Collisionless Dynamical Friction and Relaxation in a Simple Drift Wave-Zonal Flow Turbulence

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We present a study of the role of zonal flows in relaxation and transport in a reduced model of collisionless ITG turbulence. A fundamentally new constituent in the relaxation dynamics is revealed, namely that ion and electron guiding center motion togather necessitate a radial flux of polarization charge, which in turn exerts a *dynamical friction* on phase space density evolution. This effect then enters the evolution of $\langle \delta f^2 \rangle$ and the transport dynamics, as described by a Lenard-Balescu type equation. The underlying physics is similar to that which follows from conservation of potential vorticity, albeit now for a phase space fluid, and is not simple shearing or wave packet modulation. Consequences for zonal flow momentum balance are discussed.

Keywords: drift wave, zonal flow, granulation, polarization charge, collisionless dynamical friction, momentum theorem

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1. Introduction

Zonal flow (ZF) physics still remains an important issue in magnetic fusion, since ZFs reduce turbulent transport by self-regulating process of shearing and thus enhance the performance of fusion devices. [1, 2]. Recently Charney-Drazin momentum theorems [3, 4], which account for momentum balance between flows and waves in potential vorticity dynamics, were extended to fusion plasmas [5]. One can also extend the theorems to kinetic systems, based on the similarity between potential vorticity dynamics and phase space density dynamics. The detailed derivation and discussion of the kinetic Charney-Drazin theorems are beyond the scope of this work and will be discussed in the other publications. Here we point out that the kinetic Charney-Drazin theorems imply the importance of ZFs in phase space dynamics; the time evolution of δf^2 is closely tied to the ZF growth.

The extension of Charnery-Drazin theorems to kinetic systems poses the problem on the resonance. In non-resonant limit, the theorems still hold and one can interpret the result as the momentum balance between flows and *waves*. On the other hand, when one has a strong resonance between particles and waves, one cannot interpret Charney-Drazin theorems as flow-wave momentum balance; pseudomomentum may not be well-defined with resonance. In strong resonant limit, waves would grow until the effect of particle trapping becomes important. In this situation, localized *structures* in phase space, such as BGK mode [6], granulations [7], holes [8] and etc, would emerge and alter the dynamics of ZFs and transport processes. The existence of localized structures effectively act as a dynamical friction in phase space, as seen in the Lenard-Balescu equation.

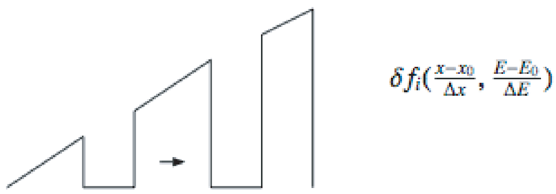

Fig. 1 Growth of a hole

In this work we will discuss the dynamics of phase space structures and their interaction with ZFs. This work can be divided into two parts depending on two numbers characterizing systems; i) the Kubo number, $K \equiv \tau_{ac}/\tau_{bounce}$, where τ_{ac} is a life time of a packet and τ_{bounce} is a bounce time in a potential trough, and ii) the Chirikov parameter, $S \equiv \Delta v/\Delta v_{ph}$, where Δv is a width of island and Δv_{ph} is the spacing between resonances. In the first part, we treat a simple single structure limit, i.e. $K \geq 1$ and $S \ll 1$, and show that the growth of structures cannot avoid ZF coupling. The connection to the momentum theorems will be discussed as well. In the second part, we treat multi-structure limit, i.e. $K \leq 1$ and $S \geq 1$ discuss the interaction between multi-structures and ZFs and calculate the resultant transport processes.

2. Single Structure in Phase Space and Zonal Flow

As the first topic, we consider a localized *single* structure in phase space with $K \geq 1$ and $S \ll 1$. In these parameter regime, coherent structures or holes [8], due to self-trapping [6] or binding [9, 10], emerge. These structures grow when propagating up-gradient, since the total

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f must be conserved along a trajectory (Fig. 1). The time evolution of holes [8] is

$$\partial_t \int \mathrm{d}E\delta f_i^2 = -2\langle \tilde{V}_r \tilde{n}_i \rangle \frac{\partial \langle f \rangle}{\partial x}|_0 \quad (1)$$

where the subscript 0 denotes the location of a hole, (x_0, E_0). The growth of a hole is tied to a particle flux of ions and a gradient at the location of a hole. Locally, by having a particle flux, one can reduce the amount of density at the point, which leads to increasing the depth of the hole and hence to growth of the hole, since *total* phase space density is conserved.

