Repeated Acceleration of Thermal Ions by an Oblique Shock Wave and Associated Whistler Instabilities

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A magnetosonic shock wave propagating obliquely to an external magnetic field can repeatedly accelerate thermal ions if $\theta_0 \simeq 45°$, where θ_0 is the angle between the wave normal and the external magnetic field. The ion energy gains in this process are theoretically analyzed, and an expression for the maximum energy is derived. This theory is verified using a two-dimensional, electromagnetic particle code. Furthermore, whistler wave instabilities generated by the accelerated ions are studied. The simulation demonstrates that whistler waves are excited in both the upstream and downstream regions, but that the whistler waves in these two regions have different frequencies and wavenumbers. It is shown that the characteristics of these waves can be explained by linear theory.

Keywords: collisionless shock, ion acceleration, whistler wave instability, particle simulation

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1. Introduction

Theory and particle simulations [1–3] have revealed that nonthermal, energetic ions that have barely entered a shock wave can be further accelerated to higher energies because while they are in the shock region their gyromotions are almost parallel to the transverse electric field. If these ions remain near the shock wave for long periods of time, the acceleration process can occur several times. The condition for this is given by [2]

$$v_{\parallel} \cos\theta_0 \simeq v_{\rm sh}, \tag{1}$$

where $v_{\rm sh}$ is the shock propagation speed, $v_{\parallel}$ is the particle velocity parallel to the magnetic field, and θ_0 is the shock propagation angle (i.e., the angle between the wave normal and the external magnetic field). If the relation $v_{\rm sh} \sim c\cos\theta_0$ is satisfied, where c is the speed of light, energetic particles could be indefinitely accelerated owing to the relativistic effects [3].

The above studies assume that energetic ions are present from the beginning. Recently, the repeated interactions of thermal ions with an oblique shock wave have been studied without assuming the presence of energetic ions at $t = 0$ [4]. When the thermal ions encounter the shock wave for the first time, some of them are energized by reflection from the shock front [5, 6]. With attention to the fact that parallel velocities in the upstream region immediately after the reflection can be estimated as $v_{\parallel} \simeq 2v_{\rm sh}\cos\theta_0$, it was theoretically predicted that if $\theta_0 \simeq 45°$, the reflected ions can be further accelerated by the shock wave with the mechanism discussed in Ref. [2] because Eq. (1) is satisfied. This theoretical prediction was verified by one-dimensional, electromagnetic particle simulations.

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In this present paper, we extend the study [4] of the repeated acceleration of thermal ions and investigate, with theory and two-dimensional particle simulations, the energies of the accelerated ions. We also study the whistler wave instabilities driven by energetic ions. In Sec. 2, we derive a theoretical expression for the maximum energy of repeatedly accelerated ions for the case when $\theta_0 \simeq 45°$. We also present a linear theory for whistler wave instabilities. In Sec. 3, using a two-dimensional, electromagnetic particle code, we confirm that the theoretical and simulation results for the maximum ion energy are in good agreement. It is also shown that the whistler waves are excited in both the upstream and downstream regions. The characteristics of these waves are compared with the linear theory. Section 4 summarizes the study.

2. Theory for Energies of Repeatedly Accelerated Ions

We theoreticaly estimate the maximum energies of ions repeatedly accelerated by a planar shock wave propagating in the x direction ($\partial/\partial y = \partial/\partial z = 0$) in an external magnetic field in the (x, z) plane,

$$\boldsymbol{B}_0 = B_0(\cos\theta_0, 0, \sin\theta_0). \tag{2}$$

The upstream region ions are considered to be in thermal equilibrium with a thermal speed much smaller than $v_{\rm sh}$. When ions encounter the shock wave, some of them are reflected at the shock front and are energized by the longitudinal electric field. The parallel and gyration speeds in the upstream region immediately after the first reflection are estimated as

$$v_{\parallel 0} \simeq 2v_{\rm sh}\cos\theta_0, \quad V_0 \simeq 2v_{\rm sh}\sin\theta_0. \tag{3}$$

We analyze ion motions after the first reflection. Because the reflected ions have gyro radii greater than the width of the transition region, we approximate the shock profile as a step funtion and write the position of the shock front as $x_w = v_{sh}t$. We also use the approximation that $B_y = 0$ in both the upstream and shock regions, since B_y is almost zero outside the transition region. The magnetic field is then written as $\boldsymbol{B} = B(\cos\theta, 0, \sin\theta)$ with $\theta = \theta_0$ in the upstream region $x > x_w$ and $\theta \simeq \pi/2$ in the shock region $x < x_w$, where B_z in the shock is assumed to become much greater than that in the upstream region, while B_x remains constant [7].

