Numerical Simulation of Contactless Methods for Measuring j_C Distribution of High Temperature Superconducting Thin Film

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The inductive method and the permanent magnet method for measuring the critical current density in a high-temperature superconducting (HTS) thin film have been investigated numerically. For this purpose, a numerical code has been developed for analyzing the time evolution of the shielding current density in a HTS sample. The results of computations show that, in the inductive method, the critical current density near the film edge cannot be accurately evaluated. On the other hand, it is found that, in the permanent magnet method, even if the magnet is placed near the film edge, the maximum repulsive force is roughly proportional to the critical current density. This means that the critical current density near the film edge can be estimated from the resulting proportionality constants.

Keywords: current distribution, current measurement, high-temperature superconductor, simulation, thin film

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1. Introduction

High-temperature superconductors (HTSs) can be used in standard applications such as power-transmission cables, flywheel systems and fusion reactor systems. As is well known, HTS materials have various characteristics to keep the superconducting state. In particular, since the critical current density j_C is one of most important parameters characterizing a superconducting property, it is necessary to accurately measure j_C.

The standard four probe method is generally used to measure the critical current density j_C. In the method, a HTS sample is coated by gold or silver, and subsequently, it should be heat-treated. This process may lead to not only the destruction of the HTS but also the degradation of the HTS characteristics. Therefore, a contactless method has been so far desired for measuring j_C.

Claassen *et al.* proposed a contactless method for measuring the critical current density j_C [1]. By applying an ac current $I(t) = I_0 \sin 2\pi f t$ to a small coil placed just above a HTS thin film, they monitored a harmonic voltage induced in the coil. From the experimental results, they found that, only when a coil current I_0 exceeds a threshold current I_T, the third-harmonic voltage V_3 develops suddenly. Moreover, it was also revealed that j_C can be evaluated from I_T. This method is called the inductive method and is widely used for the determination of j_C-distributions. On the other hand, Mawatari *et al.* elucidated the inductive method on the basis of the critical state model [2]. As a result, they derived a theoretical formula for the relation between j_C and I_T.

In contrast, Ohshima *et al.* proposed a novel contactless method [3, 4]. While moving a permanent magnet placed above a HTS film, they measure an electromagnetic force acting on the film. As a result, they found that the maximum repulsive force F_M is almost proportional to the critical current density j_C. This means that j_C can be estimated by measuring F_M. This method is called the permanent magnet method and is recently used for the measurement of the j_C-distribution [5].

In order to simulate two types of contactless methods, a numerical code has been developed by analyzing the time evolution of a shielding current density in a HTS thin film [6]. By using the code, we have succeeded in reproducing the contactless methods. However, since we adopt the cylindrical coordinate in the code [6], the center of the coil and magnet is located at just above the origin. Therefore, it is impossible to evaluate a spatial distribution of j_C in a HTS sample by use of the code.

The purpose of the present study is to develop a numerical code for analyzing the time evolution of the shielding current density in a HTS thin film for the case with the non-axisymmetric model. In addition, we simulate the inductive method and the permanent magnet method by using the code, and investigate the influence of the coil and the magnet position on the determination of the j_C-distribution.

2. Governing Equations

In measuring a critical current density j_C by means of the contactless methods, a time-dependent magnetic field is applied to a HTS sample. Throughout the present

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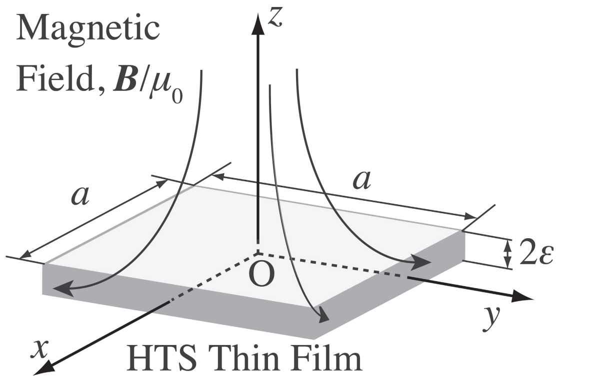

Fig. 1 A schematic view of contactless methods for measuring the critical current density j_C.