ZF coupling appears in a particle flux of ions. Since the net dipole $\int dx \sum_\alpha q_\alpha n_\alpha(x) x$ is invariant [8], one can replace a particle flux of ions by the sum of particle flux of electrons and polarization charges. Polarization charges introduces fluid vorticity into a system and a polarization charge flux can be regarded as a vorticity flux. From these observations, one can rewrite the structure growth as

$$\partial_t \left\{ \int \mathrm{d}E \frac{\delta f_i^2}{2\langle f \rangle'|_0} + \langle V_\theta \rangle \right\} = -\nu \langle V_\theta \rangle - \langle \tilde{V}_r \tilde{n}_e \rangle. \quad (2)$$

At a stationary state, one has $\langle V_\theta \rangle = -\langle \tilde{V}_r \tilde{n}_e \rangle / \nu$. Localized holes scatter off electrons and can pump ZF growth and their dynamics cannot avoid the coupling to ZFs.

It is interesting to compare the result with Charney-Drazin momentum theorems for Hasewaga-Wakatani system [5]

$$\partial_t \left\{ \frac{\langle \delta q^2 \rangle}{\langle q \rangle'} + \langle V_\theta \rangle \right\} = -\nu \langle V_\theta \rangle - \langle \tilde{V}_r \tilde{n} \rangle - \delta_t \langle \tilde{q}^2 \rangle / \langle q \rangle'. \quad (3)$$

$q = n - \nabla^2 \phi$, $\langle \delta q^2 \rangle / \langle q \rangle'$ and $\delta_t \langle \delta q^2 \rangle \equiv \partial_r \langle \tilde{V}_r \delta q^2 \rangle + D_0 \langle (\nabla \delta q)^2 \rangle$ are a potential vorticity, wave activity density and a local potential enstrophy increment, respectively. A detailed discussion on the theorems is found in [5]. One of the points we emphasize here is that wave activity density is equal to psedomomentum of quasiparticles and hence the theorems describe the momentum freezing between flows and *waves*. Here we emphasize the clear correspondence between single structure growth, Eq. 2 and momentum theorems, Eq. 3. Localized structures, as well as waves, can interact with ZFs.

3. Multi-Structures in Phase Space and Zonal Flow

We hitherto identified ZFs as a fundamental constituent in phase space structure dynamics in single structure limit. An interesting and important limit is multi-structure limit, $K \leq 1$ and $S \geq 1$, i.e. when multi-structures such as clump or granulations [7] exist and interact with each other in phase space. The interaction between phase space structures would cause relaxation and transport in the system.

As an example for the interaction of multi-structures and the associated relaxation and tranpsort, we treat a trapped ion induced ITG turbulence. In a trapped ion mode, the resonance between waves and the magnetic precession of trapped particles destabilizes a mode. One of the features of the resonance is the long coherence time between waves and trapped particles. In another word, the Kubo number approaches unity. In this situation, the evolution of structure and the failure of quasilinear theory are both likely, hence one must include the effect of phase space structures in calculating a transport.

To describe the dynamics of phase space turbulence, one can employ Dupree-Lenard-Balescu type equations;

$$\partial_t \langle \delta h(1) \delta h(2) \rangle + T(1,2) = P(1,2) \quad (4)$$

$$\partial_t \langle f \rangle = -\partial_r [-D \langle f \rangle' + F \langle f \rangle] \quad (5)$$

The first equation describes the time evolution of a two point correlation fucntion. Here δh is a non-adiabatic part in δf, $T(1,2)$ is a triplet term and $P(1,2)$ is a production term. The production, $P(1,2)$, is directly related to relaxation and transport; the physics of this term is described extensively later. The triplet term, $T(1,2)$, is effectively the lifetime of correlations and given by a relative dispersion due to the difference in drift and shear velocities at different points, a nonlinear scattering and a collisional dissipation. The detailed expression of $T(1,2)$ and $P(1,2)$ without the effect of ZFs is given in [11]. The second equation has the form of Fokker-Planck equations and describes transport process. The flux consists of two parts: one is a diffusion D and the other is a dynamical friction F. The existence of structures induces a dynamical friction in phase space. Note that a conventional quasilinear theory only predicts a diffusive part in a transport equation. An explicit form of D and F comes from the production term $P(1,2)$. A novel effect is ZF coupling via dynamical friction, F, as explained later.

