# Toroidal Flow Velocity Profile of Impurity Ions in Field-Reversed Configuration Plasma

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Particle orbits of impurity ions are calculated in order to estimate their toroidal flow velocity. If the impurity ion flow differs from the ion flow, Doppler shift measurement of impurity spectra requires a suitable modification of experimental results. At the field null, the carbon ion flow coincides with the deuterium ion flow due to the friction force between them. At the separatrix, however, impurity and plasma ions each have their own flow velocities. The diamagnetic drift, which depends on the charge and mass of each fluid species, is found to dominate the toroidal flow velocity.



Keywords: field-reversed configuration, toroidal flow velocity, Doppler shift measurement, orbit calculation, resistive flux decay

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## 1. Introduction

Rotational instability with toroidal mode number $n = 2$ limits the lifetime of field-reversed configuration (FRC) plasmas [1,2]. Several studies have shown that the application of external multipole fields suppresses time variation of the line-integrated electron density [3,4]. These experiments also reveal that elliptical deformation of FRC plasmas is inhibited by multipole fields. Internal structure, on the other hand, can be deformed to a complex dumbbell-like shape [5], even when no deformation of the separatrix shape can be found. Therefore, we need to clarify the origin of rotation in order to prolong the FRC lifetime.

Particle loss [6] and end-shorting [7] are now considered the two most promising mechanisms. Recently, Belova *et al.* claimed that particle loss associated with resistive flux decay may contribute to rotation [8]. We have shown that the flux decay can contribute directly to toroidal spin-up [9]. Suppose the FRC plasma is axisymmetric: this assumption is valid until rotational instability is triggered. In this case, the canonical angular momentum

$$P_\theta = m v_\theta r + q\psi(r, z) \tag{1}$$

of every particle is conserved, where $m$ and $q$ are the mass and charge, respectively, $v_\theta$ is the toroidal velocity component, and $\psi(r, z)$ is the poloidal flux function. If the poloidal flux decays due to resistivity, and toroidal axisymmetry is still valid, then

$$m\Delta\,(v_\theta r) = -q\Delta\psi\ . \tag{2}$$

Equation (2) shows that every ion gains angular momentum in the ion diamagnetic direction when the trapped poloidal flux decays. Generally, the separatrix radius decreases during the decay phase. If the guiding center $r$ is also decreased, the toroidal velocity $v_\theta$ is further increased.

To obtain the toroidal flow velocity experimentally, Doppler shift measurement of impurity ion spectra such as C III and C V is conventionally employed. If impurity ions rotate with the same velocity as plasma ions, then Doppler shift measurement offers accurate information regarding plasma flow. Otherwise, we need to modify the experimental result to obtain the rotational velocity.

In the present paper, we numerically study the impurity ion (carbon ion) velocity in an FRC plasma that decays resistively and therefore rotates, according to Ref. 9.

## 2. Numerical Model

Since the discussion above is based on a single-particle picture, we need to calculate a collective ion velocity. A number of super-particles are traced numerically in the decaying FRC plasma. The poloidal flux decay is reproduced by

$$\frac{\partial\psi}{\partial t} = -r\eta\, J_\theta\ . \tag{3}$$

Here, the electrical resistivity $\eta$ equals $A\eta_{\rm cl}$, where $A$ is the anomaly factor, and $\eta_{\rm cl}$ is the classical resistivity. Note that $\psi > 0$ inside the separatrix region in our study. The flux lifetime is controlled by the parameter $A$. By integrating Eq. (3) with the Runge-Kutta method, a flux function at a calculation point is found. The electromagnetic fields are then written by the obtained $\psi$ as

$$\boldsymbol{B} = \nabla\times\boldsymbol{A} = \nabla\times\left(\frac{\psi}{r}\boldsymbol{e}_\theta\right),\ \boldsymbol{J} = \frac{1}{\mu_0}\nabla\times\boldsymbol{B},\ \boldsymbol{E} = \eta\boldsymbol{J}. \tag{4}$$

The plasma and impurity ions ($C^{4+}$) are traced as

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super-particles in the field given by Eq. (4). The weight of a super-particle is given initially and then estimated from the Maxwell distribution. The friction term due to collisions is included in the equation of motion. Pitch angle scattering is simulated by a Monte-Carlo method [10]. The density and particle flux of both plasma and impurity ions are calculated by the particle-in-cell method [11].