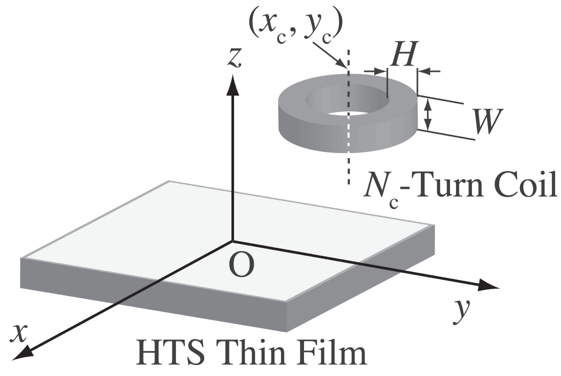

Fig. 2 A schematic view of an inductive method.

study, we assume that a magnetic field $\boldsymbol{B}/\mu_0$ is applied to a square-shaped HTS thin film of the length a and the thickness 2ε (see Fig. 1). Furthermore, we adopt the Cartesian coordinate system $\langle \mathrm{O} : \boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z \rangle$, where z-axis is the thickness direction. Note that, in the inductive method, the origin O is chosen at the center of a HTS upper surface. In the permanent magnet method, O is taken at the centroid of a HTS.

As usual, we assume that the thin-layer approximation: since the thickness of the HTS is sufficiency thin, a shielding current density can hardly flow in the thickness direction. Hereafter, a HTS film cross-section passing through z = const. and its boundary are denoted by Ω and $\partial\Omega$, respectively.

Under the above assumptions, a shielding current density in a HTS is written as

$$\boldsymbol{j} = \frac{1}{\varepsilon}\nabla S \times \boldsymbol{e}_z, \tag{1}$$

and the behavior of the scalar function $S(\boldsymbol{x}, t)$ is governed by the following integro-differential equations [7]:

$$\mu_0 \frac{\partial}{\partial t}\left[\int_\Omega d^2\boldsymbol{x}' Q\left(|\boldsymbol{x} - \boldsymbol{x}'|\right) S(\boldsymbol{x}', t) + \frac{1}{\varepsilon} S\right] + \frac{\partial}{\partial t}\langle \boldsymbol{B}\cdot\boldsymbol{e}_z\rangle + (\nabla\times\boldsymbol{E})\cdot\boldsymbol{e}_z = 0. \tag{2}$$

Here, $\boldsymbol{x}$ is defined by $\boldsymbol{x} \equiv x\boldsymbol{e}_x + y\boldsymbol{e}_y$, and $\langle\ \rangle$ is an average operator over the thickness of the HTS. The explicit form of $Q(\gamma)$ [7] is

$$Q(\gamma) = -\frac{1}{4\pi\varepsilon^2}\left(\frac{1}{\gamma} - \frac{1}{\sqrt{\gamma^2 + 4\varepsilon^2}}\right). \tag{3}$$

As is well known, the shielding current density $\boldsymbol{j}$ is closely related to the electric field $\boldsymbol{E}$. The relation is expressed by the J-E constitute equation:

$$\boldsymbol{E} = E(|\boldsymbol{j}|)\boldsymbol{j}/|\boldsymbol{j}|. \tag{4}$$

As a function $E(j)$, we employ the power law [8]:

$$E(j) = E_C (j/j_C)^{16}, \tag{5}$$

where E_C is a critical electric field. In the following, we assume that a HTS film has a uniform j_C-distribution.

For applying the initial and boundary conditions to (2), we assume $S = 0$ at $t = 0$ and $S = 0$ on $\partial\Omega$. By solving the initial-boundary problem of (2), we can obtain the time evolution of a shielding current density. A numerical code has been developed for solving the initial-boundary problem of (2). In order to simulate two types of contactless methods, the code can be executed by specifying an assumed critical current density j_C and a magnetic field $\boldsymbol{B}$ generated by a coil or a permanent magnet.

3. Simulation of Inductive Method

By performing the theoretical calculation based on the critical state model, Mawatari *et al*. have derived the following formula [2]

$$j_C^N = F(r_{max}) I_T/\varepsilon, \tag{6}$$

where j_C^N is an estimated value of the critical current density j_C. $F(r_{max})$ is the maximum of a primary coil-factor function $F(r)$ [2] which can be determined from the configuration of the coil and the HTS. Furthermore, I_T is a lower limit of a coil current I_0 above which the third-harmonic voltage V_3 begins to develop. For estimating I_T, we use the conventional voltage criterion: $V_3 = 0.1\,\mathrm{mV} \Leftrightarrow I_0 = I_T$ [2] in the present study.