In the case when $\theta_0 \simeq \pi/4$, since $v_{\|0}\cos\theta_0 \simeq v_{sh}$, the reflected ion can remain near the shock front for a while and it can hence cross the shock front, $x = x_w$, several times because of its gyromotion. We write the gyro phase as

$$\psi = \Omega(t - t_n) + \psi_n, \tag{4}$$

where t_n, with n integer, is the time when the ion crosses the shock front. Using this phase, we can write the velocities of ions as

$$v_x = (V_n\cos\psi + cE_y/B)\sin\theta + v_{\|n}\cos\theta, \tag{5}$$

$$v_y = -V_n\sin\psi, \tag{6}$$

$$v_z = -(V_n\cos\psi + cE_y/B)\cos\theta + v_{\|n}\sin\theta, \tag{7}$$

where the subscript n refers to the quantities in the upstream (or shock) region when n is even (or odd). The effects of the electric fields E_x and E_z are neglected because they are almost zero outside the transition region. In the following, based on Faraday's law, we approximate E_y in the shock region as $E_y \simeq v_{sh}B_z/c$.

Equation (5) gives the relative position between the particle and shock front as

$$x - x_w = \frac{V_n}{\Omega}\sin\theta[\sin\psi + \delta_n(\psi - \psi_n) - \sin\psi_n], \tag{8}$$

where δ_n is defined as

$$\delta_n = \frac{[v_{\|n}\cos\theta - v_{sh} + (cE_y/B)\sin\theta]}{V_n\sin\theta}. \tag{9}$$

We write the gyro phase just before $t = t_{n+1}$ as ψ'_n, which is given, from Eq. (8), as

$$\sin\psi'_n = -\delta_n(\psi'_n - \psi_n) + \sin\psi_n. \tag{10}$$

We now discuss the energy change that occurs in one gyroperiod. If we know the quantities in the upstream region, V_{2l}, $v_{\|2l}$, and ψ_{2l}, where l is an integer, we can obtain those quantities in the shock region by relating the ion motions in the two regions at $t = t_{2l+1}$. The ion is accelerated by E_y while it is in the shock region and it goes back to the upstream region at $t = t_{2l+2}$. The quantities in the upstream region become

$$v_{\|2l+2} \simeq -2V_{2l}\sin\theta_0\cos\theta_0(\cos\psi'_{2l} + \delta_{2l}) + v_{\|2l} \tag{11}$$

$$V_{2l+2}^2 = V_{2l}^2\{\sin^2\psi'_{2l} + [\cos\psi'_{2l}\cos 2\theta_0 - \delta_{2l}(1 - \cos 2\theta_0)]^2\}, \tag{12}$$

$$V_{2l+2}\sin\psi_{2l+2} = V_{2l}\sin\psi_{2l}, \tag{13}$$

which we obtained by connecting the motions in the two regions at $t = t_{2l+2}$. The value of δ_{2l+2} is then written as

$$\delta_{2l+2} = -\frac{V_{2l}}{V_{2l+2}}(2\cos^2\theta_0\cos\psi'_{2l} + \delta_{2l}\cos 2\theta_0). \tag{14}$$

These equations lead to the energy increment in one gyroperiod as

$$K_{2l+2} - K_{2l} = -2m_i\sin\theta_0(\cos\psi'_{2l} + \delta_{2l})v_{sh}V_{2l}, \tag{15}$$

which is equal to the work done by E_y in the shock wave.