## 3. Results and Discussion

The orbit of an impurity ion is calculated in the prescribed field given in Eqs. (3) and (4). To compare our results with an FRC experiment, the flux decay time obtained by our calculation is set to equal the flux lifetime of the Nihon University Compact Torus Experiment (NUCTE)-III device. As noted above, the flux decay time is controlled by the anomaly factor $A$.

The time evolution of the trapped flux in the NUCTE-III is shown in Fig. 1. The FRC plasma forms in 0-10 µs. Rotational instability with toroidal mode number $n = 2$ is caused at 35 µs, and the magnetic configuration collapses. From the end of the formation phase ($t = 10$ µs in Fig. 1), therefore, the FRC plasma maintains axisymmetry for 25 µs; this corresponds to $23t_{A0}$, where $t_{A0}$ is the Alfvén time $t_{A0} \equiv r_w/v_{A0}$, and $v_{A0} \equiv B_{ex}/\sqrt{\mu_0 m_i n_0}$ ($B_{ex}$: the initial external magnetic field, $m_i$: the plasma ion mass, $n_0$: the initial density at the field null). The decrement of the trapped flux for $23t_{A0}$ is about 0.26 mWb. For $A = 10$, we

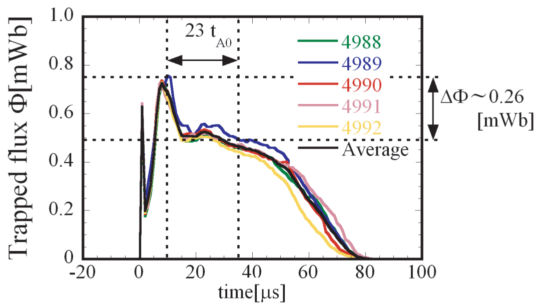


Fig. 1 The time evolution of the trapped flux in the NUCTE-III

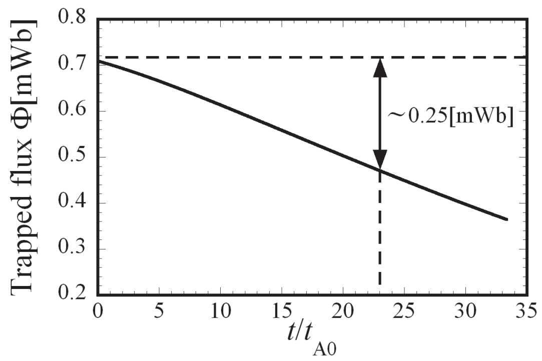


Fig. 2 The time evolution of the maximum trapped flux obtained by the numerical calculation.

numerically obtain the time evolution of the flux shown in Fig. 2, where the decrement of the flux for $23t_{A0}$ is almost the same as the experimental result.

According to Ref. [9], the presence of resistive flux decay causes rotation of the FRC plasma. The calculated plasma ion ($D^+$) density and toroidal velocity profile are shown by the color contours in Figs. 3 and 4, respectively. We can see that the plasma ions diffuse out from the separatrix by collisions. As the magnetic flux decays, ions can gain the toroidal momentum near and outside the separatrix. This rotational velocity results from particle loss and the direct effect of flux decay.

The profiles of impurity ion ($C^{4+}$) density and flow velocity obtained by our calculations are presented in Figs. 5 and 6, respectively. The initial impurity density is set at $10^{-5} n_i$, where $n_i$ is the ion density. We assume that the impurity ion temperature is the same as the plasma ion temperature, since collisions are frequent in the formation phase, leading to thermal equilibrium. The temperatures of both ions and impurity ions are 124 eV here. The impurity ions are found to concentrate gradually at the field null due to collisional diffusion. The difference seen in the density profile between plasma and impurity ions results from their collision frequency. The concentration of impurity ions has been shown by Spitzer [12]. We also found