In the inductive method, the time-dependence magnetic filed $\boldsymbol{B}/\mu_0$ is generated by applying an ac current $I(t) = I_0 \sin 2\pi f t$ to an N_c-turn coil placed just above a HTS thin film. For determining the coil position, the xy coordinates of the center of coil is given by $(x, y) = (x_c, y_c)$ (see Fig. 2). Furthermore, the cross-section of the coil is expressed as $D = \{(z, r) : |z - Z_c| \le H/2, |r - R_c| \le W/2\}$ with the cylindrical coordinate (r, θ, z). Here, H and W are height and width of the cross-section, respectively, and its center is $(z,\ r) = (Z_c,\ R_c)$. Throughout the present section, the parameters are fixed as follows: $a = 20\,\mathrm{mm}$, $2\varepsilon = 600\,\mathrm{nm}$, $x_c = 0\,\mathrm{mm}$, $W = 1.5\,\mathrm{mm}$, $H = 1\,\mathrm{mm}$, $R_c = 1.75\,\mathrm{mm}$, $Z_c = 0.7\,\mathrm{mm}$, $N_c = 400$, $f = 1\,\mathrm{kHz}$, $E_C = 1\,\mathrm{mV/m}$. For the above configuration, we obtain $F(r_{max}) =$

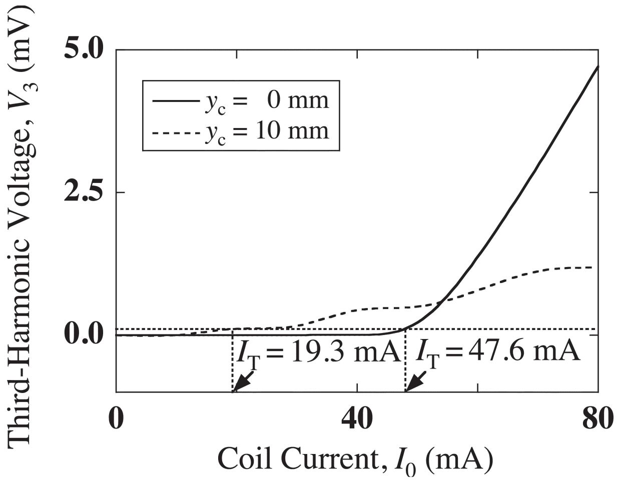

Fig. 3 Dependence of the third-harmonic voltage V_3 on the coil current I_0 for the case with $j_C = 1\,\mathrm{MA/cm^2}$.

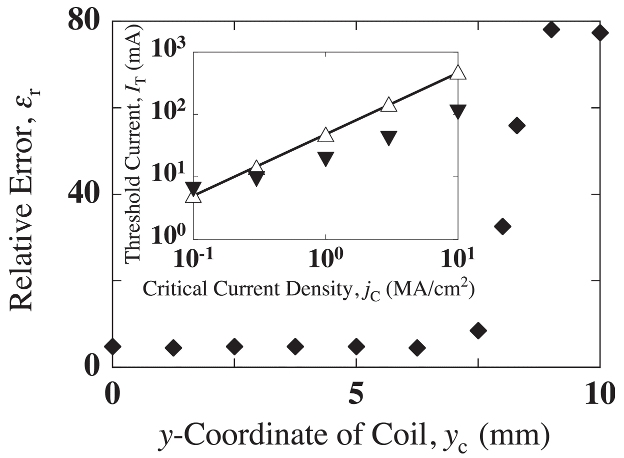

Fig. 4 Dependence of the relative error ε_r on the y-coordinate y_c of the coil. The inset shows that dependence of the threshold current I_T on the critical current density j_C. Here, △: $y_c = 0$ mm, ▼: $y_c = 10$ mm.

$6.23 \times 10^4\,\mathrm{m^{-1}}$.

Under the above conditions, let us investigate the influence of the coil position on the determination of the j_C-distribution. To this end, the y-coordinate y_c of the center of the coil is changed from 0 mm to 10 mm.

