# Reconstruction of Vacuum Magnetic Flux in QUEST

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It is important to determine the best method for reconstructing the magnetic flux when eddy currents are significantly induced during magnetic measurement in spherical tokamaks (STs). Four methods for this reconstruction are investigated, and the calculated magnetic fluxes are compared to those measured in the cavity of a vacuum vessel. The results show that the best method is the one that uses currents from virtual coils for reconstruction. In this method, the placement of the virtual coils is optimized with numerical simulations using the Akaike information criterion (AIC), which indicates the goodness of fit of models used to fit measured data. The virtual coils are set on a line 15 cm outside the vacuum vessel.



Keywords: spherical tokamak, magnetic flux reconstruction, eddy current, magnetic measurement, flux loop

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## 1. Introduction

A spherical tokamak (ST) has a small aspect ratio ($A = R_0/a$, where $R_0$ is the major radius and $a$ is the minor radius of tokamak plasma), and they are expected to be capable of high beta, good confinement, and steady-state operation in a compact configuration with a modest magnetic field [1]. It is difficult to install electrical braking to avoid toroidal eddy currents in the vacuum vessel because of a lack of central space in STs. As magnetic measurements are significantly affected by eddy currents, developing a method for reconstructing the magnetic flux under the condition that eddy currents are significantly induced is crucial.

As magnetic measuring instruments (for example, flux loops or pickup coils) are attached only to the vessel wall in most cases, numerical analysis is needed to estimate the magnetic flux distribution in the cavity of the vacuum vessel. Several methods are used to reconstruct the distribution; each uses a different technique to estimate the eddy currents, and the accuracy of reconstruction depends on the method. In this study, external coils of the device are discharged without producing plasma, and the distribution of vacuum magnetic flux in the cavity of the vacuum vessel is directly measured by flux loops fixed temporarily in the midplane of the device. These measured magnetic flux data are compared with the calculated reconstruction results for each method to determine the most suitable reconstruction method using data from the vessel wall.

In section 2, the experimental device and magnetic measurement system are introduced. The four methods for reconstructing the magnetic flux are discussed in section 3. In section 4, the results of measurement are shown, and the most suitable method is determined. We present a summary and conclusion in section 5.

## 2. Experimental Apparatus

Figure 1 shows a schematic view of Q-shu University Experiment with Steady-State Spherical Tokamak (QUEST), Kyushu University's ST device [2]. QUEST has 11 poloidal field coils and a pair of cancel coils (CCs). The PF4 coil, which is the central solenoid (CS) coil in a tokamak, has three parts, PF4-1, PF4-2, and PF4-3, and PF4-2 is in turn composed of two parts, a and b. The CCs are usually connected to the PF4 coils in series to make a null point in the cavity of the vacuum vessel.

In QUEST, 61 flux loops set on the inside surface of the vacuum chamber's wall are available as instruments for magnetic measurement. Because these loops are used in a vacuum, special cables are used. The core of the cables is made of Cu, which is covered by an insulator (MgO). The outermost shell is made of 0.25-mm-thick SUS316. Flux loops have the advantage of relatively easy and economical setup and are used for magnetic flux measurements in many devices [3–5].

The tools for measuring the magnetic flux distribu-

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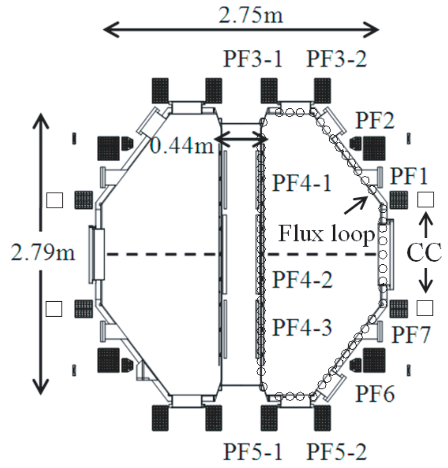


Fig. 1 Cross-sectional side view of QUEST including PF coils. Dashed line inside the vacuum vessel shows the position of the board with measurement tools shown in Fig. 2.

