Fokker-Planck Simulation of Multi-Species Heating in Tokamak Plasmas

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Wave-particle interactions can produce non-Maxwellian momentum distribution functions, which modify the propagation and absorption of the wave. Kinetic analysis including deformation of the momentum distribution function is required for quantitative wave heating analysis. The deformation strongly depends on the loss mechanism and radial transport of energetic ions. In our previous work, we studied time evolution of the momentum distribution function by local ICRF heating analysis. In the present analysis, we describe global analyses of multi-species heating including loss mechanisms and radial transport.

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1. Introduction

Plasma heating and current drive deform the momentum distribution functions of the heated species. The distortion of the momentum distribution functions from Maxwellian affects the heating mechanism, such as the wave propagation and absorption. Therefore, kinetic analysis which includes the effects of deformation of the momentum distribution functions is required for a quantitative description of heating and current drive. In the present paper, results of multi-species heating analysis in tokamak plasma using the integrated code TASK are reported.

The TASK code is composed of several components. The full wave component, TASK/WM [1], calculates the wave electric field by solving Maxwell's equations including the plasma dielectric tensor. The bounce-averaged Fokker-Planck component, TASK/FP [2], analyzes the time evolution of the momentum distribution functions for electrons and ions by solving the Fokker-Planck equation including the quasi-linear diffusion terms calculated from the wave electric field. The dielectric tensor component, TASK/DP, calculates the plasma dielectric tensor by numerically integrating the momentum distribution functions. By repeating this cycle, we can describe the time evolution or the steady state of the wave heating and current drive.

We analyze multi-species heating of plasma which consists of four species: electron, deuteron, triton and α particle. TASK/FP calculates the time evolution of the momentum distribution function for each species simultaneously. For the cases we have studied, neutral beam injection, ICRF wave, and 3.5 MeV α particles which are generated by nuclear reaction are used as heat sources. The ICRF waves generate energetic ions at the fundamental and higher harmonic cyclotron resonance; a high-energy tail of the momentum distribution function, which affects the absorption of the ICRF waves, is formed. In the case of the second cyclotron harmonic, a small fraction of ions is strongly accelerated and a stronger energetic tail is generated. The electron momentum distribution function is flattened in the vicinity of the parallel wave-particle resonance, and the modification leads to the power absorption through Landau damping or transit-time magnetic pumping. Neutral beam and fusion reaction sources lead to non-Maxwell distribution functions, and these distribution functions affect the other species through collisions.

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2. Model of Fokker-Planck Analysis

First, we describe the Fokker-Planck module, TASK/FP, which solves the bounce averaged Fokker-Planck equation:

$$\frac{\partial f_s}{\partial t} = E(f_s) + C(f_s) + Q(f_s) + D(f_s) + L(f_s) + S \quad (1)$$

where the terms E, C, Q, D, L, and S are the acceleration term by the toroidal DC electric field, collision term due to Coulomb collision, quasi-linear diffusion term due to wave-particle interaction, radial transport term, loss term, and source term, respectively. f_s denotes the momentum distribution function for species s, $f_s(p_\parallel, p_\perp, \rho, t)$, where $p_\parallel$ and $p_\perp$ are the parallel and perpendicular momentum at the minimum magnetic field point on the magnetic surface and ρ is the normalized minor radius of the surface. TASK/FP includes the trapped particle effect by bounce averaging with zero banana width. This module can calculate the time evolution of distribution functions not only for the mainly heated species but also for the other species.

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Several models for the Coulomb collision terms ($\boldsymbol{D}_{\rm cl}$ and $\boldsymbol{F}_{\rm cl}$) are available in TASK/FP. The linear collision model [3] assumes a Maxwellian distribution for field particles. It doesn't conserve the total momentum or energy and is not adequate when various heating processes occur simultaneously. In the non-linear collision model [3–5], the distribution functions of field particle species are expanded in terms of Legendre polynomials, $P_l(\cos\theta)$:

$$f_s(\boldsymbol{v}_s) = \sum_{l=0}^{L} f_s^{(l)}(v_s)P_l(\cos\theta_s) \tag{2}$$

where θ is the pitch angle at the minimum magnetic field point on the magnetic surface. By integration over v_s, we can calculate the non-linear collision terms. If we keep the lowest order term ($L = 0$), the non-linear term conserves only the number of particles. The non-linear Coulomb collision model satisfies momentum ($L \geq 1$) and energy ($L \geq 2$) conservation. Relativistic effects are also included according to the formulation by Braams [5].