First, for estimating the threshold current I_T, the third-harmonic voltage V_3 is calculated as functions of the coil current I_0 and is plotted in Fig. 3. We see from this figure that, for $y_c = 0$ mm, V_3 begins to develop from a certain value of I_0, and after that, V_3 monotonously increases with I_0. By applying the voltage criterion to the I_0-V_3 curve for $y_c = 0$ mm, we get $I_T = 47.6$ mA. By substituting the value of I_T to (6), we can obtain $j_C^N = 0.99\ \mathrm{MA/cm^2}$. This value fairly agrees with the assumed critical current density $j_C = 1\ \mathrm{MA/cm^2}$. On the other hand, it is found that, for $y_c = 10$ mm, the behavior of V_3 greatly differs in $y_c = 0$ mm. According to the voltage criterion, the value of I_T is 19.3 mA for $y_c = 10$ mm.

Next, let us investigate the relation between the threshold current I_T and the critical current density j_C. To this end, I_T is calculated as functions of j_C and is depicted in the inset of Fig. 4. We see from this figure that, for $y_c = 0$ mm, I_T is roughly proportional to j_C. This tendency quantitatively agrees with Mawatari's theoretical formula (6). On the other hand, it is found that, for $y_c = 10$ mm, the proportional relation between I_T and j_C no longer hold.

Finally, let us numerically investigate a limit of the measurement of the critical current density j_C. In order to quantitatively evaluate the accuracy of the threshold current I_T, we define a relative error

$$\varepsilon_r \equiv \|I_T^A - I_T^N\|_\infty / \|I_T^A\|_\infty. \quad (7)$$

Here, I_T^N, a estimated value of I_T, is obtained from the voltage criterion, and I_T^A, a theoretical value, is expressed as $I_T^A = j_C \varepsilon / F(r_{max})$. Furthermore, $\|f\|_\infty$ is denoted by $\|f\|_\infty = \mathop{\mathrm{Max}}_{j_C \in J} |f(j_C)|$. Here, J is defined by $J \equiv \{0.1\,\mathrm{MA/cm^2} \le j_C \le 10\,\mathrm{MA/cm^2}\}$. The relative error ε_r is calculated as a function of y_c and is plotted in Fig. 4. We see from this figure that, for $y_c > 7.5$ mm, the accuracy of the inductive method is drastically degraded with y_c. An important point is that, for $y_c = 7.5$ mm, the sum of y_c and the outer radius $R_c + W/2$ is equal to $a/2$. From this result, we conclude that, until the outer radius of the coil is equal to the film edge, the critical current density can be accurately evaluated from Mawatari's theoretical formula.

4. Simulation of Permanent Magnet Method

In the permanent magnet method, the time-dependence magnetic field $\boldsymbol{B}/\mu_0$ is generated by a cylindrical permanent magnet placed above a HTS thin film. Here, the radius and the height of the magnet are r_m and h_m, respectively, and the xy coordinates of the center of the magnet is denoted by $(x, y) = (x_m, y_m)$. A distance L between a magnet bottom and a film surface is controlled as follows:

(i) From $L = L_{max}$ to $L = L_{min}$, the magnet is moved toward the film at the constant speed: $v = (L_{max} - L_{min})/\tau_0$. Here, τ_0 is a constant.

(ii) From $L = L_{min}$ to $L = L_{max}$, the magnet is moved away from the film at the same speed v.

Furthermore, for determining the strength of the magnet, we employ a magnetic flux density B_F at $(x, y, z) = (0, 0, \varepsilon)$ for the case with $L = L_{min}$. Throughout the present section, the parameters are fixed as follows: $a = 40$ mm, $2\varepsilon = 200$ nm, $x_m = 0$ mm, $r_m = 2.5$ mm, $h_m = 3$ mm, $\tau_0 = 39$ s, $L_{max} = 20$ mm, $L_{min} = 0.5$ mm, $E_C = 0.1$ mV/m, $B_F = 0.3$ T.