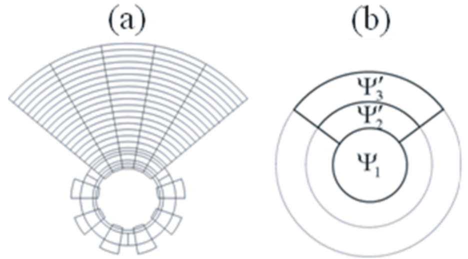


Fig. 2 Top schematic view of tools for measuring the magnetic flux distribution in the midplane of the vacuum vessel. Twenty-one flux loops are set on a board in meshes (a). $\Psi_i$ is the flux value defined by the ($R$, $Z$) value of the outside of each loop, and $\Psi'_i$ is the flux measured by the $i$-th loop (b).

tion in the midplane of the vacuum vessel are shown in Fig. 2 (a). Twenty-one flux loops are set on a board in meshes. The innermost and outermost loops are numbered 1 and 21, respectively. The innermost part of the first flux loop is located at $R = 0.26$ m, where $R$ is the major radius. The one-turn voltage can be measured, and the flux value is obtained by numerically integrating the signal. The other loops are set every 5 cm along the major radius, and the loops cover a 100° region in the toroidal direction, as shown in Fig. 2 (b). Thus, one can measure the flux inside the solid line in Fig. 2 (b). The flux defined by the position of loop number $n$, $\Psi_n$, is the measured flux value in the midplane, and is written as

$$\Psi_n = \Psi_1 + \frac{360}{100}\left(\sum_{i=2}^{n} \Psi'_i\right). \tag{1}$$

$\Psi'_i$ is the flux measured by the$i$-th loop. In this paper, flux loops set on the midplane are called loops on the midplane (LOMs), and those set on the chamber wall are called loops on the wall (LOWs).

# 3. Method of Reconstructing Magnetic Flux

It is difficult to measure the magnetic flux distribution in the cavity of a vacuum chamber with plasma directly. Thus, it is necessary to find an accurate method of estimating the distribution that includes the effect of eddy currents using the signals obtained by the LOWs and the coil currents. In this section, four such methods are introduced. The magnetic fluxes calculated by these methods are compared with those measured by the LOMs, and the most suitable method is determined in section 4.

## 3.1 Solving the electrical circuit equation (method A)

The flux is reconstructed using the measured coil currents and the eddy currents estimated by numerically solving the electrical circuit equation.

The vacuum vessel is modeled as a large collection of conducting toroidal rings, each of which is called a segment. The one-turn voltage, which is the source of the eddy currents, is generated by the time evolution of the currents in external coils and segments. This condition can be expressed by the circuit equation,

$$\sum_{j=1}^{n} M^{\mathrm{e}}_{ij} \frac{\mathrm{d}I^{\mathrm{e}}_j}{\mathrm{d}t} + \sum_{k=1}^{m} M^{\mathrm{c}}_{ik} \frac{\mathrm{d}I^{\mathrm{c}}_k}{\mathrm{d}t} + RI^{\mathrm{e}}_i = 0, \tag{2}$$

where $n$ is the total number of segments, $m$ is the total number of coils whose currents are measured, $M^{\mathrm{e}}_{ij}$ is the mutual inductance between the segments, $M^{\mathrm{c}}_{ik}$ is the mutual inductance between external coils and segments, $I_{\mathrm{c}}$ is the known current in the coils, and $I_{\mathrm{e}}$ is the eddy current. Equation (2) is modified with vector notation as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{I}^{\mathbf{e}} = (\boldsymbol{M}^{\mathbf{e}})^{-1}\left(-\boldsymbol{M}^{\mathbf{c}}\frac{\mathrm{d}\boldsymbol{I}^{\mathbf{c}}}{\mathrm{d}t} - R\boldsymbol{I}^{\mathbf{e}}\right). \tag{3}$$

This simultaneous differential equation can be solved numerically, and we can obtain the estimated value of the eddy current. We call this method A in this paper.

## 3.2 Estimation of eddy current with loop voltage (method B)

In this method, the flux is reconstructed using the measured coil currents and the eddy currents estimated by a technique introduced in Ref. [6]. The vacuum vessel is separated into segments, each of which corresponds to one LOW. The eddy current induced in each segment, $I_{\mathrm{eddy}}$, is estimated from the loop voltage measured by the LOW, which corresponds to the segment,

$$V_{\mathrm{meas}} = I_{\mathrm{eddy}} R_{\mathrm{seg}}, \tag{4}$$

where $V_{\mathrm{meas}}$ is the loop voltage measured by the LOW, and $R_{\mathrm{seg}}$ is the resistivity of the segment. We call this method B in this paper.