We now derive the expression for the maximum energy of the repeatedly accelerated ions. Equation (15) is rewritten as

$$K_{2l+2} = K_0 - 2m_i v_{sh}\sin\theta_0(V_{2l}\cos\psi'_{2l} + V_0\delta_0) + 2m_i v_{sh}\sin\theta_0\cos 2\theta_0\sum_{j=0,l-1}V_{2j}(\cos\psi'_{2j} + \delta_{2j}). \tag{16}$$

In the case when $\theta_0 \sim \pi/4$, the second term in the right-hand side is almost zero. We thus expect that the energy takes its maximum value when $\cos\psi'_{2l} \simeq -1$ and V_{2l} is the greatest. From Eqs. (12) and (14), we can see that there exists an upper limit of V_{2l}, which is given by

$$V_{2l}^2 \le V_{2l-2}^2 + V_{2l-4}^2\cos^2\psi_{2l-4}, \tag{17}$$

which follows that

$$V_{2l}^2 \le V_2^2 + V_0^2\cos^2\psi_0 = V_0^2(1 + \delta_0^2). \tag{18}$$

We can thus obtain the maximum energy as

$$K_{max} = K_0 + 2m_i\sin\theta_0(1 - \delta_0 + \delta_0^2)v_{sh}V_0. \tag{19}$$

This is determined by V_0, $v_{\|0}$, and δ_0 and is independent of the interaction times of the particle with the shock wave. When the ion gains this energy, the parallel speed becomes

$$v_{\|max} \simeq V_0 + 2v_{sh}\cos\theta_0. \tag{20}$$

3. Linear Theory for Whistler Instabilities

We consider whistler wave instabilities caused by the accelerated ions. Whistler waves can be excited both in the upstream and shock regions, but the characteristics of the waves in these two regions are different.

In the upstream region, some ions are accelerated in the parallel direction. These ions can excite whistler waves through ion cyclotron resonance. The resonance condition is given by

$$k_\| v_\| + \Omega_i = \omega_w, \tag{21}$$

where $v_\parallel$ is the parallel velocity of the accelerated ions, and ω_w are the frequencies of the whistler waves, which are given as

$$\omega_w = v_A k(1 + c^2k^2/\omega_{pe}^2)^{-1/2} \times\{1 + c^2k^2/\omega_{pe}^2[1 + (m_i/m_e)\cos\theta_w^2]\}, \quad (22)$$

with θ_w being the angle between the wave vector $\boldsymbol{k}$ of the whistler wave and the external magnetic field. Assuming that the kinetic effects of the accelerated and background plasmas can be neglected, we can roughly estimate the growth rate as

$$\gamma \sim \left(\frac{n_h}{n_0}\right)^{1/2}\left(\frac{\Omega_i}{\omega}\right)^{1/2} v_A k(1+\cos\theta_w), \quad (23)$$

where n_h and n_0 are the densities of the energetic and background ions, respectively. Equation (23) indicates that whistler waves propagating parallel to the ambient magnetic field are most unstable.

In the shock region where the magnetic field is almost parallel to the z direction, the whistler waves are excited through Cherenkov resonance. In the frame moving with v_{sh}, the accelerated ions move almost in the z direction, while the background ions and electrons move in the negative x direction. Consequently, the relative velocity between the accelerated ions and background plasmas has a component perpendicular to the magnetic field, and it can excite whistler waves propagating obliquely to the magnetic field through Cherenkov resonance. The resonance condition is written as

$$k_\parallel v_\parallel = \boldsymbol{k}\cdot\boldsymbol{u} + \omega_w, \quad (24)$$

where $\boldsymbol{u}$ is the velocity of the background plasma in the wave frame. The growth rate is then given as

$$\gamma \sim \left[\frac{n_h}{n_0}\frac{\omega_{pi}^4 v_0^4}{\Omega_i c^4}\frac{\sin^2\phi\cos^2(\theta_w-\phi)}{\cos\theta}\right]^{1/3}, \quad (25)$$

where ϕ is the angle between the magnetic field and the relative velocity of the accelerated ions to the background plasma, and $v_0 = (v_\parallel^2 + u^2)^{1/2}$. The equation indicates that if $\phi \ll 1$, the waves with $\theta_w \sim 2\phi$ will have the greatest growth rates.