The source term $S(f_s)$ includes particle sources such as NBI and α particles generated by fusion reaction. If there is no radial transport and particle loss, the plasma stored energy continuously increases in time by external heating. In order to obtain a steady state solution, we introduce two mechanisms. One is an artificial particle loss with relaxation time τ_L. The loss term $-f/\tau_L$ and compensating source term $f_{\rm maxwell}/\tau_L$ are implemented. The other is a more realistic radial transport term. The radial transport term $D(f_s)$ is composed of radial diffusion and pinch terms. Presently, TASK/FP implements radial diffusion with fixed radial profiles, without energy dependence, and with radial pinch, to sustain the initial radial profile.

3. Calculation Results of Multi Species Heating

We carried out numerical analysis of multi species heating in three cases. The first and second cases are multi-species heating analysis without radial diffusion. The third case includes preliminary radial diffusion. The difference between the first and second case are the existence of the artificial loss term. In these analyses, we used the parameters in Table 1, simulating ITER like tokamaks. We used a 55 MHz ICRF wave, 1 MeV deuterium NBI, and 3.5 MeV α particles as heat sources. The ICRF wave accelerates electrons by Landau damping and tritons by the second cyclotron harmonic resonance. Therefore, fast electron and triton generated by ICRF, fast deuterium injected by NBI, and fast α particle generated by fusion reaction collide with each other and transfer their energy through collision. Figure 1 shows wave absorption profiles calculated by TASK/WM. Resonance surface for triton is found in Fig. 1 (a). In the present analyses, ICRF wave absorption for deuteron and He is neglected because fundamental resonance absorption is small.

Table 1 Plasma parameters

major radius	R_0	6.2 m
minor radius	a	2.0 m
elongation	κ	1.7
triangularity	δ	0.33
magnetic field on axis	B_0	5.3 T
temperature on axis	T_0	20.0 keV
temperature on surface	$T_{\rm s}$	2.0 keV
density on axis	n_0	$1\times10^{20}/{\rm m}^3$
density on surface	$n_{\rm s}$	$1\times10^{19}/{\rm m}^3$
deuterium ratio	$n_{\rm D}/n_{\rm e}$	0.5
tritium ratio	$n_{\rm T}/n_{\rm e}$	0.5
NBI energy	$E_{\rm NBI}$	1 MeV
α particle energy	E_α	3.5 MeV
ICRF wave frequency	$f_{\rm RF}$	55.0 MHz
relaxation time	τ_L	100 ms

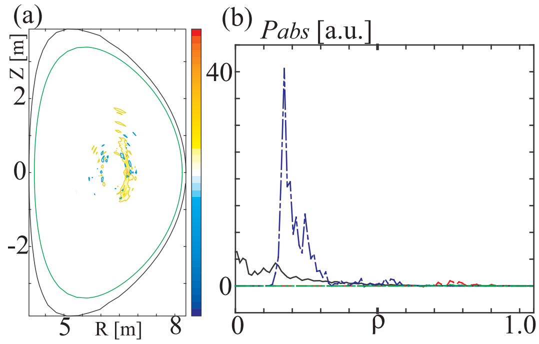

Fig. 1 ICRF wave absorption profiles. (a) Wave absorption for T on poloidal cross section. (b) Radial profile of absorbed power density $P_{\rm abs}$.

3.1 Case without loss

First, we studied the case of multi-species heating of plasma without the loss term. In this analysis, the NBI power was 31.6 MW, the α particle heating power was 55.4 MW, and the initial value of the ICRF heating power was 18.8 MW (6.39 MW for electrons and 12.4 MW for triton). Since the wave amplitude was fixed in this calculation, the absorbed power depends on the population of high-energy ions. Figure 2 shows the contours of distribution functions for electron, deuteron, triton, α particle at 200 msec after the onset of heating. Anisotropy of the deuteron and triton distribution functions (Figs. 2 (b) and (c)) is due to NBI and ICRF wave absorption. Cyclotron harmonic resonance modifies the perpendicular momentum of particles. There are two tips in Fig. 2 (c) because of trapped particle effects. Fig. 2 (d) shows that the distribution function of He has a peak at 3.5 MeV ($p \sim 19$) . From Fig. 2 (a) and 2 (c), it was found that the deformation of the electron distribution function is much less than that of triton. This is because that electrons absorb less ICRF wave power than triton, and the collisional relaxation time of electrons is shorter than that of triton.