### 3.3 Reconstruction using the virtual coil current at the position of the vessel wall (method C)

In this method, the currents of the segments are chosen to satisfy the signals measured by the LOW. These segments are regarded as virtual coils at the position of the vessel wall whose currents are chosen by fitting. The flux is reconstructed using these estimated currents and the coil currents. In this analysis, the total number of virtual coils is the same as the number of LOWs. Thus, the relationship between the measured values of the fluxes and the currents in the virtual coils is written as

$$\Psi_i^{\mathrm{e}} = \Psi_i^{\mathrm{m}} - \Psi_i^{\mathrm{c}} = \sum_j M_{ij} I_j, \tag{5}$$

where $\Psi_i^{\mathrm{e}}$ is the magnetic flux due to the eddy current, $\Psi_i^{\mathrm{m}}$ is the measured flux, and $\Psi_i^{\mathrm{c}}$ is due to the coils. The subscript $i$ identifies the LOW. $M_{ij}$ is the mutual inductance between LOW $i$ and virtual coil $j$, and $I_j$ is the current of virtual coil $j$. Equation (5) is modified with vector notation as follows:

$$\boldsymbol{I} = \boldsymbol{M}^{-1}\boldsymbol{\Psi}. \tag{6}$$

We call this method C in this paper.

### 3.4 Reconstruction using the virtual coil current at an arbitrary position (method D)

In this method, the virtual coils are set outside the vacuum vessel; the positions and number of the virtual coils are arbitrary. The flux is reconstructed using the currents of the virtual coils and the currents of discharged coils. The currents of the virtual coils are chosen using the least squares method to satisfy $\Psi_i^{\mathrm{e}}$ in method C. The placement of the coils is optimized according to the results of the Akaike information criterion (*AIC*) in numerical simulations, as follows. The flux distribution in this simulation arises from the currents of the CS coil, the CCs, and the eddy currents calculated by the circuit equation as shown in method A. The *AIC* indicates the goodness of fit of models used to fit measured data; the model that has a lower value of the *AIC* is better [7]. In this study, the *AIC* is defined as

$$AIC = n\left(\ln\left(2\pi\frac{S_{\mathrm{e}}}{n}\right) + 1\right) + 2\,(p+2)\,, \tag{7}$$

where $n$ is the number of data points, $S_{\mathrm{e}}$ is the sum of the squared error, and $p$ is the number of free parameters. The relationship between the *AIC* and the number of virtual coils in the numerical simulation is shown in Fig. 3. In this simulation, the virtual coils are set on a line 5-19 cm from the line made by connecting the positions of the LOWs. When the number of virtual coils is increased, the priority for setting a new virtual coil is decided by the deviation between the calculated and given magnetic flux values. Once a virtual coil layout is chosen, the fluxes at the positions of the LOWs are calculated, and the deviation between the calculated and given values is obtained. A new virtual coil is added at the position closest to the loop whose deviation is largest, and another coil is added at the symmetric position in the $z$ direction. According to Fig. 3, the most suitable distance between virtual coils and loops on the wall is 15 cm, and the total number of virtual coils is 11. The layout of virtual coils determined by numerical simulation is shown in Fig. 4.

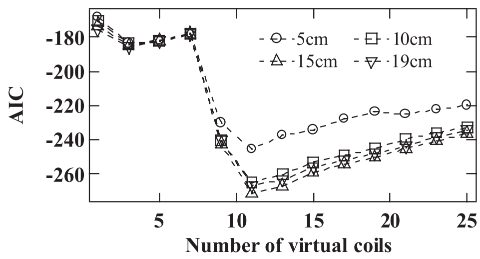


Fig. 3 Relationship between *AIC* and number of free parameters. Symbols indicate the distance between the virtual coils and loops on the wall (circles: 5 cm, squares: 10 cm, upward triangles: 15 cm, downward triangles: 19 cm). For the same number of virtual coils, the difference in *AIC* between each pair of distances corresponds to that of the sum of squared error, $\chi^2$.

