Detrapping Mechanism of Ultrarelativistic Electrons from an Oblique Shock Wave

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Multi-dimensional effects on electron motion in a magnetosonic shock wave propagating obliquely to an external magnetic field are studied by means of a two-dimensional (two space coordinates and three velocities), relativistic, electromagnetic particle code. The simulations demonstrate that after trapping and energization in the main pulse of the shock wave, some electrons are detrapped from it while maintaining their ultrarelativistic energies. The detrapping is caused by magnetic fluctuations propagating along the wave front. Furthermore, some of the detrapped electrons are found to be accelerated by the shock wave to much higher energies because they can enter and exit the shock wave several times as a result of their gyromotions.

Keywords: particle acceleration, collisionless shock, whistler wave, particle simulation

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1. Introduction

Particle simulations have revealed [1] that prompt electron acceleration to ultrarelativistic energies can occur in a magnetosonic shock wave propagating obliquely to an external magnetic field with $|\Omega_e|/\omega_{pe} \gtrsim 1$, where $\Omega_e (< 0)$ and ω_{pe} are the electron gyro and plasma frequencies, respectively. In such a wave, some electrons are reflected near the end of the main pulse of the shock wave, get trapped and are energized in the main pulse region. The electrons then oscillate in the main pulse region, and their kinetic energies take maxima near the position of the magnetic field peak. The energy is extremely high when the propagation speed of the shock wave v_{sh} is close to $c \cos\theta$, where c is the speed of light and θ is the propagation angle of the shock wave. A physical picture of this acceleration mechanism has been given [1], and a quantitative theory has been developed [2].

Simulations have also demonstrated that once electrons are trapped, they cannot readily escape from the wave and are trapped deep in the main pulse region, which indicates that the number of trapped electrons increases continually with time [2]. Reference [3] discussed the mechanism for the deep trapping, noting that in this acceleration mechanism, the electric field parallel to the magnetic field $E_\parallel$ and its integral along the magnetic field, $F = -\int E_\parallel \mathrm{d}s$, play essential roles. It was shown using theory and simulations that if $\partial F/\partial t > 0$ at particle positions, the parallel energies of the reflected electrons decrease, causing deep trapping. The reason for the increase in F was, however, not found.

Recently, the effects of trapped electrons on wave evolution have been studied with theory and simulations [4]. It has been shown that the trapped electrons strengthen $E_\parallel$ and F and that as a result, the magnitude of F increases with time. This result indicates that the electrons are trapped more deeply and accelerated to higher kinetic energies because of the electromagnetic fields they themselves produce.

Although the above extensive studies have examined electron trapping and acceleration by an oblique shock wave, the theory and simulations in these studies were one-dimensional, and multi-dimensional effects have not been investigated. In this paper, we study this topic with two-dimensional (two spatial coordinates and three velocities), particle simulations. We find that some electrons can be detrapped from the main pulse because of multi-dimensional effects. In Section 2, we briefly describe a physical picture of the detrapping and suggest that it is caused by magnetic fluctuations propagating along the wave front. In Section 3, this is confirmed by two-dimensional, particle simulations. It is furthermore demonstrated that after detrapping, some electrons can be accelerated by the mechanism studied in Refs. [5, 6]. The generation of magnetic fluctuations due to whistler waves is also discussed. Section 4 summarizes our work.

2. Multidimensional Effects on Electron Motion

Here we present a physical picture of electron detrapping due to multidimensional effects. We consider electron motion in a magnetosonic shock wave propagating in the x direction in an external magnetic field in the (x, z) plane, $\boldsymbol{B}_0 = B_0(\cos\theta, 0, \sin\theta)$. Using drift approximation, we write the velocity of a trapped electron as

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$$\boldsymbol{v} = v_{\parallel}\frac{\boldsymbol{B}}{B} + c\frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2}, \tag{1}$$

where we have neglected the ∇B-drift (and other unimportant drifts) and gyration velocity. The gyration velocity is unimportant in this trapping and acceleration mechanism [1].