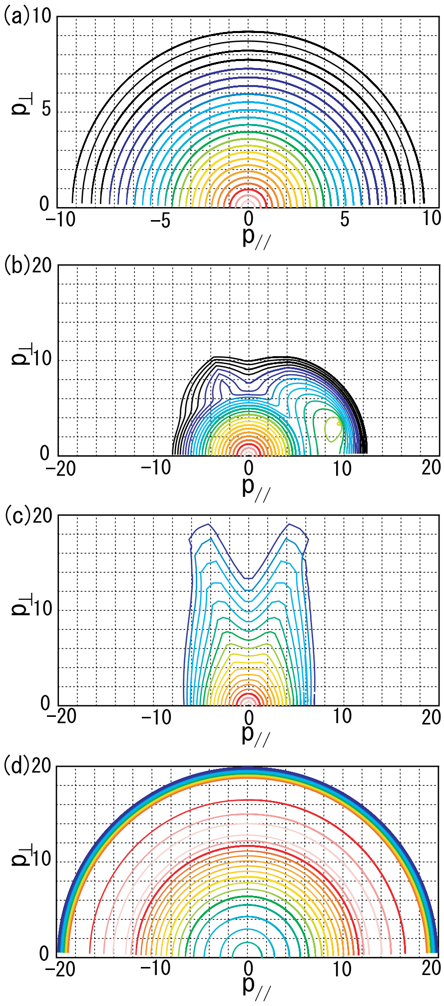

Fig. 2 Contours of distribution functions at the peak of wave absorption in 2D momentum space. (a) electron, (b) D, (c) T, (d) He. Vertical and horizontal axes are perpendicular and parallel normalized momentum. $p \sim 10$ correspond to 1 MeV. The contours have been chosen such that they are evenly spaced when the distribution functions are Maxwellian.

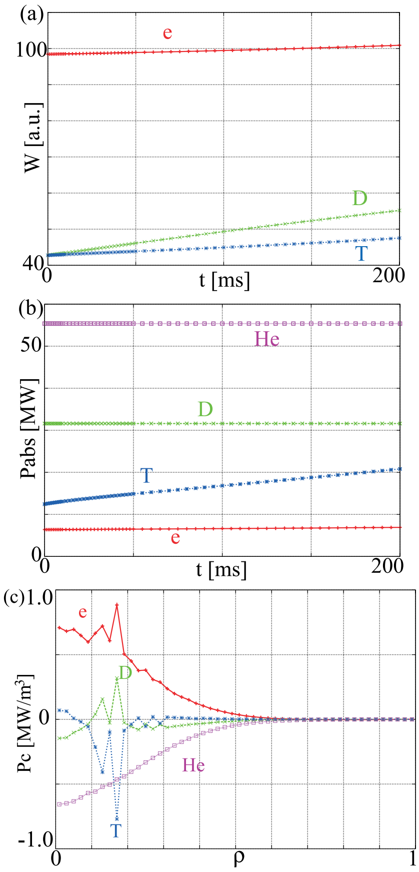

Fig. 3 (a) Time evolution of stored energy, W. (b) Time evolution of absorption power, P_{abs}. (c) radial profile of collisional power transfer density, P_{c}. In Fig. 3 (b), each line indicates ICRF absorption for electron and triton, deuteron NBI power, and fusion reaction power (He).

Figure 3 shows the time evolution and radial profile of several quantities. Without a loss mechanism, the stored energy continuously increases for each species (Fig. 3 (a)). Also, fast ions continue to be generated. Because of the generation of these fast ions, the absorption power for triton strongly increases with time (Fig. 3 (b)), and the collisional power transfer from tritons to electrons is dominant.