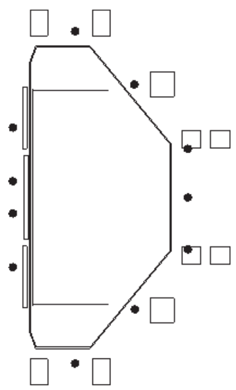

Fig. 4 Layout of virtual coils (black dots) in the model with the minimum *AIC* value in numerical simulations.

## 4. Experimental Results and Analysis

The coil is discharged under the condition that PF4-1, PF4-2-a, PF4-3, and the CCs are connected in series and only these coils are working. In this configuration, a very large eddy current is induced. Next, PF4-1, PF4-2-a, and PF4-3 are regarded as one coil and called the CS coil. The time evolution of the current of the CS coil and the total eddy current estimated by the circuit equation (method A) are shown in Fig. 5. In Fig. 6, the sum of the squared errors between fluxes measured by the LOMs and the calculated values is shown. At 0.475 s, the squared error is lower than that at other times in methods A, B, and C because the eddy current is relatively small. However, when the

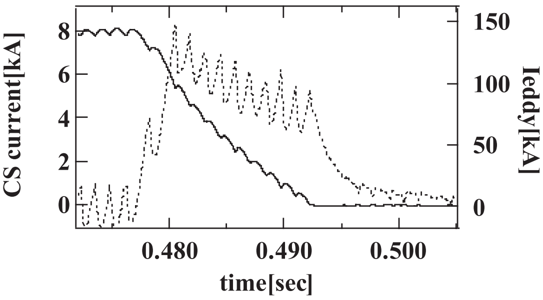


Fig. 5 Time evolution of CS coil current (solid line) and total eddy current estimated by the circuit equation (dashed line) in an analyzed shot.

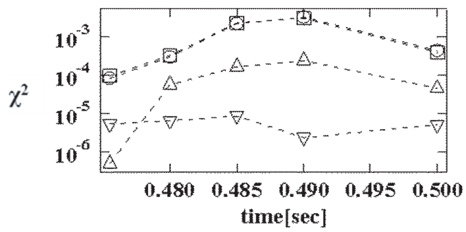


Fig. 6 Time evolution of the sum of squared error, $\chi^2$. Circles: results of calculation by method A, squares: results of method B, upward triangles: results of method C, and downward triangles: results of method D.

eddy current becomes large, reconstruction becomes inaccurate, especially in methods A and B. In contrast, method D maintains its accuracy even when the eddy current is large, and the squared error is lower than that of the other methods from 0.48 to 0.5 s. Figure 7 shows the distribution of measured and calculated fluxes at 0.48 and 0.5 s. The magnetic flux distribution is reconstructed well by method D. The difference between the peak point of the measured magnetic fluxes, which is called the null point, and that of the calculated magnetic fluxes in method D is less than 10 cm. In the other methods, the differences are relatively large. Hence, the most accurate method of reconstructing the distribution of vacuum magnetic flux is method D.

## 5. Summary and Conclusion

To determine the best method of reconstructing the magnetic flux when eddy currents are induced, the fluxes in the cavity of the vacuum vessel are measured, and the measured values are compared with the values calculated by four different methods. The virtual coil currents chosen to satisfy the flux values measured by the LOWs reconstruct the magnetic field in the cavity of the vacuum vessel with good accuracy. The virtual coils are set on a line 15 cm outside of the vessel wall.

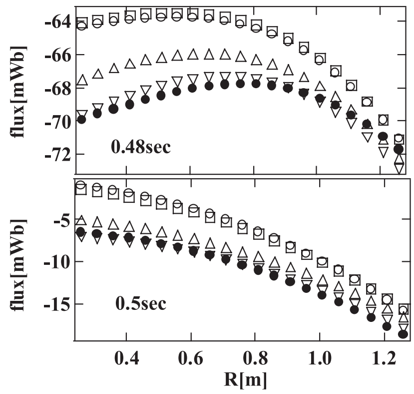


Fig. 7 Distribution of flux on the midplane. Horizontal axis shows the radius of each LOM, and vertical axis shows the flux values. Closed circles represent measured values, and open symbols represent calculated values as in Fig. 6.

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