In order for Eqs. (21) and (24) to be satisfied for whistler waves with $\Omega_i \ll \omega \ll |\Omega_e|$, the velocity of the accelerated ions relative to the background plasma must be in the range

$$v_A < v_0\cos(\theta-\phi) < \frac{v_A}{2}\sqrt{\frac{m_i}{m_e}}\cos\theta. \quad (26)$$

4. Simulation

We study the ion acceleration using a two-dimensional (two spatial coordinates and three velocity components), electromagnetic particle code with full ion and electron dynamics. We simulate the (x, z) plane with the size $L_x = 8192\Delta_g$ and $L_z = 512\Delta_g$, where Δ_g is the grid spacing, which enables us to simulate whistler wave instabilities. The total number of simulation particles is $N \sim 5\times10^8$. The ion-to-electron mass ratio is $m_i/m_e = 100$. The thermal speeds are $v_{Te}/(\omega_{pe}\Delta_g) = 1.0$ and $v_{Ti}/(\omega_{pe}\Delta_g) = 0.1$. The electron skin depth is $c/(\omega_{pe}\Delta_g) = 8$. The ratio of the electron gyrofrequency to the electron plasma frequency is $|\Omega_e|/\omega_{pe} = 1.0$ in the upstream region; the Alfvén speed is then $v_A/(\omega_{pe}\Delta_g) = 0.8$. The propagation angle is $\theta_0 = 45°$, for which repeated acceleration of thermal ions is expected.

Figure 1 shows ion phase space plots (x, K), where K is the kinetic energy, and profiles of B_z, averaged over the z direction, for a shock wave with $v_{sh} = 2.6v_A$. At $\omega_{pe}t = 240$, some ions are energized via the reflection at the shock front at $x/(c/\omega_{pe}) \simeq 160$. At $\omega_{pe}t = 1020$, ions with much higher energies are present near the shock front. At $\omega_{pe}t = 1960$, a great number of energetic ions exist over a wide region from the shock front to the upstream region. These ions have been produced by repeated interaction with the shock wave.

We now demonstrate that the simulation results for the energies of repeatedly accelerated ions are consistent with theory. Figure 2 displays the energy distribution of ions that are in the upstream region at $\omega_{pe}t = 1800$ after they were accelerated by the shock. The upper and lower pan-

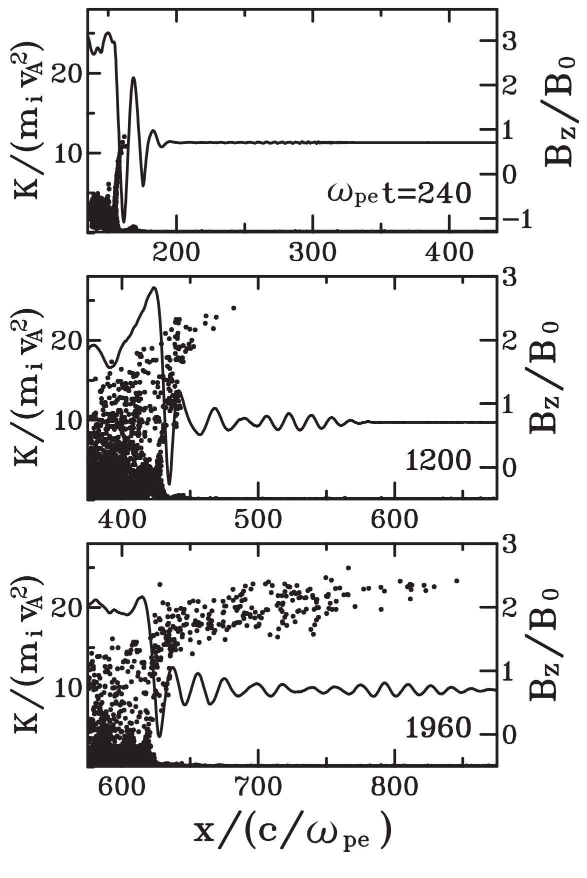

Fig. 1 Ion phase space plots (x, K) and profiles of B_z for a shock wave with $\theta_0 = 45°$.

els are for ions that have been accelerated twice and three times by the shock waves, respectively; ions that have been accelerated more times are not plotted since they are much fewer. The dashed line indicates the theoretical value of $K_{\max}$ given by Eq. (19), which is independent of the number of times an ion is accelerated. Figure 2 clearly shows that the theory provides a good estimate of the upper limit of the energies observed in the simulation.