Suppose that the wave is one-dimensional ($\partial/\partial y = \partial/\partial z = 0$). Then, B_x is constant, $B_x = B_{x0}$, and the relative velocity between the trapped electrons and the shock wave is

$$v_x \simeq (c\cos\theta - v_{sh})B_0/B, \tag{2}$$

where we have used $v_{\parallel} \simeq c$ for the trapped electrons. Equation (2) indicates that once the electrons get trapped, it is almost impossible for them to escape from the shock wave if v_{sh} is close to $c\cos\theta$.

Considering multi-dimensional effects, we write B_x, which can vary with time and space, as

$$B_x = B_{x0} + \delta B_x(x, y, z, t). \tag{3}$$

Then, v_x is given by

$$v_x \simeq (c\cos\theta - v_{sh})B_0/B + \delta v_x, \tag{4}$$

with

$$\delta v_x \simeq c(\delta B_x - \delta E_y)/B, \tag{5}$$

where δE_y is the fluctuation of E_y along the wave front. This predicts that if the particle moves in the region where $\delta v_x > 0$ (< 0), it is detrapped from the wave in the upstream (downstream) region because v_x can take large positive (negative) values.

Regarding the mechanism for generating δB_x and δE_y, whistler wave instabilities driven by the relative motion of trapped and untrapped electrons through Cherenkov resonance may be important. The resonance condition is

$$\boldsymbol{k}\cdot\boldsymbol{v}_t = \boldsymbol{k}\cdot\boldsymbol{v}_0 + \omega_0, \tag{6}$$

where $\boldsymbol{v}_t$ and $\boldsymbol{v}_0$ are the velocities of the trapped and untrapped electrons, respectively, and $\boldsymbol{k}$ and ω_0 are the wave vector and frequency of the whistler wave, respectively. The dispersion relation of whistler waves is

$$\omega_0^2 = \frac{\Omega_e^2}{\omega_{pe}^4}\frac{(\omega_{pi}^2 + c^2k^2)(\omega_{pi}^2 + c^2k^2\cos^2\theta_w)}{(1 + c^2k^2/\omega_{pe}^2)^2}, \tag{7}$$

where θ_w is the angle between $\boldsymbol{k}$ and the ambient magnetic field. If whistler waves with $k_y \neq 0$ or $k_z \neq 0$ are excited, δB_x and δE_y would be produced, which may detrap electrons. Assuming that the amplitude of the whistler wave is small, the relationship between δB_x and δE_y is

$$\delta E_y/\delta B_x \sim \omega/(ck\cos\theta_w), \tag{8}$$

which indicates that the magnitude of δE_y is much smaller than that of δB_x in a whistler wave propagating obliquely to the ambient magnetic field because $\omega/(ck) \ll 1$. We therefore consider the effects of δB_x on electron trapping.

3. Particle Simulations

In this section, using a two-dimensional (two spatial coordinates and three velocities), relativistic, electromagnetic particle code with full ion and electron dynamics, we study electron motion in an oblique shock wave. The system size of the simulation is $L_x \times L_y = 16384\Delta_g \times 128\Delta_g$, where Δ_g is the grid spacing; using this L_y, we can include the effects of whistler waves propagating along the y direction. The number of ions and electrons are $N_i = N_e \simeq 1.3\times 10^8$. The external magnetic field is $\boldsymbol{B}_0 = B_0(\cos\theta, 0, \sin\theta)$ with $\theta = 45°$. The ratio of the gyro and plasma frequencies of electrons is $|\Omega_e|/\omega_{pe} = 3.0$ in the upstream region. The mass ratio is $m_i/m_e = 100$. Light speed is $c/(\omega_{pe}\Delta_g) = 4.0$ and the electron and ion thermal velocities in the upstream region are $v_{Te}/(\omega_{pe}\Delta_g) = 0.5$ and $v_{Ti}/(\omega_{pe}\Delta_g) = 0.05$, respectively. The Alfvén speed is then $v_A/(\omega_{pe}\Delta_g) = 1.2$. We present the simulation results for a shock wave propagating in the x direction with a v_{sh} value that is 96 % of $c\cos\theta$.

