Nonlinear Mode Couplings in a Cylindrical Magnetized Plasma

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Nonlinear mode couplings in plasma turbulence generate mesoscale global structures, such as zonal flows and streamers. A streamer is a poloidally localized structure that is radially elongated. In the Large Mirror Device-Upgrade, a streamer structure was observed in the ion saturation-current fluctuation measured with a 64-channel poloidal Langmuir probe array. The bunching of waves was poloidally localized and rotated slowly in the ion diamagnetic direction. The frequency and poloidal mode number of its envelope corresponded to one particular mode, which was strongly coupled with many other modes. By calculating the radial biphase profile of three pronounced modes, including the particular mode, the streamer structure was confirmed to be radially elongated.

Keywords: linear plasma, turbulence, nonlinear coupling, streamer, bispectral analysis

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1. Introduction

Turbulence is a state of plasma in which nonlinear interactions occur. Turbulence has recently become important for understanding anomalous transports in magnetized plasmas. Nonlinear couplings in drift wave turbulence should generate mesoscale structures, such as zonal flows and streamers, which strongly influence anomalous transport [1]. Zonal flow is a radially localized, poloidally elongated structure that suppresses radial transport. In contrast, streamers are poloidally localized, radially elongated structures that enhance radial transport [2–5]. Streamer structure lives longer than the characteristic turbulence correlation time and is a global structure generated by nonlinear phase locking of several modes.

Zonal flows have been studied theoretically [6] and experimentally [7–11]. Zonal flow was first observed by a heavy ion beam probe in the Compact Helical System [8]. In contrast, there are very few observations of streamers. The streamer was first observed in linear plasma in the Large Mirror Device-Upgrade (LMD-U) [12]. Ref. [12] reported poloidally localized structure or bunching of waves in turbulence. The structure was measured by a poloidal probe array fixed at a constant measuring radius of 40 mm. In addition, two results indicated that the poloidally localized structure was elongated in the radial direction. First, the poloidally localized structure was simultaneously measured by two poloidal probe arrays with measuring radii of 40 mm and 20 mm, respectively. The structure appeared at almost the same poloidal angle at both measuring radii. Second, the mode components generating mesoscale activity were shown to have radially elongated structures by calculating the cross-spectra between the data from a two-dimensionally (2D) movable probe and a reference probe. Although these two results suggest that the poloidally localized structure is radially elongated, they are not sufficient proof of it. This paper reports phase locking of the modes composing poloidally localized structure as observed inside plasma using a bispectral analysis. The structure was confirmed to be radially elongated by calculating each phase difference of the phase locking inside the plasma. A brief explanation of bispectral analysis is given in the following section. The mesoscale structure observed in the LMD-U is introduced in Section 3. The radial elongation of this structure is confirmed in Section 4. Section 5 presents a summary.

2. Bispectral Analysis

Bispectral analysis [13] examines the relationship among three modes that satisfy the matching condition. Such modes should exist independently and satisfy the matching condition randomly, or should exist depending on the necessity, for reasons such as nonlinear mode couplings. Bispectral analysis distinguishes these two cases. With a single point measurement, the matching condition is usually considered for the frequency, f, and it becomes

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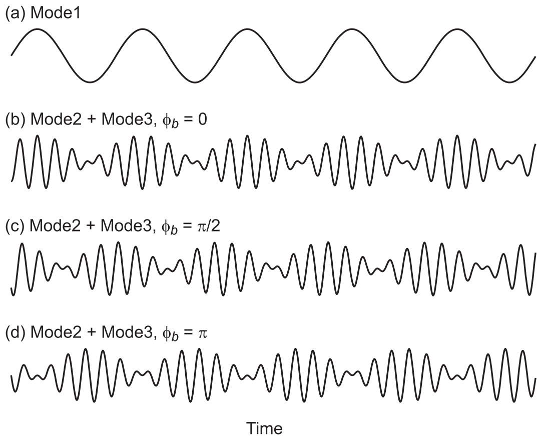

Fig. 1 Drawing showing ideal cases of the phase relationship between three coupling modes, 1, 2, and 3. (a) Mode 1 (−1.2 kHz) and (b–d) the envelope generated by modes 2 (7.8 kHz) and 3 (6.6 kHz). Biphases of the three coupling modes are (b) 0, (c) $\pi/2$, and (d) π.