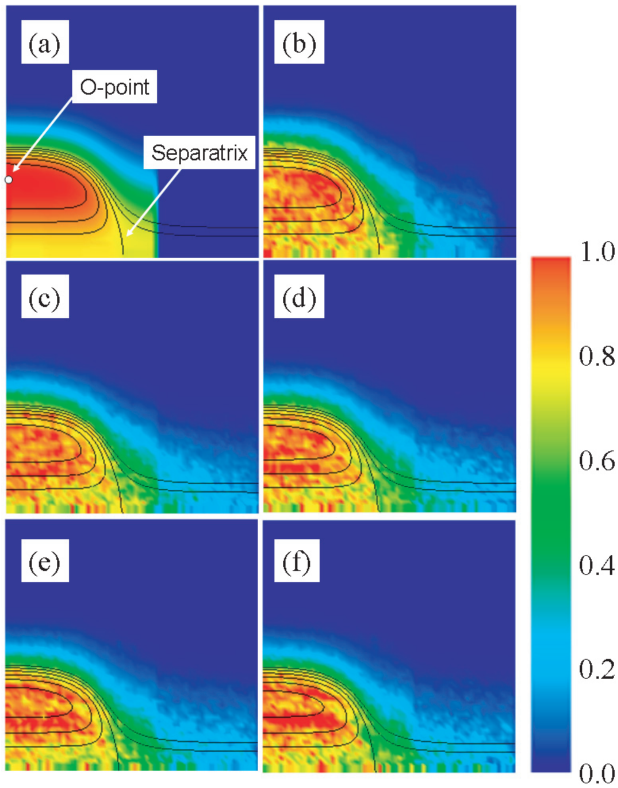


Fig. 3 Color contours of the plasma ion density, values of which are normalized by the initial density at the field-null. The figures are shown at (a) $t = 0$, (b) $t = 2t_{A0}$, (c) $t = 4t_{A0}$, (d) $t = 6t_{A0}$, (e) $t = 8t_{A0}$, and (f) $t = 10t_{A0}$.

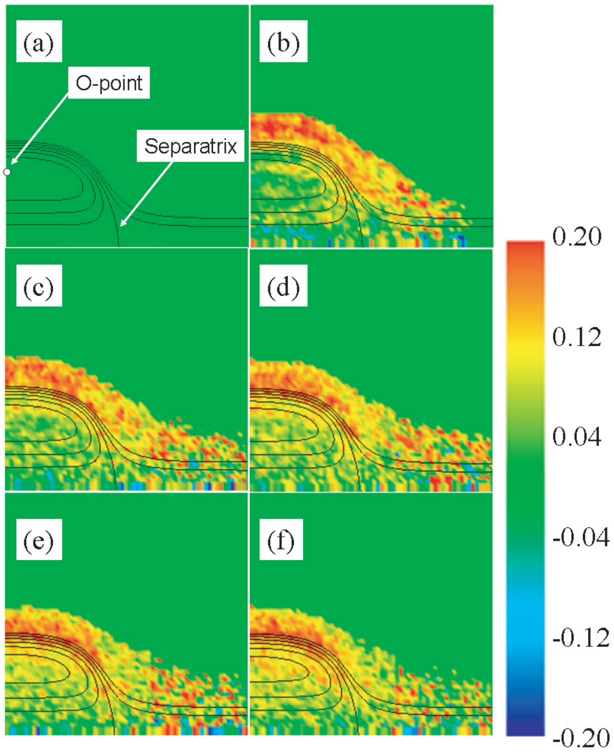


Fig. 4 Color contours of the toroidal flow velocity for plasma ions. The velocity is normalized by $r_w/\tau$, where $\tau \equiv m_i r_w^2/(q_i|\psi_w|)$. The output time is the same as in Fig. 3.