3.2 Case with artificial loss

Next, multi-species heating withan artificial loss term simulating radial diffusion was analyzed. For this case, we used the same heating power as in the previous subsection. Figure 4 is analogous to Fig. 3 for this case. With the loss term, the stored energy saturates for each species (Fig. 4 (a)), and the fast ion population in this case is less than it is in the case without loss. Because of this decrease of fast ion density, ICRF absorption power for electron and triton also saturates (Fig. 4 (b)), and collisional power transfer from triton to deuteron becomes dominant (Fig. 4 (c)).

3.3 Case with radial transport

Finally, we used the radial transport term $D(f_s)$ instead of the loss term $L(f_s)$. The spatial diffusion coefficient D_r was taken to be constant in momentum space and parabolic in the radial direction ($D(\rho = 0) = 0.1\,\mathrm{m/s^2}$ and $D(\rho = 1) = 1\,\mathrm{m/s^2}$). The pinch effect which sustains the density profile was introduced through the radial friction coefficient F_r, which satisfies the following equation:

$$\frac{\partial n}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho D_r \frac{\partial n}{\partial \rho} - F_r n\right) = 0. \qquad (3)$$

Figure 5 (b) shows the temperature profile with radial transport. From these figures, it is found that the temperature peak of triton is broadened by radial diffusion.

4. Conclusion

We have updated the Fokker-Planck component of TASK, TASK/FP, for kinetic analysis of heating and trans-

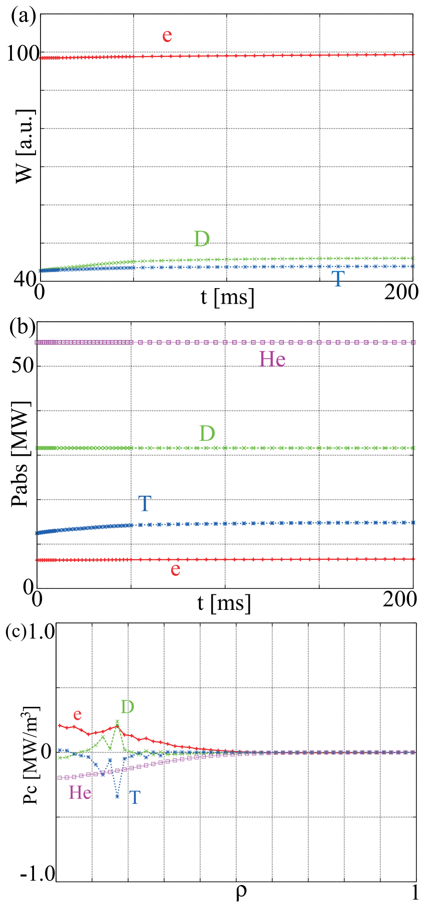

Fig. 4 Time evolution of (a) stored energy W and (b) absorption power P_{abs} and (c) radial profile of collisional power transfer P_c with loss term.

port. The present analysis describes the simultaneous time evolution of the momentum distribution functions for a multi-species plasma. We have confirmed that plasma heating depends on the loss mechanism. More realistic radial transport models, as used in conventional transport simulation and with momentum dependence, are essential for quantitative comparison with experimental observation. The implementation is under way.

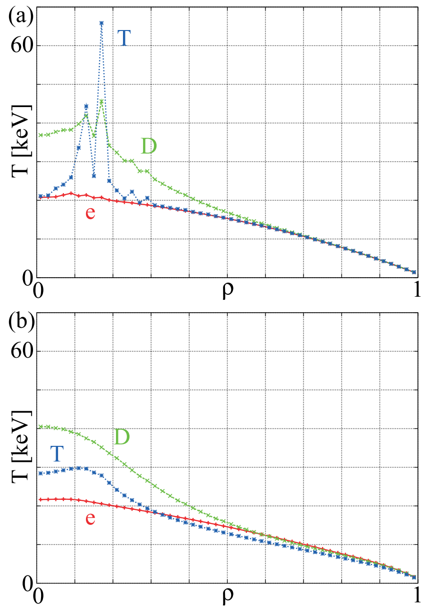

Fig. 5 Radial profile of temperature (a) without loss and (b) with radial transport.

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