$f_1 + f_2 = f_3$, where the subscripts indicate the three modes. When the Fourier transformed expressions, $Z(f)$, of the three modes are $Z_1 = Z(f_1)$, $Z_2 = Z(f_2)$, and $Z_3 = Z(f_3)$, the bispectrum, B, of the three modes is expressed as $B = \langle Z_1\, Z_2\, Z_3^* \rangle$. When the three modes exist independently, the absolute value, $|B|$, converges to zero. When the phases of the three modes are connected by a certain relationship, $|B|$, becomes finite. The bicoherence, b, is a normalized value of B, and the biphase, ϕ_b, indicates the relationship among the phases of the three modes. They are expressed as

$$b^2 = \frac{|B|^2}{\langle |Z_1\, Z_2|^2 \rangle \langle |Z_3|^2 \rangle}, \tag{1}$$

$$\phi_b = \tan^{-1} \frac{\mathrm{Im}(B)}{\mathrm{Re}(B)}. \tag{2}$$

Figure 1 shows examples of the phase relationships between three ideally coupled modes, 1, 2, and 3, with frequencies $(f_1, f_2, f_3) = (-1.2\,\mathrm{kHz}, 7.8\,\mathrm{kHz}, 6.6\,\mathrm{kHz})$. This combination of frequencies will be discussed later when analyzing the experimental data in the following section. In this example, the amplitudes of modes 2 and 3 are equal, and f_2 and f_3 are close, so that the two modes form a bunching of waves, the beat frequency of which is the same as f_1. Comparing mode 1 with the envelope of the bunching waves show that the phase is locked, since nonlinear coupling exists among the three modes. The biphase, ϕ_b, directly indicates the phase difference between mode 1 and the envelope. Figures 1 (b), 1 (c), and 1 (d) show the cases when the biphases of the three coupling modes are 0, $\pi/2$, and π, respectively. For $\phi_b = 0$, the amplitude of the envelope is maximum when mode 1 is maximum. For $\phi_b = \pi/2$, the envelope becomes large before mode 1 approaches the maximum. For $\phi_b = \pi$, the envelope is maximum when mode 1 is minimum. On the basis of this fact, the spatial phase structure of the bunching waves inside the plasma can be investigated through a certain reference mode.

Bispectral analysis can also be extended to multiple dimensions. For example, with a poloidal probe array measurement, the matching condition becomes $f_1 + f_2 = f_3$ and $m_1 + m_2 = m_3$, where m is the poloidal mode number. When the 2D Fourier transformed expressions, $Z(f, m)$, of the three modes are $Z_1 = Z(f_1, m_1)$, $Z_2 = Z(f_2, m_2)$, and $Z_3 = Z(f_3, m_3)$, the bispectrum, B, of the three modes is also $B = \langle Z_1\, Z_2\, Z_3^* \rangle$.

3. Poloidally Localized Structure

Poloidally localized structure in a linear plasma was observed in the LMD-U plasma [12]. The LMD-U vacuum vessel has an axial length of 3.74 m [14]. The plasma is generated by radio-frequency waves (7 MHz/3 kW) inside a quartz tube with an inner diameter of 95 mm. When the argon pressure inside the tube was 0.2 Pa and the axial magnetic field inside the vacuum vessel was 0.09 T, the peak electron density of the plasma was $8 \times 10^{18}\,\mathrm{m}^{-3}$ and the electron temperature was about 3 ± 0.5 eV inside the plasma.

