Full Wave Simulation of Lower Hybrid Waves in ITER Plasmas Based on the Finite Element Method

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The first lower hybrid (LH) full wave simulation of an ITER-scale plasma is presented. LHEAF [O. Meneghini et al., Phys. Plasmas 16, (2009)], an efficient LH full wave solver based on Finite Element Method (FEM) was used. In this study the scalability of the LHEAF approach was investigated, and the possibility of using massive parallel computer for solving extremely large problems was shown. In reactor scale plasmas, LH waves having a typical $n_{\|} \approx 2$ are expected to be absorbed in the periphery of the plasma. In order to exploit the spatial localization of the LH waves, LHEAF is modified to consider only the region of plasma where the wave fields are non-zero. By this approach, the size of the computational domain was reduced by more than a factor of 10. In this simulation, the magnetic equilibrium and the density and temperature profiles proposed for AT operation scenario on ITER are used. In addition, the wide SOL is supposed to play an important role in the propagation of the LH waves on ITER, and its presence was included in the simulation. For a Maxwellian plasma the power deposition profile is narrow and peaks at $r/a \approx 0.7$.

Keywords: LHEAF, COMSOL, ITER, LH, lower hybrid

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1. Introduction

Until recently numerical modeling of LHCD has consisted, for the most part, of toroidal ray tracing codes and Fokker Planck (FP) solvers [1–3]. Ray tracing approximates the wave as a bundle of rays under the assumption of the validity of Wenzel-Kramers-Brillouin (WKB) approximation, and is used to calculate the path of waves and the evolution of their wave vector. FP packages combined with ray tracing modules are used for the evaluation of the power absorbed and of the driven current.

This approach has been successful at correctly modeling some of the experimental results of LHCD [4]. Nonetheless the assumptions on which these codes rely inherently exclude some physics which may be important for a more accurate modeling. For example, ray tracing approach is known to be questionable in particular regions of plasma where the WKB approximation does not hold. Also, ray tracing does not describe phenomena such as interference and diffraction and does not allow the propagation of the waves in regions where the wave is cutoff.

These limitations can be addressed by full wave simulation, that is by directly solving Maxwells equations inside of the plasma region. Full wave simulations are commonly treated in the spectral domain [5, 6], where spatial dispersion effects are treated more easily. However spectral solvers represent the solution in terms of basis functions which are defined over the whole computational domain. Consequently they have difficulty at accurately representing the tokamak vessel or the launching antenna structure, and their use is mostly limited to the description of the core plasma. Also, they are extremely computationally demanding and in general they must run of massive parallel computers.

To address all of these issues simultaneously, we developed a new full wave simulation code for LH waves named LHEAF (Lower Hybrid wavE Analysis based on FEM). LHEAF instead uses a Finite Element Method (FEM) approach which is more efficient than the spectral approach followed by other core plasma wave solvers [7]. In this paper we investigate the scalability of LHEAF by running for the first time a full wave simulation of LH waves in a realistic ITER plasma.

2. Modeling of LH Waves Using LHEAF

The propagation of the LH waves is well described by the magnetized cold plasma approximation, while their absorption is governed by the electron landau damping (ELD) process. Within the cold plasma approximation the time harmonic wave equation is a conventional Partial Differential Equation (PDE). However, the ELD term depends on the parallel wave number k_z, meaning that this effect is non-local and consequently, the wave equation has an integro-differential form. Directly solving this equation corresponds to solving a large dense matrix and requires

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a large computational effort.

LHEAF deals with this issue by adopting two analysis techniques. First it assumes that plasma is symmetric in the toroidal direction and decomposes the electric field into the toroidal mode number. This technique is known as single mode analysis and is commonly used by full wave spectral codes [8]. However its implementation in a 3D FEM is new and it is based on the use of Floquet-Bloch like periodic boundary to specify the toroidal mode number, as described in detail in Ref. [9]. Second, LHEAF treats the convolution integral in an explicit manner by splitting the original integro-differential equation into two coupled equations which are solved iteratively, the idea being that the ELD term is approximated by an effective local damping. The resulting electromagnetic problem has the form of a conventional PDE which can be efficiently solved by a 3D FEM solver.

