Relativistic Degeneracy Effect on Propagation of Arbitrary Amplitude Ion-Acoustic Solitons in Thomas-Fermi Plasmas

Abdolrasoul ESFANDYARI-KALEJAHI, Massoud AKBARI-MOGHANJOUGHI and Ehsan SABERIAN

Department of Physics, Faculty of Sciences, Azarbaijan University of Tarbiat Moallem, 51745-406, Tabriz, Iran

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Arbitrary amplitude ion-acoustic solitary waves (IASWs) are studied using Sagdeev-Potential approach in electron-positron-ion plasma with ultra-relativistic or non-relativistic degenerate electrons and positrons and the matching criteria of existence of such solitary waves are numerically investigated. It has been shown that the relativistic degeneracy of electrons and positrons has significant effects on the amplitude and the Mach-number range of IASWs. Also it is remarked that only compressive IASWs can propagate in both non-relativistic and ultra-relativistic degenerate plasmas.

Keywords: Sagdeev-potential, ion-acoustic soliton, degenerate plasma, relativistic plasma, Fermi statistics

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1. Introduction

Of the nonlinear excitations, ion-acoustic solitary waves (IASWs) are of the most important and well-understood characteristics of plasma environments. Theoretical studies of main properties of these solitary structures date back to 1961 using Sagdeev pseud-potentials method [1]. Another method which is widely used to investigate the collective wave phenomenon in plasma is the so-called multi-scales perturbation method [2–8]. However, the latter method, which is based on approximation, is used only for the small-amplitude treatment of plasma in a state away from thermodynamic equilibrium. Therefore to obtain a good agreement with experiments, in this method, one needs to take higher-orders in perturbation amplitudes. In recent years there have been many investigations on solitary IASWs as well as solitary electrostatic waves (ESWs) in diverse plasma environments using Sagdeev pseudo-potential approach [9–13]. The small amplitude propagation and interaction of IASWs with relativistic degeneracy pressure effects have been recently considered in Ref. [14].

Among different kinds, pair-plasmas have attracted special attention in recent years, a special cases of which can be electron-positron (EP) and electron-positron-ion (EPI) [15–19] plasmas. Electron-positron-ion plasma exists in places such as active galactic nuclei [20], pulsar magnetospheres [21] and in many dense astronomical environments, namely, neutron stars and white dwarfs [22] and may play a key role in understanding the beginning and evolution of our entire universe [23]. This kind of plasma may also be practically produced in laboratories [24–27]. More specifically when positrons, due to their significant lifetimes, are used to probe particle transport in Tokamaks, two component electron-ion (e-i) plasma behaves as three component (e-p-i) plasma [25]. Furthermore, the wave properties such as stabilities of a two component electron-ion (EI) plasma solitary excitations may be radically altered by inclusion of low amounts of positrons.

Owing to their wide applicabilities in micro- and nano-electronic miniaturization, dense-plasmas is becoming one of the interesting fields of theoretical as well as experimental fields of plasma research [28–34]. Dense plasma or the so-called quantum plasma are characterized by high densities and low temperatures, however, a dense plasma may be realized in such hot places as planet interiors and white dwarfs [35]. More recently, quantum hydrodynamics model has been applied to study the electron-hole dynamics in semiconductors [36, 37]. The quantum effects in collective behavior of a plasma system becomes important when the inter-particle distances are comparable or less than the de Broglie thermal wavelength $\lambda_B = h/(2\pi m_e k_B T)^{1/2}$ or equivalently when the thermal energies of plasma species are less than Fermi-energies [38]. In such cases the plasma becomes degenerate, in which the plasma ingredients are under effective influence of Pauli exclusion principle and classical statistical assumptions break down. Quantum effects also play important role in the nonlinear processes of white-dwarfs [39]. For instance, for a cold neutron star the densities can be as high as $10^{15}\,\mathrm{gm/cm^{-3}}$ in the core, which is several times the density of an atomic nuclei. In extreme conditions such as the middle of a supernova or the core of a massive white dwarf the densities can be even catastrophically higher. At these very high densities the electrons and positrons may become ultra-relativistic giving rise to the collapse of star

author's e-mail: M_Akbari@azaruniv.edu

under its giant gravitational force [40,41].

The present study is devoted to investigation of propagation of IASWs in an unmagnetized EPI plasma using Sagdeev pseudo-potential method in such extreme condition, taking into account the relativistic degeneracy effects for electrons and positrons. The basic normalized plasma equations are introduced in Sec. 2. Nonlinear arbitrary-amplitude solution is derived in Sec. 3. Section 4 devotes to short argument about small amplitude IASWs. Numerical analysis and discussion is given in Sec. 5 and final remarks are presented in Sec. 6.