With this discharge condition, poloidally localized structures in the ion saturation-current fluctuation waveforms at radii of 40 mm and 20 mm were observed by two poloidal probe arrays (64 channels [15] and 24 channels, respectively). With simultaneous measurement by these two probe arrays, the observed localized regions appeared at almost the same poloidal angles. Figures 2 (a) and 2 (c) show the spatiotemporal waveforms of the ion saturation-current fluctuations measured simultaneously with the poloidal probe arrays. Figures 2 (b) and 2 (d) are the temporal waveforms of Figs. 2 (a) and 2 (c), respectively, at the zero poloidal angle positions. Both waveforms consist of bunching waves with beat frequencies of −1.2 kHz and slow sinusoidal waves with $m = 1$ and $f = -1.2\,\mathrm{kHz}$, where negative frequency corresponds to propagation in the ion diamagnetic direction. The bunching waves are poloidally localized; i.e., they are separated into high-amplitude and low-amplitude regions in poloidal space, and those envelopes form structures with the poloidal mode number of 1. These $m = 1$ structures slowly rotate with a frequency of −1.2 kHz, which is the same property as the sinusoidal waves. High-amplitude regions observed in both Figs. 2 (a) and 2 (c) are at almost the same poloidal positions, as indicated by black lines in the figures. This fact suggests that the poloidally localized structure is radially elongated inside the plasma. Note that the phases of the −1.2 kHz sinusoidal waves of the two measuring radii do not match; only the phases of the envelopes of the bunching waves agree.

4. Spectral and Bispectral Analyses

Figure 3 shows the 2D power spectrum of the spatiotemporal waveform observed in Fig. 2 (a). The sam-

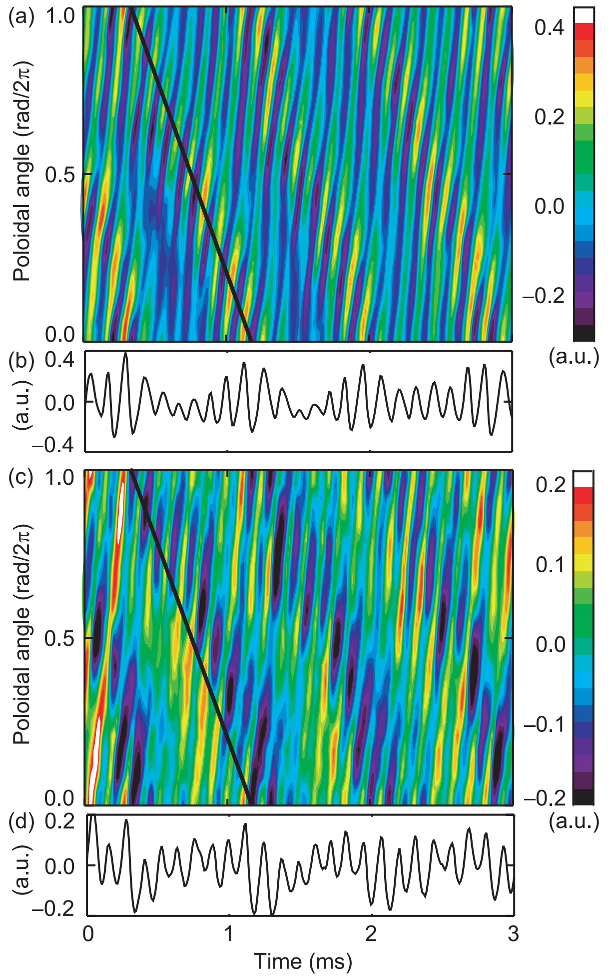

Fig. 2 Spatiotemporal waveforms of ion saturation-current fluctuations measured at radii of (a) 40 mm with a 64-channel poloidal probe array and (c) 20 mm with a 24-channel poloidal probe array. (b), (d) Temporal waveforms of (a), (c) at zero poloidal angle positions. Black lines in (a) and (c) indicate the same timings and positions. Both lines trace the high-amplitude regions of the poloidally localized structures.