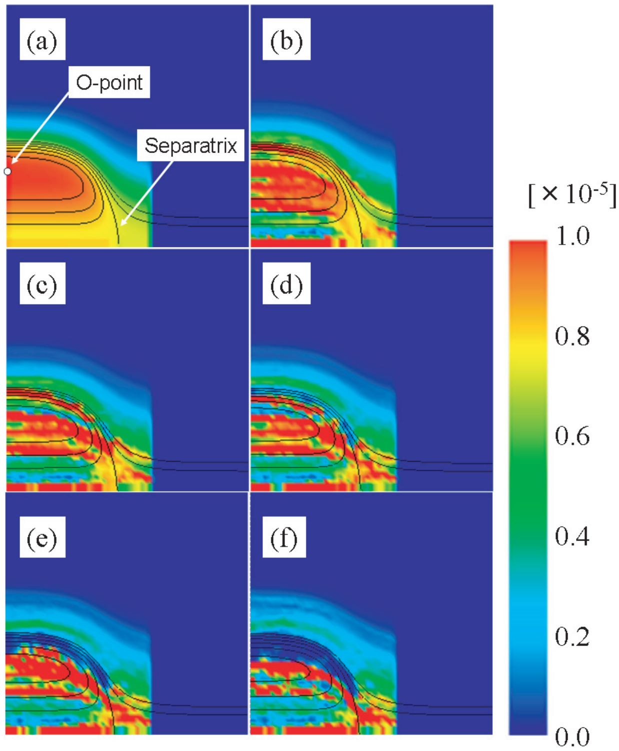


Fig. 5 Color contours of the impurity ion density. Figures are shown in the same manner as in Fig. 3.

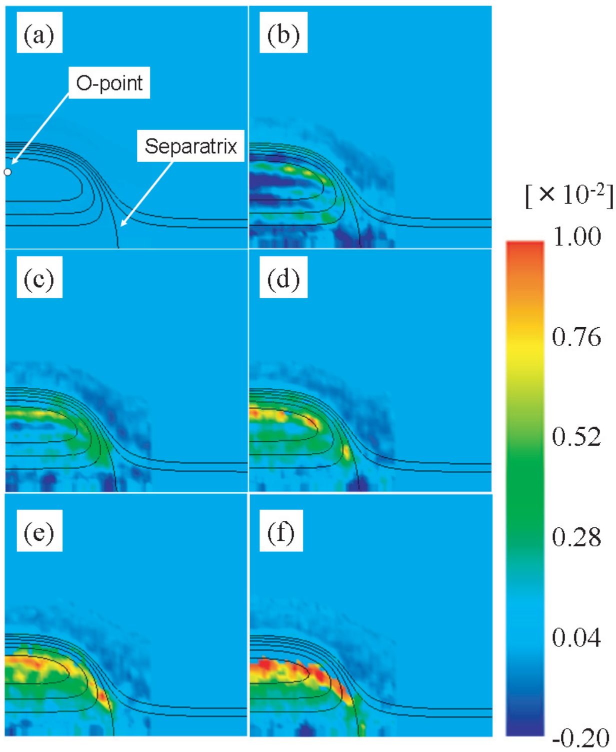


Fig. 6 Color contours of the toroidal flow velocity for impurity ions. Figures are shown in the same manner as in Fig. 4.

that the impurity ion density at the separatrix reduced significantly; this reduces the diamagnetic drift velocity

$$\boldsymbol{u}_{d\alpha} = -\frac{\nabla p_\alpha \times \boldsymbol{B}}{Z_\alpha e n_\alpha B^2}, \tag{5}$$

where $Z_\alpha$, $p_\alpha$, and $n_\alpha$ are the charge number, pressure, and density, respectively, for the $\alpha$-species ions. From Fig. 6, we find toroidal velocity peaks at the field-null point. At the separatrix, on the other hand, the rotational velocity vanishes after $t = 6t_{A0}$. The time evolution of toroidal velocity at the field null is presented in Fig. 7. It appears from Fig. 7 that the toroidal velocity of impurity ions follows the time-averaged (coarse-grained in time) ion velocity. We can say that Doppler shift measurement of impurity ion spectra is valid at the field-null point. In contrast to Fig. 7, the separatrix velocity of impurity ions deviates considerably, as shown in Fig. 8. Regarding the diamagnetic drift (5), the flow velocity $C^{4+}$ is a quarter of the velocity for $D^+$ when the effects of the density and the pressure gradient profile are neglected. Furthermore, although the ion density gradient is sustained, the impurity ions near the separatrix are distributed uniformly. Therefore, the diamagnetic drift velocity is reduced for the impurity ions. At the field-null point, the friction force acting on the impurity ions dominates the toroidal velocity, and thus the velocity tends to coincide with the plasma velocity. At the separatrix, however, a strong magnetic field perpendicular to the friction force restricts the impurity flow to radial motion.

