Helical Pitch Parameter Dependency of High Beta Equilibrium of Helical Plasmas

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The helical pitch parameter ($\gamma$) dependency of equilibrium and the stability of the high beta plasma in the LHD type magnetic configuration is studied numerically. It is confirmed that the small $\gamma$ configurations are favorable for the LHD-type fusion reactors in the point of robustness of high beta equilibrium, compatibility of easy ignition and high output power of core plasma, in addition to a sufficient space for blankets.

Keywords: LHD, high beta, stability, equilibrium

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1. Introduction

LHD type magnetic configuration is produced by continuous helical coil systems. The helical pitch parameter $\gamma \equiv a_c \cdot k$ characterizes the magnetic configuration of helical systems, where $a_c$ and $k$ are the current center radius and the wave number of the helical coils, respectively.

The LHD experimental results of achieving average beta value 5% without the beta collapse suggests the possibility of the helical equilibriums with ultrahigh beta MHD stable core plasmas. The existence of the MHD stable high beta core plasma lead the way for the realization of economic fusion power systems.

LHD-type reactors require a large major radius ($R_c$) to attain the self-ignition condition with a sufficient space for blankets. To reduce the major radius size, small $\gamma$ configuration is considered [1]. On the other hand, small $\gamma$ configuration requires relatively high current density ($J_c$) for helical coils and the volume of the last closed magnetic flux surface ($V_{lcfs}$) of vacuum field become small (Fig. 1).

The helical pitch parameter dependency of equilibrium and the stability of the high beta plasma is studied numerically. For this purpose, we have developed a new numerical scheme based on Biot-Savart law. For simplicity, we describe the method for the straight helical systems (Fig. 2), from here.

2. Numerical Method to Solve Helical Equilibrium

MHD equations $\nabla P = J \times B$ are possible to be solved without approximations, under the rotating helical coordinate system ($X, Y, \zeta$) [2], which rotates in synchronization with helical coils. $\zeta$ is the axial coordinate. Arbitrary functions $P(\Psi)$ and $I(\Psi)$ are introduced and plasma current is expressed as follows,

$$J = \frac{1}{\mu_0} I'(\Psi) B + P'(\Psi) \begin{pmatrix} -kY \\ kX \\ 1 \end{pmatrix},$$

where $k$ is the axial wavenumber of helical coils, $\Psi$ is the flux function and $P(\Psi)$ is the plasma pressure distribution. The first term of the right-hand side of eq. (1) is the driven current term, which is independent to the plasma pressure. $X$ and $Y$ components of the second term of eq. (1) are the diamagnetic currents, which produce magnetic field on the inside of the plasma column, mainly. The $\zeta$ components of the second term of eq. (1) is the bootstrap currents, which produce magnetic field on the outside of the plasma column, mainly. Magnetic field $B$, vector potential $A$ and the magnetic flux function $\Psi$ can be calculated by Biot-Savart...
law as follows,

\[
B(r) = \frac{\mu_0}{4\pi} \int d^3r' \left\{ \frac{(r' - r) \times J_s(r')}{|r' - r|^3} \right\},
\]

(2)

\[
A(r) = \frac{\mu_0}{4\pi} \int d^3r' \left\{ \frac{J_s(r')}{|r' - r|} - \frac{J_s(r)}{|r'|} \right\},
\]

(3)

\[
\Psi(r) = A(r) + k (XAY - YAX),
\]

(4)

\[
J_s(r') \equiv J(r') + J_c(r').
\]

(5)

Plasma equilibrium is reduced to the following relation,

\[
\Psi(r) = \Psi^0(r) + \Psi^c(r),
\]

(6)

where \(\Psi^0(r)\) is the flux function produced by the plasma current whose profile is determined by the “total” flux function \(\Psi(r)\) through the eq. (1). \(\Psi^c(r)\) is the flux function produced by the helical coil currents. Plasma equilibrium computations are reduced to solve eq. (6) self-consistently for \(\Psi\). The relaxation scheme is possible to solve eq. (6). In the following we have assumed that the driven current is zero (\(I(\Psi) = 0\)) and pressure profile is one of a flat top type, using the value of the flux function at separatrix, \(\Psi_s\).

\[
P(\Psi) = \beta_{ax} \frac{B_{ax}^2}{2\mu_0} \exp \left\{ -D \left( \frac{\Psi}{\Psi_s} \right)^2 \right\}, \quad D = 7
\]

(7)

3. MHD Stability and its Helical Pitch Parameter Dependency

Plasma stability is determined by the MHD potential energy [3],

\[
W = \int dV \left( \frac{3}{2} P + \frac{1}{2\mu_0} B^2 \right) \equiv W_T + W_B.
\]

(8)

\(W\) minimum configuration is an MHD stable equilibrium. When \(\delta W < 0\) \(\delta W = W - W_0, W_0 \equiv \int dV \frac{1}{2\mu_0} B_{ext}^2\), transition to vacuum state is energetically prohibited. Beta collapse of core plasma does not occur.

When the bootstrap current cancels, partially, the magnetic field outside the helical coils, the magnetic field energy \(W_B\) can be reduced extensively, as shown in Fig. 3. The variation of potential energy become negative (\(\delta W < 0\)) by plasma sustainment and bootstrap transition to high beta equilibrium occur.

Helical pitch parameter dependency of high beta equilibrium is summarized in Fig. 4.

When \(\gamma\) is small, the size of the magnetic surface become small, since the axial magnetic field decreased relatively. However, the ability of MHD stability increases since the role of bootstrap current is increased. This ten-
Fig. 5 (a) Pressure, bootstrap current, flux function and magnetic field intensity along X coordinate. (b) Specific volume $U$ and rotational transform $\iota/2\pi$. Pressure profile is superimposed in this graph.

The value of the MHD potential energy, $W$, depend on the functional form of equilibrium pressure distribution $P(\Psi)$. The core plasma has a possibility of transition to an equilibrium distribution function $P(\Psi)$, which minimize the potential energy $W$, at the specified value of the stored thermal energy $W_T$. Computations of the MHD potential energy, $W$, has predicted that peaked pressure profile,

$$P(\Psi) = \beta_{ax} \frac{B_{ax}^2}{2\mu_0} \exp \left\{ -D \left( \frac{\Psi}{\Psi_s} \right) \right\}, \quad D = 7,$$

has a possibility of collapsing to the flat top type pressure profile given by eq. (7). However, the ultrahigh beta equilibrium with the flat top type pressure profile given by eq. (7) will be almost minimize the MHD potential energy $W$, because local linear stability criterion [4] is satisfied by the combination of the high magnetic shear at peripheral region and the magnetic well at inner region of the magnetic surface, as shown in Fig. 5.

Figure 5 shows profiles of equilibrium quantities along the long axis of magnetic surface for the case of $\beta_{ax} = 176\%$. The magnetic surface volume $V_{lcfs}$ grows by 2.4 times compared with the magnetic surface volume at the vacuum state.

4. Summary

Summary is as follows.

- Small helical pitch parameter configuration has a lot of advantages for the LHD-type fusion reactors.
- The heating systems for the ignition become small because the vacuum magnetic surface volume is small.
- After the ignition, fusion output power is large due to the large volume of the high beta core plasma.
- Core plasma is robust against MHD perturbations.
- The ultrahigh beta equilibrium has enough intensity in the magnetic field for the alpha-particle confinement.
- The space for the blanket is large.

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