Under the above conditions, we investigate the influence of the magnet position on the determination of the j_C-distribution. For this purpose, the y-coordinate y_m of

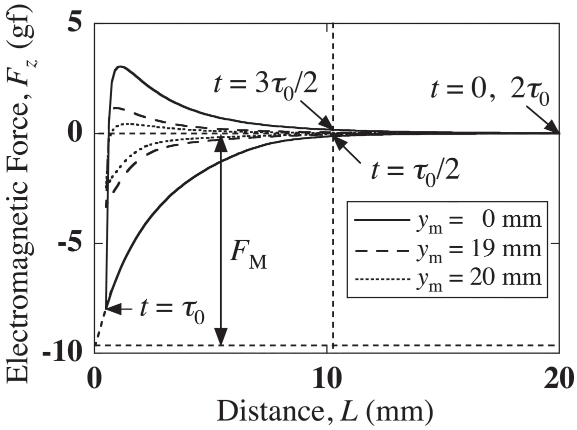

Fig. 5 Dependence of the electromagnetic force F_z on the distance L for the case with $j_C = 3.85\,\mathrm{MA/cm^2}$.

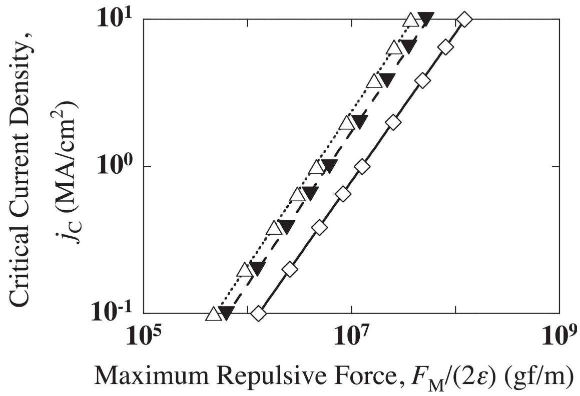

Fig. 6 Dependence of the critical current density j_C on the maximum repulsive force F_M. Here, ◇: $y_c = 0\,\mathrm{mm}$, ▼: $y_m = 19\,\mathrm{mm}$, △: $y_m = 20\,\mathrm{mm}$.

the center of the magnet is changed from 0 mm to 20 mm.

Let us first investigate an electromagnetic force F_z acting on the film. For the various values of y_m, the electromagnetic force is calculated as functions of the distance L and are depicted in Fig. 5. We see from this figure that a repulsive force gradually increases as the magnet moves toward the film ($0 \le t \le \tau_0$). On the other hand, an attractive force decreases to zero when the magnet moves away from the film ($\tau_0 < t \le 2\tau_0$). These tendencies do not change regardless of the magnet position. The electromagnetic force for $L = 0$ can be easily determined by extrapolating the L-F_z curve (see Fig. 5). In the following, this value is called a maximum repulsive force F_M.

Next, we investigate the relation between the maximum repulsive force F_M and the critical current density j_C. Note that the experimental results were obtained for the case with only $y_m = 0\,\mathrm{mm}$ [3, 4]. For various values of y_m, F_M is evaluated as functions of j_C and is plotted in Fig. 6. This figure indicates that, for $y_m = 0\,\mathrm{mm}$, F_M increases in proportion to j_C. This result is in qualitatively agreement with Ohshima's experimental one. On the other hand, the results of computations show that, even when the magnet is located at y_m = 19 mm and 20 mm, F_M is almost proportional to j_C. In other words, the relation can be expressed as $j_C = K(x_m, y_m)(F_M/2\varepsilon)$, where K is a proportionality constant.

From this result, we conclude that, even if the magnet is placed near the film edge, the critical current density j_C can be determined. Therefore, the j_C-distribution in the HTS film can be estimated from the proportionality constants determined by the resulting F_M-j_C lines.

5. Conclusion

We have developed a numerical code for analyzing the time evolution of the shielding current density in a HTS sample for the case with the non-axisymmetric model. By using the code, simulating the inductive method and the permanent magnet method, we investigate the influence of the coil and the magnet position on the determination of the distribution of the critical current density j_C. Conclusions obtained in the present study are summarized as follows:

(1) In the inductive method, the critical current density j_C near the film edge cannot be accurately measured. In other words, until the outer radius of the coil is equal to the film edge, j_C can be evaluated from Mawatari's theoretical formula.

(2) In the permanent magnet method, even if the magnet is located near the film edge, the maximum repulsive force F_M is almost proportional to j_C. From this result, j_C near the film edge can be estimated from the proportionality constant determined with the resulting $F_M - j_C$ lines.

Therefore, we conclude that the measurement of j_C near the film edge is suitable for the permanent magnet method.

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