We next study whistler instabilities driven by the accelerated ions. Figure 3 shows a contour map of B_x in the (x, z) plane at $\omega_{\rm pe}t = 1400$ with the profile of B_z averaged over the z direction. We can clearly see the structure of B_x due to the whistler waves. In the upstream region, $x/(c/\omega_{\rm pe}) > 490$, the wave propagating parallel to the ambient magnetic field has grown so that it has the greatest amplitude; the wavenumber of this mode is $ck_\parallel/\omega_{\rm pe} \simeq 0.30$ and $ck_\perp/\omega_{\rm pe} \simeq 0.035$. However, in the downstream region, $x/(c/\omega_{\rm pe}) < 470$, the wave propagating obliquely to the magnetic field becomes dominant; the wavenumber of this mode is $ck_\parallel/\omega_{\rm pe} \simeq 0.18$ and $ck_\perp/\omega_{\rm pe} \simeq 0.074$. Figure 4 shows the frequency spectra for these modes. The frequencies of the modes dominant in the upstream and downstream regions are $\omega \simeq 9.0\Omega_{\rm i}$ and $\omega \simeq 3.9\Omega_{\rm i}$, respectively.

We compare the characteristics of these whistler waves with linear theory. Figure 5 displays the ion distributions for the parallel velocity $v_\parallel$ at $\omega_{\rm pe}t = 1800$; the upper panel is for ions in the shock transition region, $429 \le x/(c/\omega_{\rm pe}) \le 442$, while the lower panel is for ions in the downstream region, $404 < x/(c/\omega_{\rm pe}) < 417$. The distribution in the transition region has three peaks: a peak

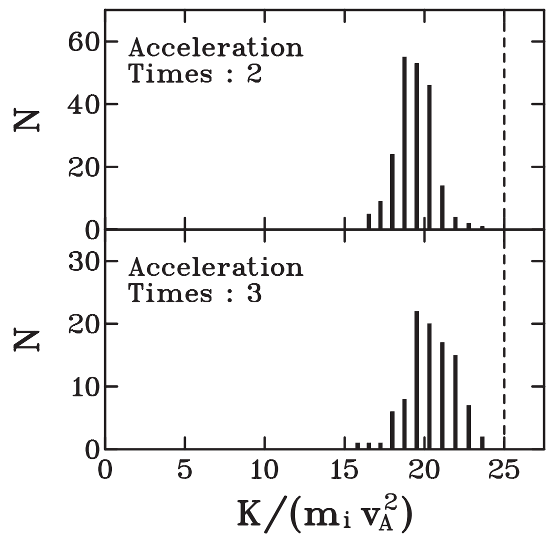

Fig. 2 Energy distribution of ions that have been accelerated twice and three times. The vertical dashed line indicates the theoretical maximum energy in Eq. (19).

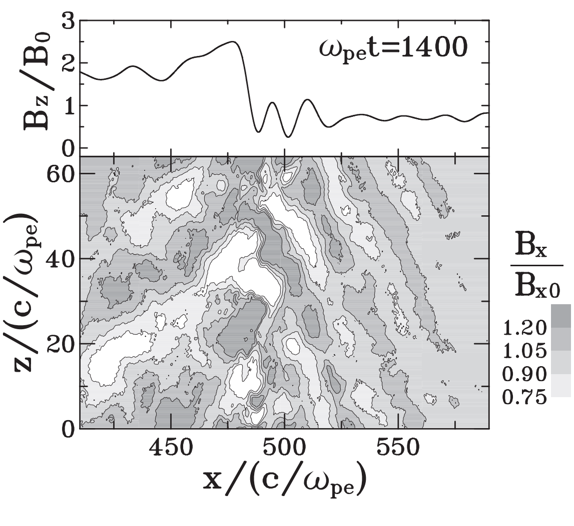

Fig. 3 Contour map of B_x in the (x, z) plane at $\omega_{\rm pe}t = 1400$ with the profile of B_z averaged over the z direction.

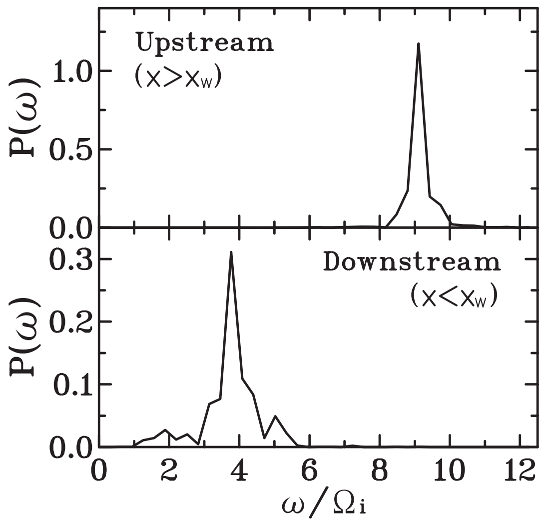

Fig. 4 Frequency spectra of the modes dominant in the upstream and downstream regions.