The current implementation of LHEAF uses the RF Module of COMSOL Multiphysics and MATLAB. In this integrated environment, the former solves the EM problem at each step, while the latter sets up the iterative solution and calculates the effective local damping. The non-Maxwellian electron velocity distribution arising from the interaction of the LH waves with the plasma, can be taken into account by a one dimensional Fokker Plank code (1DFP) which is included in the iteration loop. The 1DFP code is integrated in LHEAF by updating the parallel distribution function at each step of the iteration, before the effective local damping is calculated.

LHEAF results have been validated with ray tracing and the TORIC-LH code [10] for a Maxwellian plasmas [11]. Validation of LHEAF for non-Maxwellian plasma is under way. Hence, in the following a Maxwellian plasma is assumed, no calculations of driven current are presented and we will focus solely on power deposition.

3. Solution of Large LH Waves Problems

The numerical problem arising from the FEM technique scales more favorably to large scale plasmas when compared with spectral techniques. In the following we study how different full wave approaches scale, as a function of the linear size of the device (i.e. its major radius R, assuming a constant aspect ratio $\epsilon = a/R$). The scaling of the solution time of solvers other than FEM was deduced under the conservative assumption that the computational requirements are dominated by the inversion time of the linear system and not by the filling of the system itself.

3.1 FEM

In LHEAF, the solution to the electromagnetic problem is found via 3D FEM usually based on a 1st order polynomial associated to a uniform rectangular mesh. In the case of electromagnetic waves propagating into an anisotropic medium (the magnetized plasma) FEM methods lead to sparse, complex, non-symmetric linear systems.

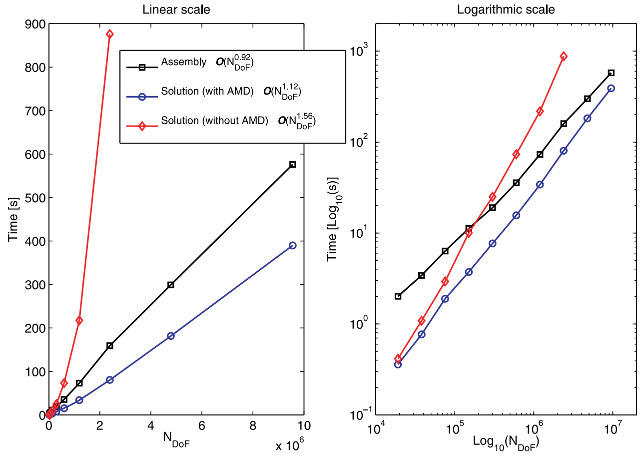

Fig. 1 Linear and logarithmic plot of the time required for assembly and for the solution of the FEM problem as a function of the number of DoF. We compared the time required to solve the problem with the MATLAB UMFPACK solver with and without the Approximate Minimum Degree (AMD) preordering scheme. We found PARDISIO and MUMPS to give similar result to UMFPACK with preordering.

A common way to solve large sparse linear systems is to use iterative techniques. However we opted for using direct solvers, the reason being that electromagnetic problems in plasma lead to very ill conditioned linear systems(due to the high contrast of refractive index) thus making the convergence rate very slow. Furthermore, we found iterative methods to not converge at all when "Floquet-Bloch" periodic boundary conditions were present.

The time required for inverting a sparse matrix strongly depends on the fill-in of the system. Sparse linear system solvers such as UMFPACK [12], PARDISIO [13] and MUMPS [14] use advanced preordering algorithms to minimize the fill-in. We compared the time required to solve the problem with the MATLAB UMFPACK solver with and without the Approximate Minimum Degree (AMD) [15] preordering scheme. We found the solution time to scale almost linearly $O(N_{\mathrm{DoF}}^{1.12})$ with the number of degrees of freedom (DoF) when AMD is used. The PARDISIO and MUMPS solvers gave similar or better result than UMFPACK with preordering. Since the number of DoF is proportional to the cross-sectional area, we conclude that the FEM approach scales as R^2, where R is the major radius of the device.