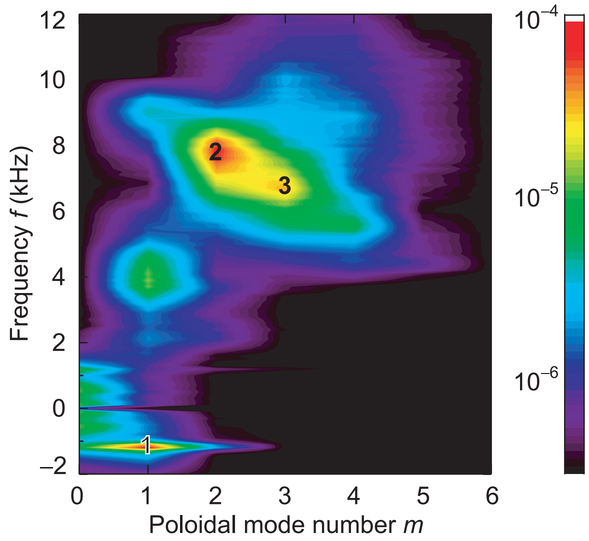

Fig. 3 2D power spectrum of the ion saturation-current fluctuation observed in Fig. 2 (a). Frequencies and poloidal mode numbers of three pronounced modes labeled “1”, “2”, and “3” are $(f, m) = (-1.2\,\mathrm{kHz}, 1)$, $(7.8\,\mathrm{kHz}, 2)$, and $(6.6\,\mathrm{kHz}, 3)$. These three modes satisfy the matching conditions.

pling frequency is 1 MHz, the frequency resolution of the Fourier transformation is 0.1 kHz, and the result is an ensemble of 390 time windows. The poloidal mode number, m, is set to $0 \leq m \leq 32$, and the frequency, f, is $|f| \leq 500\,\mathrm{kHz}$.

Three pronounced modes appear in the 2D power spectrum, which are labeled “1”, “2”, and “3” in the figure. The frequencies and poloidal mode numbers of the modes are $(f_1, m_1) = (-1.2\,\mathrm{kHz}, 1)$, $(f_2, m_2) = (7.8\,\mathrm{kHz}, 2)$, and $(f_3, m_3) = (6.6\,\mathrm{kHz}, 3)$. Here, mode 1 is a slow sinusoidal wave, and also (f_1, m_1) is equivalent to the behavior of the envelope of the bunching waves observed in Fig. 2. In addition, the three modes satisfy the matching conditions $f_1 + f_2 = f_3$ and $m_1 + m_2 = m_3$. These facts suggest that the three modes are nonlinearly coupled. In particular, the frequencies of the modes are the same as in the brief example shown in Fig. 1; i.e., modes 2 and 3 generate wave bunching, and the frequency of the envelope is the same as that of mode 1. Whether phase locking exists between mode 1 and the envelope of mode 2 plus mode 3 should be examined by bispectral analysis. The same argument can be extended to poloidal space. Poloidally localized structure generated by modes 2 and 3 form an $m = 1$ structure that rotates in the poloidal direction with the same frequency as mode 1.

To investigate the phase locking between the mode and the envelope estimated above, bispectral analysis was applied to the Fourier transformed expressions $Z_1 = Z(f_1, m_1)$, $Z_2 = Z(f_2, m_2)$, and $Z_3 = Z(f_3, m_3)$ of the three pronounced modes. The results for 390 ensembles yield a squared bicoherence, b^2, of 0.57, which clearly proves that the three modes are nonlinearly coupled, since it is higher than the confidence level, 0.003 (= 1/390). In other words, the three modes exist independently, and one mode should be excited by nonlinear mode coupling of the other two. The same calculation indicates that the biphase, ϕ_b, is 0.08 (rad/2π), which is slightly higher than zero. This agrees with the observation in Fig. 2 (a) that a large-amplitude region appears just before the maximum of mode 1.

Note that the poloidally localized structure is not generated by only modes 2 and 3, but many modes and even broadband fluctuations contribute. This is because mode 1 couples with many other modes and broadband fluctuations with nearly constant values of biphases, which is almost the same as the biphase for modes 1, 2, and 3 [12]. Mode 1 is an important mode and can be called a mediator for mesoscale formation [6].