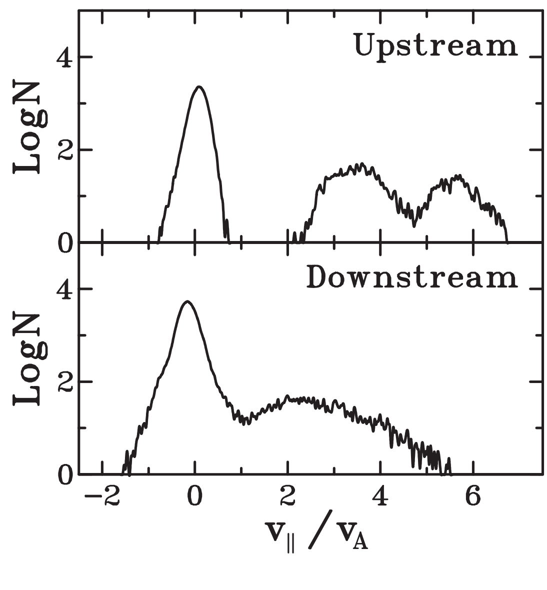

Fig. 5 Ion distributions for the parallel velocity $v_\parallel$ at $\omega_{\rm pe}t$ = 1180.

at $v_{\parallel} \simeq 0$ due to background ions that have not yet encountered the shock wave, a peak at $v_{\parallel} \simeq 3.4v_A$ due to ions that have been reflected at the shock front, and a peak at $v_{\parallel} \simeq 5.5v_A$ due to ions that have been accelerated more than twice. The whistler wave in the upstream region is excited by ions with $v_{\parallel} \simeq 3.4v_A$. Using $v_{\parallel}$, we can estimate from Eqs. (21) and (22), the wavenumber and ω of the most unstable mode as $ck_{\parallel}/\omega_{pe} \simeq 0.4$, $ck_{\perp}/\omega_{pe} \simeq 0.0$, and $\omega \simeq 9.0\Omega_i$, which is in good agreement with the observed values (see upper panel in Fig. 4). Ions with $v_{\parallel} \simeq 5.5v_A$ cannot excite whistler waves because $v_{\parallel}$ is too high to satisfy Eq. (26).

The dominant mode in the downstream region can also be explained by linear theory. We roughly estimate the value of $v_{\parallel}$ of the accelerated ions as $v_{\parallel} \simeq 2.0v_A$ (see Fig. 5). The velocity of transmitted ions in the downstream region (not shown here) is $u_x \simeq -0.5v_A$ in the frame moving with v_{sh}. The relative velocity between the accelerated and transmitted ions has a perpendicular component, and the angle between the relative velocity and the magnetic field is estimated to be $\phi \simeq 7^\circ$. Equation (25) predicts that the propagation angle of the most unstable mode is $\theta_w \sim 2\phi \sim 14^\circ$, which is 70% of that of the dominant mode in the simulation, $\theta_w \simeq 21^\circ$. From Eqs. (24) and (22), we obtain ω of the most unstable mode with $\theta_w \simeq 21^\circ$ as $\omega = 4.0\Omega_i$, which is in good agreement with the simulation result shown in the lower panel of Fig. 4.

5. Summary

A magnetosonic shock wave propagating obliquely to the external magnetic field can repeatedly accelerate some thermal ions if $\theta_0 \simeq 45^\circ$, where θ_0 is propagation angle of the shock wave. We studied the ion energies and whistler instabilities by the accelerated ions. We theoretically analized ion motions and derived an expression for the maximum energy. This theory is in good agreement with simulation results obtained by a two-dimensional, electromagnetic particle code. The simulation demonstrates that whister waves are excited in both the upstream and downstream regions. However, the whistler waves in these two regions have different frequencies and wavenumbers. It is shown that these characteristics can be explained by linear theory.

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