Figure 1 shows electron phase space plots (x, γ) and profiles of B_z, where γ is the Lorentz factor, and the value of B_z is averaged over the y direction. At $\omega_{pe}t = 480$, some electrons are trapped and energized in the main pulse region, $835 < x/(c/\omega_{pe}) < 845$. At $\omega_{pe}t = 2280$, many energetic electron exist in a broader region, $2000 < x/(c/\omega_{pe}) < 2080$, from the upstream to the downstream regions. Some have been detrapped from the main pulse. At $\omega_{pe}t = 3920$, many detrapped and energetic electrons appear. Further, we reveal that the energies of detrapped electrons reach $\gamma \simeq 700$. Such electrons have undergone the enhanced acceleration by the shock wave after detrap-

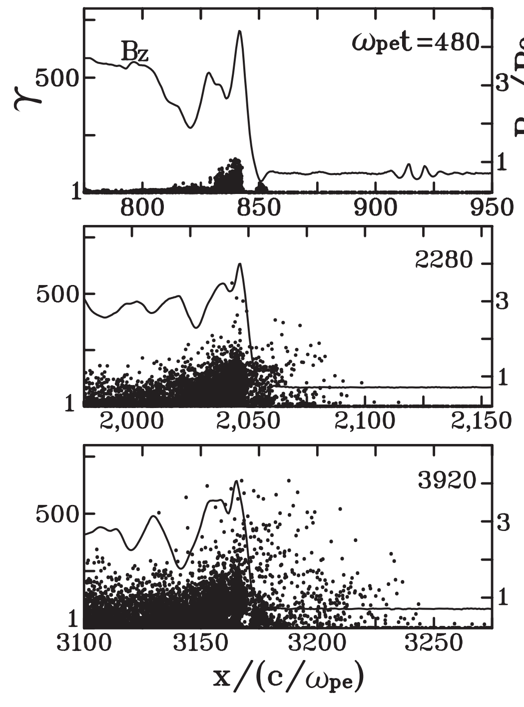

Fig. 1 Phase space plots (x, γ) of electrons and magnetic field profiles at $\omega_{pe}t = 480$, 2280, and 3920.

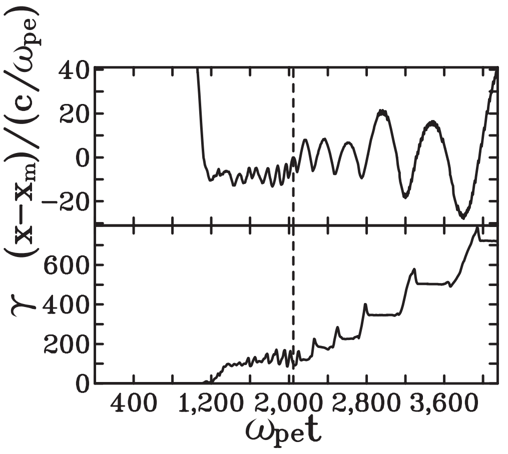

Fig. 2 Time variations in x and γ of an electron that underwent trapping, detrapping, and enhanced acceleration.

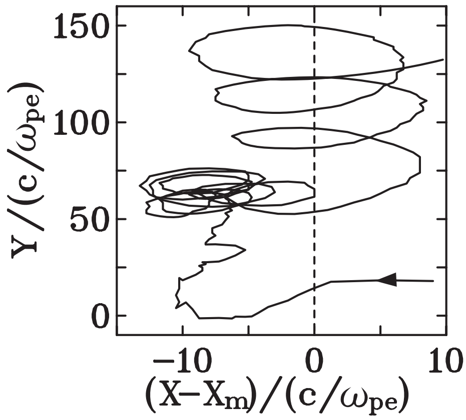

Fig. 3 Electron orbit in the (x, y) plain.