The poloidally localized structure was investigated through the radial direction by means of bispectral analy-

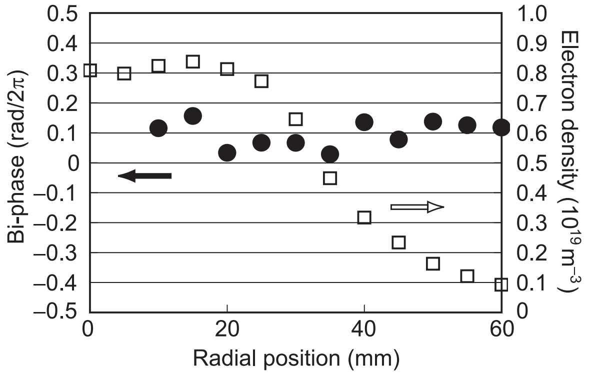

Fig. 4 Radial profile of the biphase between modes 1, 2, and 3 (closed circles). $Z_1 = Z(f_1, m_1)$, measured with 64-channel poloidal probe array (reference signal), and $Z_2 = Z(f_2)$ and $Z_3 = Z(f_3)$, measured with a radially movable probe, were used for the calculation. Result indicates that the poloidally localized structure is almost radially independent. Open squares indicate the electron density profile.

sis. For this measurement, a radially movable probe was used in addition to the 64-channel poloidal probe array. The structure is mainly generated by modes 2 and 3. The biphase between modes 1, 2, and 3 indicates the phase difference between mode 1 and the envelope. When mode 1 measured at a radius of 40 mm with the poloidal probe array is regarded as a reference signal, and the biphases between this and modes 2 and 3 at different radii are calculated, the phase structure of the envelope along the radial direction can be determined. Here, the phase of mode 1 changes with the measuring radius, as shown in Fig. 2, but this does not imply that the phase of the envelope also changes. Since the radially movable probe makes a single-point measurement, $Z_1 = Z(f_1, m_1)$, $Z_2 = Z(f_2)$, and $Z_3 = Z(f_3)$ are considered for the biphase calculation. Figure 4 shows the radial profile of the biphase between mode 1 at a radius of 40 mm and the local modes 2 and 3. The radially movable probe was scanned from 10 to 60 mm along the radial direction. The electron density profile is also plotted for better understanding. The biphase was almost constant around 0.1 (rad/2π) inside the plasma. This differs slightly from the value 0.08 (rad/2π) at 40 mm, since the probe array and the radially movable probe are installed at different axial positions (the difference is 510 mm). Thus, radial elongation of the poloidally localized structure is clearly proven, and the structure is apparently a streamer.

5. Summary

In summary, the spatiotemporal behavior of the ion saturation-current fluctuation was measured with a 64-channel poloidal Langmuir probe array. A bunching of waves, which is localized in the poloidal direction, was observed. The envelope of the bunching waves has a structure with a poloidal mode number $m = 1$ and slowly rotates in the poloidal direction with a frequency $f = -1.2$ kHz, where a negative frequency indicates propagation in the ion diamagnetic direction. There exists a particular mode (mode 1), $(f, m) = (-1.2\,\text{kHz}, 1)$, that has the same frequency and poloidal mode number as the envelope. Bispectral analysis shows that mode 1 is strongly coupled with many other modes; i.e., modes coupled with mode 1 generate a wave bunching of which the envelope is phase locked with mode 1. The biphase indicates the phase difference between mode 1 and the envelope. By calculating the radial biphase profile between mode 1 measured with the 64-channel poloidal probe array and the two other modes with frequencies of 7.8 kHz and 6.6 kHz measured with a radially movable probe, the biphase profile was found to be almost constant inside the plasma. Thus, radial elongation of the poloidally localized structure was confirmed, and the structure was proven to be a streamer.

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