform and the electrons move radially from a small radius to a large radius and the magnetic field grows in time, an increase in radius results in a decrease in the magnetic field. In other words, if the plasma fills the gap between two coaxial cylindrical conductors when the cathode is at the inner conductor (electrons move towards larger radius), the magnetic field penetrates into the plasma. When, however, the cathode is at the outer conductor (electrons move towards smaller radius), the magnetic field does not penetrate into the plasma [4].

We next suppose that the electrons can move in z direction. We then obtain

$$\frac{\partial}{\partial t}\left(\frac{B_\theta}{n}\right) = -\frac{V_{ez}}{n}\frac{\partial B_\theta}{\partial z}. \tag{20}$$

Penetration of the magnetic field into the plasma along the $z$axis is due to electrons motion along the z direction. When$V_{ez} \gg V_A$, we obtain fast penetration of the magnetic field into the plasma along the z direction.

At the early times, when the magnetic field arrives at the plasma boundary that is separated from the vacuum by a non-neutral sheath of width c/ω_{pe}, electrons move radially and the space-dependent electric field induces the magnetic field as shown in Fig. 4. By considering $\omega \ll \omega_{ce}$, the electrons motion perpendicular to the magnetic field can be described by drift theory. Electrons therefore obtain the axial velocity V_z due to the magnetic field curvature

$$V_z = \frac{mV_r^2}{erB}. \tag{21}$$

Consequently, a decrease in radius r results in an increase in the drift velocity V_z. The magnetic field penetrates into the plasma as long as $r < c/\omega_{pi}$. This condition can be satisfied, when the electron velocity is larger than a specific value.

4. Conclusion

In this paper we studied the mechanism of the magnetic field penetration into the plasma, in cylindrical geometry. The resistivity was negligible, so that the magnetic field diffusion is much slower than the magnetic field penetration. Numerical study showed that the penetration occurs in the plasma near of cathode and during the penetration of the magnetic field a gap appears between the cathode and the plasma. The presented mechanism was based on the electrons motion. At the early times, when the magnetic field arrives at the plasma boundary, electrons move radially and the space-dependent electric field induces the magnetic field. Electrons then obtain the axial velocity V_z due to the magnetic field curvature that can lead to fast penetration of the magnetic field into the plasma.

[1] D.D. Rytuov, M.S. Derzon and M.K. Matzen, Reviews of Modern Physics **72**, 167 (2000).
[2] M. Tuszewski, Phys. Rev. Lett. **77**, 1286 (1996).
[3] F.H. Shu, *The Physics of Astrophysics: Gas Dynamics* (University Science Books, Mill Valley, CA, 1992).
[4] A. Fruchtman, Phys. Fluids B **3**, 1908 (1991).
[5] A. Fruchtman, Phys. Fluids B **4**, 3446 (1992).
[6] A. Fruchtman and Y. Maron, Phys. Fluids B **3**, 1546 (1991).
[7] A. Fruchtman, Phys. Fluids B **4**, 855 (1992).
[8] N. Chakrabarti, A. Fruchtman, R. Arad and Y. Maron, Phys. Lett. A **29**, 92 (2002).
[9] V. YU. Zaburdaev, Plasma Physics Reports **26**, 462 (2000).
[10] A. Fruchtman and L.I. Rudakov, Phys. Rev. Lett. **69**, 2070 (1992).
[11] H.R. Strauss, R. Doron, R. Arad, B. Rubinstien and Y. Maron, Phys. Plasma **14**, 053504 (2007).
[12] R. Arad, K. Tsigutkin and Y. Maron, Phys. Plasma **10**, 112 (2003).
[13] L. Spitzer, *Physics of Fully Ionized Gases* (Interscience, New York, 1962).
[14] F. Trintchouk, M. Yamada, H. Ji, R.M. Kulsrud and T.A. Carter, Phys. Plasma **10**, 319 (2003).
[15] C.A.J. Fletcher, *Computational Techniques for Fluid Dynamics* (Springer-Verlag, 1991).