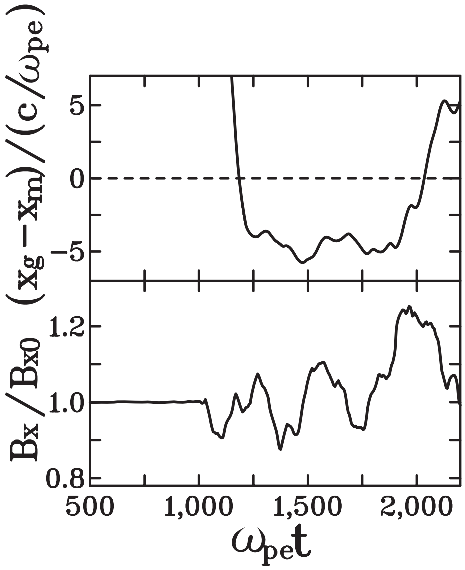

Fig. 4 Time variations in the position of the electron guiding center x_g and B_x at that position.

ping from the main pulse.

Figure 2 shows the time variations in x and γ of an electron that underwent trapping, detrapping, and enhanced acceleration. Here, x_m is the position where the magnetic field has its peak value; the upstream region is $x - x_m > 0$. The electron encounters the shock wave and is trapped at $\omega_{pe}t \simeq 1200$. It is energized to $\gamma \sim 100$ in the main pulse. The electron is then detrapped and exits to the upstream region at $\omega_{pe}t \simeq 2000$, maintaining its ultrarelativistic energy and $v_{\parallel} \simeq c$; this time is indicated by the dashed line in Fig. 2. Even after detrapping, the electron stays near the shock front because $v_{sh} \simeq c\cos\theta$, and it enters the shock region and returns to the upstream region as a result of its gyromotion. Because of this, the electron is accelerated to a much higher energy by the mechanism discussed in Refs. [5, 6]. That is, it is further accelerated by the transverse electric field in the shock wave since its gyromotion while it is in the shock is nearly parallel to the transverse electric field; the electron energy increases once in the gyroperiod. This process is repeated several times, producing an electron with extremely high energy $\gamma \simeq 700$. Figure 3 displays the electron orbit in the (x, y) plane. After detrapping, the electron clearly crosses the shock front several times as a result of its gyromotion with a large gyroradius.

We expect that in the case of the real mass ratio, $m_i/m_e = 1836$, enhanced acceleration of electrons would occur. It has been theoretically shown [2, 4] that the Lorentz factor of trapped electrons can be estimated as $\gamma \sim (\Omega_e^2/\omega_{pe}^2)/(1 - v_{sh}/c\cos\theta)$, which is independent of the value of m_i/m_e. This suggests that when $m_i/m_e = 1836$, electrons with ultrarelativistic energies, such as $\gamma \gtrsim 100$, would be produced in the main pulse if $v_{sh} \simeq c\cos\theta$. Suppose that the electrons can be detrapped from the main pulse while maintaining high γ values, as shown in Fig. 2. Then, the gyroradii of detrapped electrons, which are proportional to γ, can be of the order of the width of the shock transition region, and the electrons can enter and exit the shock wave as a result of their gyromotions.

We now study electron detrapping in detail. As described in Section 2, the change in B_x may be important in the detrapping mechanism. Figure 4 shows the time variations in the position of the electron guiding center x_g and B_x at that position. The electron gets trapped in the main pulse at $\omega_{pe}t \simeq 1200$ and is detrapped from it to the upstream region at $\omega_{pe}t \simeq 2100$. We see that before detrapping, the electron undergoes a significant increase in B_x; the value of B_x at $\omega_{pe}t \simeq 2000$ is $B_x \simeq 1.2B_{x0}$. The increase in B_x causes an increase in v_x, which leads to the detrapping. We also confirm that theory (4) can quantitatively explain the simulation; substituting the observed value of δB_x in Eq. (4) yields $v_x \simeq 0.04c$, which agrees well with the simulation value of v_x averaged over the period from $\omega_{pe}t = 1900$ to 2100, $v_x \simeq 0.05c$.