In the presence of multistructures, production $P(1,2)$ consists of two parts, $P = P^c + \tilde{P}$; P^c is a coherent part and $\tilde{P}$ is an incoherent part. A coherent part P^c comes from a coherent response, as the name stands, and represents quasilinear production. An incoherent part $\tilde{P}$, on which we focus extensively here, represents the effect of structures. The explicit expression of an incoherent part is

$$\tilde{P}(1,2) = -\sum_{k\omega} (\omega - \omega_*^i(1)) \mathrm{Im}\epsilon(2) \hat{\epsilon}^{-1}(1) \hat{\epsilon}^{*-1}(2) \times \left\langle \frac{\widetilde{\delta n}(1)}{n_0} \widetilde{\delta h}^*(2) \right\rangle_{k\omega} \langle f_i(1) \rangle e^{i\mathbf{k}\cdot\mathbf{x}_-} + (1 \leftrightarrow 2), \quad (6)$$

where $(1 \leftrightarrow 2)$ denotes the term with arguments 1 and 2 replaced. As one can see, an incoherent part $\tilde{P}(1,2)$ is proportional to $\mathrm{Im}\epsilon$. The physical interpretation of this term is an effective 'wake' in phase space and hence leads to a dynamical friction.

$\mathrm{Im}\epsilon = \mathrm{Im}\epsilon_e + \mathrm{Im}\epsilon_i + \mathrm{Im}\epsilon_{pol}$ originates from any dissipation in plasmas, either collisional or collisionless. For example, in [11] and our model, $\mathrm{Im}\epsilon_e$ comes from collisional dissipation in electron ($\mathrm{Im}\epsilon_e \propto \nu_c$) and $\mathrm{Im}\epsilon_i$ originates from collisionless damping via wave-particle resonance. A novel effect enters via a polarization charge coupling $\mathrm{Im}\epsilon_{pol}$. By including a polarization charge contribution and allowing an envelope coupling, i.e. modulation in amplitude, one can have $\mathrm{Im}\epsilon_{pol} \propto k_r\partial_r$. In fact, this term represents ZF coupling in phase space dynamics, as one can show this term is a 'disguised' Reynolds forcing. This result should not be surprising, because one can expect from a single structure analogy, Eq. 2. Note that ZF coupling occurs due to a polarization charge flux, via GK Poisson equation.

The full expression of $P(1,2) = P^c(1,2) + \tilde{P}(1,2)$ is

$$\begin{aligned} P(1,2) = &-\sum_{k\omega}(\omega-\omega_*^i(1))\mathrm{Im}\epsilon_e(2)\hat{\epsilon}^{-1}(1)\hat{\epsilon}^{*-1}(2) \\ &\times\left\langle\frac{\widetilde{\delta n}(1)}{n_0}\widetilde{\delta h}^*(2)\right\rangle_{k\omega}\langle f_i(1)\rangle e^{i\mathbf{k}\cdot\mathbf{x}_-} \\ &+\sum_{k\omega}(\omega-\omega_*^i(1))\left(2k_r\frac{\partial}{\partial r_2}\right)\hat{\epsilon}^{-1}(1)\hat{\epsilon}^{*-1}(2) \\ &\left\langle\frac{\widetilde{\delta n}(1)}{n_0}\widetilde{\delta h}^*(2)\right\rangle_{k\omega}\langle f_i(1)\rangle e^{i\mathbf{k}\cdot\mathbf{x}_-} + (1\leftrightarrow 2). \end{aligned} \tag{7}$$

A coherent part and incoherent drag on ions cancele with each other. This is because we treated linear response as a resonant delta function. In this limit, ions cannot relax alone for the case of a localized structures. This is because in the 1D problem, to conserve energy and momentum during effective collisions, particles cannot change their velocity. The explicit calculation of the cancellation is given in [11]. Note that the cancellation of a coherent and incoherent part is rather special, and they do not cancel each other in general; for example, when one retains the effect of the resonance broadening in the response function, there exist a finite contribution due to inelasticity. In this paper, however, we treat a simple delta function response and the effect of such inelasticity is not considered. The first term is from collisional dissipation on electrons and the second term represents ZF coupling via a polarization charge flux. One would explain the result based on the net charge balance: Ions are dressed by the cloud of electrons and polarization charges in order to satisfy quasi-neutrality, hence any dissipation on the cloud, i.e. collisions on electrons and ZF couplings, must lead to a friction on ions. The friction due to ZF coupling is understood as a *collisionless* friction.