Figure 1 shows the time required to assemble and solve the linear system as a function of the DoF on a desktop computer equipped two 3.0 GHz quad-core CPUs and with 96 GB of RAM. Models with more than 40 million unknowns were solved by the aid of the MUMPS library, using the massive parallel computing resources at NERSC.

3.2 Spectral solvers

Spectral domain solvers represent the solution in terms of basis functions which are defined over the computational domain they are applied to, thus resulting in linear systems which are full. A fully spectral approach, as implemented in the AORSA [6] code, uses a spectral basis set that is (k_x, k_y), and will result in full linear system. For a matrix of size $N \times N$, the cost of performing Gaussian elimination is equal to $2/3N^3$. Consequently we can expect the solution time to scale as R^6, the number of DoF being proportional to R^2 and the solution time of a full linear system scaling as the number of matrix elements cubed. On the other hand, a code such as TORIC-LH [10], is based on a (r, m) basis set and uses a 1D finite element approach in the radial direction and a 1D spectral approach in the poloidal direction. The resulting numerical system for this approach is block-diagonal. The complexity of solving such system is $N_f M^2$, where N_f is the number of flux surfaces and M is the number of poloidal modes used to represent the solution. Since the number of poloidal modes required to represent the solution with a given spatial accuracy scales linearly with the size of the device, we can deduce that this approach scales as R^3.

4. LHCD System on ITER

The ITER LHCD system is planned to operate at 5 GHz and launch 20 MW into the plasma via a Passive Active Multijunction (PAM) antenna [15] with a parallel launched spectrum peaked at $n_{\|} = 2 \pm 0.1$.

The LH launcher will be made of 4 identical PAM blocks containing 12 rows of waveguides each. Each row will consist of 24 active waveguides (9.25 × 58 mm) alternated to 25 passives waveguides (7.25 × 58 mm). All waveguides are separated by 3 mm toroidal and 1cm poloidal septa. With this arrangement, the forward power density is of about 11 MW/m^2.

In ITER the distance from the last closed magnetic flux surface ($R_{LCFS} \approx 8.18$ m) to the first wall will be at least 12 cm in order to avoid excessive heat load from the plasma. The wide SOL is expected to play an important role in the coupling and propagation of the LH waves. At the launcher location, the electron density is expected to be below the cutoff density of the slow wave ($n_c = 3.1 \times 10^{17}$ m^{-3} at 5 GHz). Experiments on ASDEX [17] and JET [18] used local gas injection in the SOL to increase the electron density in front of the launcher above the slow wave cutoff density and improve the slow wave coupling. Similar techniques are likely to be used in ITER.

5. Simulation Results

We present the single toroidal mode simulation of LH waves as they propagate in a poloidal cross section of the ITER tokamak. For this simulation the plasma is assumed to be Maxwellian, and the power absorption is self consistently retained in the simulation by means of LHEAF iterative routine. For our simulations we used the magnetic equilibrium and the core density and temperature profiles for the lower hybrid scenario [19] as provided by the ITER-LH task force (Shown in Fig. 2). Assuming that gas puffing will be used in ITER to improve long distance coupling, we used the SOL profiles consistent with the results from JET [17] and the ITER PAM coupling studies. In particular an exponential falloff of the density and temperature with a scale length of $\frac{n_e}{dn_e/dx} = 2$ cm was used.

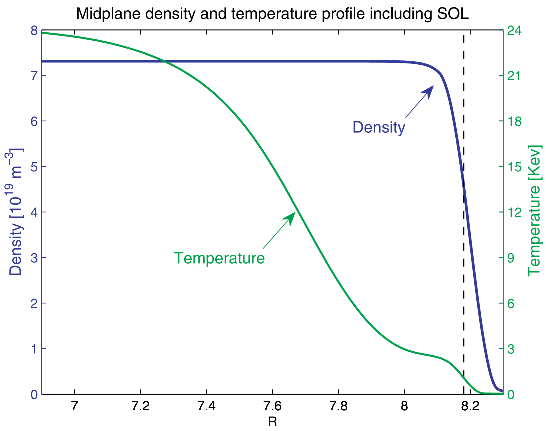

Fig. 2 Midplane density and temperature profiles for the ITER lower hybrid scenario, including SOL.