2. Basic Plasma State Equations

Consider a dense plasma consisting of electrons, positrons and positive-ions. Also, suppose that the electrons and positrons follow the zero-temperature Fermi-gas statistics, while, ions behave as classical fluid. In such plasma electrons and positrons may be considered collision-less due to Fermi-blocking process caused by pauli exclusion principle. Therefore, the semi-classical description of nonlinear dynamics and interaction of waves in such plasma can be studied in the framework of conventional hydrodynamics model. The basic normalized equations describing plasma dynamical state may be written as

$$\begin{aligned} &\frac{\partial n_{\mathrm{i}}}{\partial t}+\frac{\partial}{\partial x}n_{\mathrm{i}}u_{\mathrm{i}}=0,\\ &\frac{\partial u_{\mathrm{i}}}{\partial t}+u_{\mathrm{i}}\frac{\partial u_{\mathrm{i}}}{\partial x}=-\frac{\partial \varphi}{\partial x},\\ &\frac{\partial^2\varphi}{\partial x^2}=n_{\mathrm{e}}-n_{\mathrm{p}}-n_{\mathrm{i}}, \end{aligned} \tag{1}$$

where, electrons and positrons are of Thomas-Fermi type

$$n_{\mathrm{e}}=(1+\varphi)^{\frac{3}{2}}, n_{\mathrm{p}}=\alpha(1-\sigma_{\mathrm{F}}\varphi)^{\frac{3}{2}}, \alpha=\frac{n_{\mathrm{p}0}}{n_{\mathrm{e}0}}, \sigma_{\mathrm{F}}=\frac{T_{\mathrm{Fe}}}{T_{\mathrm{Fp}}}. \tag{2}$$

In obtaining the normalized set of equations following scalings are used

$$x\to\frac{v_{\mathrm{Fe}}}{\omega_{\mathrm{pi}}}\bar{x}, t\to\frac{\bar{t}}{\omega_{\mathrm{pi}}}, n\to\bar{n}n^{(0)}, u\to\bar{u}v_{\mathrm{Fe}}, \varphi\to\bar{\varphi}\frac{2k_{\mathrm{B}}T_{\mathrm{Fe}}}{e}, \tag{3}$$

where, $\omega_{\mathrm{pi}}=\sqrt{e^2 n_{\mathrm{e}}^{(0)}/\varepsilon_0 m_{\mathrm{i}}}$ and $v_{\mathrm{Fe}}=\sqrt{2k_{\mathrm{B}}T_{\mathrm{Fe}}/m_{\mathrm{i}}}$ are characteristic plasma-frequency and electron Fermi-speed, respectively, and $n_{\mathrm{e}}^{(0)}$ denotes the equilibrium electron density ($n_{\mathrm{e}}^{(0)}=(8\pi/3\hbar^3)p_{\mathrm{Fe}}^3$ with p_{Fe} being the electron linear Fermi-momentum). In a fully degenerate Fermi gas one may write the electron degeneracy pressure in the following general form [40]

$$P=\frac{\pi m_{\mathrm{e}}^4 c^5}{3h^3}\left[r\left(2r^2-3\right)\sqrt{1+r^2}+3\sinh^{-1}r\right], \tag{4}$$

where, h and c are Plank constant and light-speed, respectively, and $r=p_{\mathrm{Fe}}/m_{\mathrm{e}}c$ is the normalized relativity parameter. The electron number density can then be defined in terms of the relativity parameter ($n_{\mathrm{e}}=(8\pi m_{\mathrm{e}}^3c^3/3h^3)r^3$). It is noted that in the limits of very small and very large values of the relativity parameter we obtain

$$P=\left\{\begin{array}{l}\frac{1}{20}\left(\frac{3}{\pi}\right)^{\frac{2}{3}}\frac{h^2}{m_{\mathrm{e}}}n_{\mathrm{e}}^{\frac{5}{3}}(r\to 0)\\ \frac{1}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}}hcn_{\mathrm{e}}^{\frac{4}{3}}(r\to\infty)\end{array}\right\}. \tag{5}$$

Therefore, in a three-dimensional *non-relativistic* zero-temperature Fermi-gas for degenerate electrons and positrons from standard definitions we obtain $E_{\mathrm{Fj}}=\hbar^2k_{\mathrm{Fj}}^2/2m_{\mathrm{j}}$ (j = e, p) or $E_{\mathrm{Fj}}\propto n_{\mathrm{j},0}^{2/3}$, which follows that $\sigma_{\mathrm{F}}=\alpha^{-2/3}$. On the other hand, three-dimensional *ultra-relativistic* Fermi-gas, we have $E_{\mathrm{Fj}}=c\hbar k_{\mathrm{Fj}}$ or $\sigma_{\mathrm{F}}=\alpha^{-1/3}$. It is noted that in our model the inertial ions are always non-relativistic, hence, the non-relativistic hydrodynamics equation has been used in Eqs. (1). Therefore, the Poisson equation reads as

$$\frac{\partial^2\varphi}{\partial x^2}=(1+\varphi)^{\frac{3}{2}}-\alpha(1-\sigma_{\mathrm{F}}\varphi)^{\frac{3}{2}}-n_{\mathrm{i}}. \tag{6}$$