The triplet term $T(1,2)$ is, after a lengthy algebra,

$$\begin{aligned} T(1,2) = &(v_d(E_1) - v_d(E_2) + \langle V_y\rangle(x_1) - \langle V_y\rangle(x_2)) \\ &\times\frac{\partial}{\partial y_-}\langle\delta h(1)\delta h(2)\rangle - \left(D_{x_-}\frac{\partial^2}{\partial x_-^2}\right. \\ &\left.+2D_{x_-y_-}\frac{\partial^2}{\partial x_-\partial y_-} + D_{y_-}\frac{\partial^2}{\partial y_-^2}\right)\langle\delta h(1)\delta h(2)\rangle \\ &-\langle\delta h(1)C(\delta h(2))\rangle + (1\leftrightarrow 2). \end{aligned} \tag{8}$$

The first term represents the effect of relative dispersion due to the difference in a drift and shear velocity. Shear flows introduce a new time scale, i.e. a shear enhanced dispersion rate. The second term originates from scattering due to fluctuations and

$$D_{x_-} \sim \mathrm{Re}g_{k\omega}\left|\frac{q\tilde{\phi}}{T_i}\right|^2_{k\omega}(1-\cos k_x x_-), \tag{9}$$

where $g_{k\omega} = i/(\omega - \bar{\omega}_D\bar{E} - k_\theta\langle\mathbf{v}_{E\times B}\rangle + i\tau_c^{-1})$. D_{x_-} consists of two parts; a quasilinear type diffusion part, the first term in the parenthesis, and a relative diffusion part, the second term in the parenthesis. In the large separation limit, the relative diffusion part oscillates rapidly and cancels out on average; Quasilinear type diffusion dominates in this limit. In the small separation limit, $D_{x_-} \sim \mathrm{Re}g_{k\omega}|\tilde{\phi}|^2_{k\omega}k_x^2x_-^2$ and the diffusion coefficient depends on a relative separation, not only on an amplitude. $D_{x_-y_-}$ and D_{y_-} have a similar expression. The third term is a collisional cut-off.

We showed the form of $P(1,2)$ and $T(1,2)$ and discussed their physics; the full expression for the evolution of a 2 point correlation function, $\langle\delta h(1)\delta h(2)\rangle$, in the limit of $1\to 2$, is

$$\begin{aligned} &\partial_t\langle\delta h^2\rangle + T\langle\delta h^2\rangle \\ &= \sum_{k\omega}\left\{-(\omega-\omega_*^i)\frac{\mathrm{Im}\epsilon_e}{|\epsilon|^2}\langle f\rangle\left\langle\frac{\widetilde{\delta n}}{n_0}\widetilde{\delta h}^*\right\rangle_{k\omega}\right. \\ &\left.+\frac{2(\omega-\omega_*^i)k_r}{|\epsilon|^2}\langle f\rangle\left\langle\frac{\widetilde{\delta n}}{n_0}\partial_r\widetilde{\delta h}^*\right\rangle_{k\omega}\right\}. \end{aligned} \tag{10}$$

Again, the evolution of structures cannot avoid ZF coupling; in this case, the coupling is between ZFs and *multi*-structures, as compared to *single*-structures.

Relaxation and transport associated with structure dynamics is described by Eq. 5. The result for production term $P(1,2)$ yields a relevant flux in phase space $J(r) \equiv -D\partial_r\langle f\rangle + F\langle f\rangle$;

$$\begin{aligned} J(r) = \sum_{k\omega}&\left\{-k_\theta\frac{\mathrm{Im}\epsilon_e}{|\epsilon|^2}\left(\frac{\widetilde{\delta n}}{n_0}\right)_{k\omega}\widetilde{\delta h}_{-k-\omega}\right. \\ &\left.+\frac{2k_\theta k_r}{|\epsilon|^2}\left(\frac{\widetilde{\delta n}}{n_0}\right)_{k\omega}\partial_r\widetilde{\delta h}_{-k-\omega}\right\}. \end{aligned} \tag{11}$$

A relevant flux, i.e. flux of particle and energy, is obtained by taking the velocity moment. Two points are worth remarking. First of all, the transport process is drastically different from quasilinear description. Both terms originate from a dynamical friction; a diffusive part, which also shows up in quasilinear theory, cancelled with the contribution from $\mathrm{Im}\epsilon_i$ in a dynamical friction. The second point is that ZFs affect transport differently from usual fashions; the effect of ZFs shows up explicitly as a relaxation driver in the presence of structures, whereas in a usual fashion the effect of ZFs shows up more implicitly, i.e. via a decoupling of cross phase and suppresion of amplitude.

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