In the following we will focus only one top part of the antenna system (12 top rows), as proposed for the first phase of the ITER LH system [20]. The design of the LH launcher is still under way and for this simulation we assumed the launcher profile to be conformal to the flux surfaces and to be located at $R_{ANT} = 8.3$ mm at the midplane, 12 cm away from the LCFS.

In typical reactor scale plasmas, LH waves having $n_{\|} \approx 2$ are expected to be strongly damped and to be localized in the outer region of the plasma. Hence the possibility to take advantage of the flexibility of the FEM approach, by considering only the region of plasma where the wave fields are expected to be non-zero. In our simulations, the spatial localization of the LH waves was inferred prior of the full wave calculation by means of ray tracing. By this approach the original problem size was reduced by from ≈ 25.5 m^2 to ≈ 2.0 m^2.

Figure 3 shows the logarithmic plot of the parallel wave electric fields propagating through the final section of the launcher, through the SOL into the core plasma, where they are finally absorbed by ELD. Contour plots of the magnetic flux surfaces and of the full wave computational domain are over plotted. About 6% of the power is reflected at the waveguide-plasma interface, resulting in a standing wave pattern in the waveguide section. Figure 4 shows that the radial power deposition profile is peaked at $r/a \approx 0.7$ in agreement with ray tracing simulations. The power reflected accounts for the slight difference in the absolute magnitude of the radial power deposition profile be-

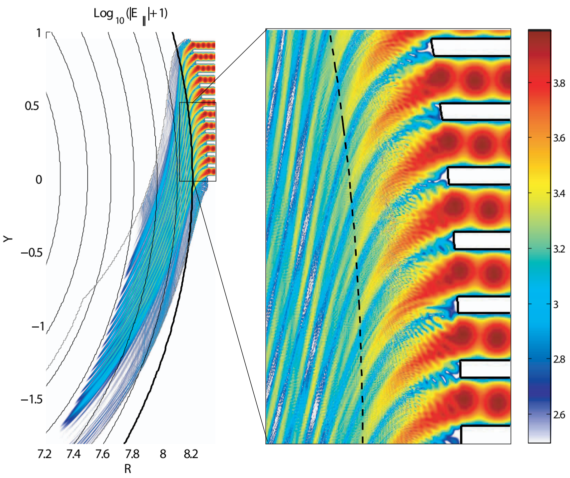

Fig. 3 Logarithmic magnitude of the electric field of LH waves as they propagate in an ITER poloidal cross section.

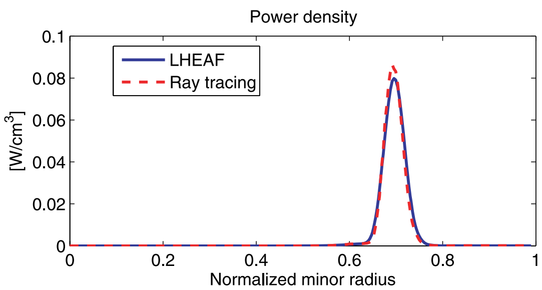

Fig. 4 Radial power deposition profile as calculated by LHEAF and ray tracing.

tween LHEAF and ray tracing.

6. Summary

The possibility of using LHEAF, a new efficient full wave simulation code for LH waves, for ITER LHCD experiments was explored. The scalability of the FEM approach used by LHEAF has been investigated and compared to other full wave codes. The computation time of the full wave calculation was found to scale almost linearly with the number of degrees of freedom, which is approximately proportional to the poloidal cross-sectional area of a model for a give mesh size. The good scalability of the LHEAF code permitted the first ever full wave simulations of LH wave propagation in ITER scale plasmas.

The magnetic field topology and the realistic density and temperature profiles for the ITER lower hybrid scenario were used. One of the most attractive feature of LHEAF is its improved description of edge plasmas and LH waves propagation in the SOL. The 12 cm SOL region which is important to reproduce the realistic long distance launching condition was included in the simulation.

LHEAF and ray-tracing simulations results show that for a Maxwellian plasma the power deposition profile peaked and centered at $r/a \approx 0.7$.

Acknowledgments

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