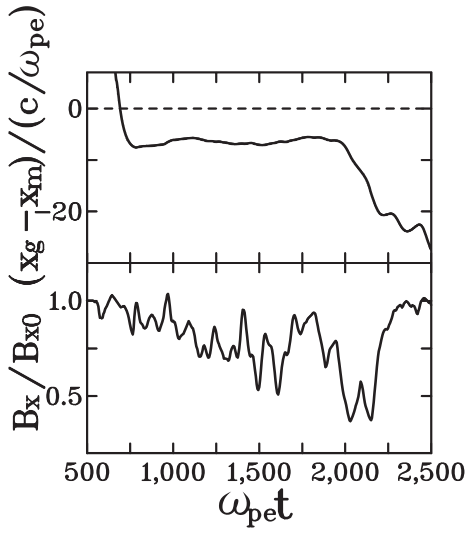

Fig. 5 Time variations in x_g and B_x for an electron that is detrapped from the main pulse to the downstream region.

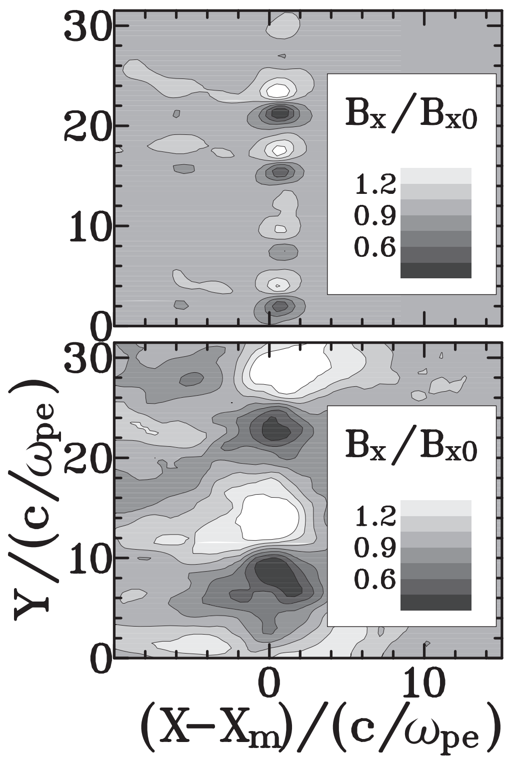

Fig. 6 Contour maps of B_x in the (x, y) plane at $\omega_{pe}t = 200$ and 1160.

Figure 5 shows the time variations in the position x_g and B_x for an electron that is detrapped from the pulse to the downstream region. The electron undergoes a decrease in B_x unlike the particle shown in Fig. 4. Therefore, v_x decreases, and the electron moves backward relative to the main pulse.

Next we discuss the structure of B_x. Figure 6 shows a contour map of B_x in the (x, y) plane at $\omega_{pe}t = 200$ and 1160. Large-amplitude fluctuations of B_x are clearly produced in the main pulse region. These are due to whistler waves excited by trapped electrons. At $\omega_{pe}t = 200$ (upper panel), the dominant mode has its wavelength along the y direction, $\lambda \simeq 7(c/\omega_{pe})$. This is consistent with the theory; substituting the observed values of the velocities of trapped and passing electrons and of the magnetic fields at $x \simeq x_m$ at this time in Eqs. (6) and (7), we have $\lambda \sim 7(c/\omega_{pe})$. At $\omega_{pe}t = 1160$ (lower panel), the characteristic wavelength of δB_x is longer, and its magnitude is greater, which can enhance electron detrapping. (It is likely that the evolution of δB_x is caused by nonlinear interaction between the whistler waves and current filaments. Detailed study of this is, however, beyond the scope of a brief paper.)

4. Summary

A magnetosonic shock wave propagating obliquely to an external magnetic field can trap electrons and accelerate them to ultrarelativistic energies if v_{sh} is close to $c \cos\theta$. Assuming that the wave is one-dimensional, the electrons cannot readily escape from it and are deeply trapped in the main pulse region. We studied multi-dimensional effects on electron motion using two-dimensional (two space coordinates and three velocities), particle simulations. The simulations show that some electrons are detrapped from the main pulse after trapping and energization in the main pulse. The detrapping is caused by magnetic fluctuations propagating along the wave front. Some detrapped electrons are further accelerated to much higher energies by the shock wave because they can enter the shock wave several times as a result of their gyromotions.

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