Our calculation results suggest that the toroidal flow velocity obtained by Doppler shift measurement needs modification. When the spectral line of $C^{4+}$ is used for a

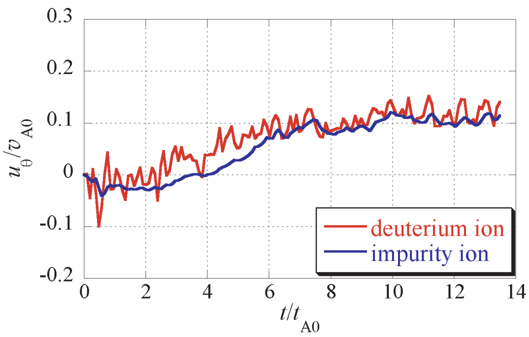


Fig. 7 The time evolution of the toroidal velocity at the field-null point for the deuterium ions (the red line) and the impurity ions (the blue line).

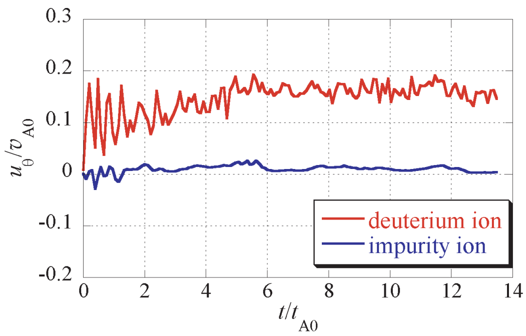


Fig. 8 The time evolution of the toroidal velocity at the separatrix and midplane for the deuterium ions (the red line) and the impurity ions (the blue line).

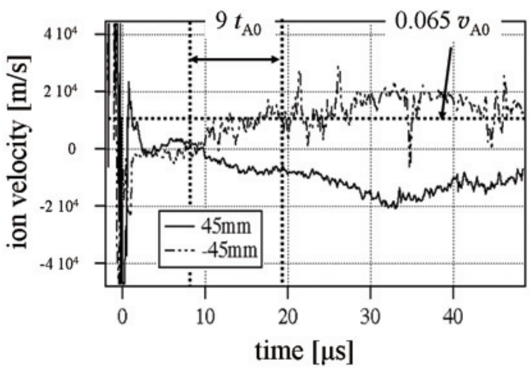


Fig. 9 The time evolution of the toroidal velocity of impurity ions that is measured by the Doppler shift measurement.

deuterium plasma, the toroidal ion velocity is expected to be less than a quarter of the measured value.

The toroidal velocity of impurity ions measured by the Doppler shift of the spectral line is shown in Fig. 9. The line of sight is shifted away from the geometric axis by 4.5 cm; it is located between the field null and the separatrix. We find that the toroidal velocity of the impurity ions is about $0.065v_{A0}$ at $t = 9t_{A0}$ after the FRC plasma forms. This velocity, measured experimentally, sits between the field-null and separatrix flow velocities obtained by our calculation. However, a detailed comparison of the flow profiles should be done in the near future. Also, we need to add the electron pressure gradient term in the electric field for a more valid estimation of the impurity flow velocity.

## 4. Summary

We have numerically calculated the orbits of impurity ions ($C^{4+}$) to study the difference in toroidal rotation velocity between impurity and plasma ions in a field-reversed configuration (FRC) plasma. The flux of the FRC plasma decays resistively, and plasma ions gradually gain the toroidal momentum. The impurity ions also spin up at the field-null point due to the friction force from the plasma ions, and their rotational velocity is in close agreement with the flow velocity of the plasma. The separatrix flow velocity for impurity ions, however, tends to deviate from the plasma ion velocity. Because of a relatively strong field at the separatrix, the friction force between the plasma and impurity ions is found to have little effect on their rotational velocity. Therefore, we consider that modification of the Doppler shift measurement is necessary for the toroidal velocity near the separatrix.


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