At the equilibrium situation the overall neutrality condition gives rise to the following relation

$$\beta=1-\alpha, \quad \beta=\frac{n_{\mathrm{i}0}}{n_{\mathrm{e}0}}. \tag{7}$$

3. One-Dimensional Arbitrary-Amplitude Analysis

In this section we derive an appropriate Sagdeev pseudo-potential describing the dynamics of arbitrary-amplitude IAWs in plasma containing classical heavy positive ions and inertial-less relativistic or non-relativistic Thomas-Fermi electrons and positrons, obeying the three-dimensional distributions in Eqs. (2). Using one-dimensional version of Eqs. (1), in a reduced coordinate $\eta=x-Mt$ (M being Mach number which is a measure of soliton speed relative to ion-sound speed), from continuity and momentum equations we obtain

$$n_{\mathrm{i}}=\frac{1-\alpha}{\sqrt{1-\frac{2\varphi}{M^2}}}, \tag{8}$$

where, we have used the fact that $\varphi\to 0$, $u_{\mathrm{i}}\to 0$ and $n_{\mathrm{i}}\to\beta$ at $\eta\to\pm\infty$. Now, substituting Eq. (8) and Eq. (2) in Poisson's equation in Eq. (1), multiplying by $\mathrm{d}\varphi/\mathrm{d}\eta$ and integrating with boundary conditions $\{\varphi, \mathrm{d}\varphi/\mathrm{d}\eta\}\to 0$ for $\eta\to\pm\infty$, we derive

$$\frac{1}{2}\left(\frac{\mathrm{d}\varphi}{\mathrm{d}\eta}\right)^2+V(\varphi)=0, \tag{9}$$

where, the Sagdeev pseudo-potential $V(\varphi)$ reads as

$$\begin{aligned} V(\varphi)=&\frac{2}{5}\left[1-(1+\phi)^{5/2}\right]+\frac{2\alpha}{5\sigma_{\mathrm{F}}}\left[1-(1-\sigma_{\mathrm{F}}\phi)^{5/2}\right]\\ &+M^2\beta\left[1-\sqrt{1-\frac{2\phi}{M^2}}\right]. \end{aligned} \tag{10}$$

For the reality of $V(\varphi)$ to be ensured we must have $\varphi \leq M^2/2$ and $\varphi \leq \sigma_F^{-1}$. The possibility of IAWs, therefore, require that

$$V(\varphi)|_{\varphi=0} = \left.\frac{dV(\varphi)}{d\varphi}\right|_{\varphi=0} = 0, \left.\frac{d^2V(\varphi)}{d\varphi^2}\right|_{\varphi=0} < 0, \quad (11)$$

it is further required that a φ_m exists such that $V(\varphi_m) = 0$ and for every $\varphi_m > \varphi > 0$ then $V(\varphi) < 0$.

4. Small Amplitude Theory

Let us consider the small-amplitude limit in the above analysis. Expanding the potential $V(\varphi)$ in (10) near $\varphi = 0$, we obtain

$$V(\varphi) = \frac{V_0''}{2}\varphi^2 + \frac{V_0'''}{6}\varphi^3, \quad (12)$$

where, $V_0'' = V''(\varphi = 0)$ and $V_0''' = V'''(\varphi = 0)$ are computed from Eq. (10) as

$$V_0'' = -\frac{3}{2} - \frac{3\alpha\sigma_F}{2} + \frac{1-\alpha}{M^2}, \quad (13)$$

$$V_0''' = -\frac{3}{4} + \frac{3\alpha\sigma_F^2}{4\alpha} + \frac{3(1-\alpha)}{M^4}. \quad (14)$$

Inserting into Eq. (9) and integrating, we obtain (provided that $V_0'' < 0$) a solitary solution in the form (see [42])

$$\varphi(\eta) = -3\frac{V_0''}{V_0'''}\frac{1}{\cosh^2(\frac{1}{2}\sqrt{-V_0''}\eta)}. \quad (15)$$

This pulse profile is identical to the soliton solution of the Korteweg-de Vries (KdV) equation, which is obtained by use of the reductive perturbation method, for example see [43]. It is important to notice that the soliton width $L = 2/\sqrt{-V_0''}$ and amplitude $\varphi_0 = -3V_0''/V_0'''$ satisfy $\varphi_0 L^2 = 12/V_0'''$ = constant, as known from the KdV theory.

5. Numerical Analysis and Discussion

As it was pointed out in Sec. 3, the ion-acoustic solitary waves (IASWs) exist if the Sagdeev pseudo-potential satisfies the following conditions: (i) $d^2V(\varphi)/d\varphi^2|_{\varphi=0} < 0$, which reveals that the fixed point $\varphi = 0$ is unstable; (ii) $V(\varphi_m) = 0$, where, φ_m is the maximum value of φ; and (iii) $V(\varphi) < 0$ when $\varphi_m > \varphi > 0$.

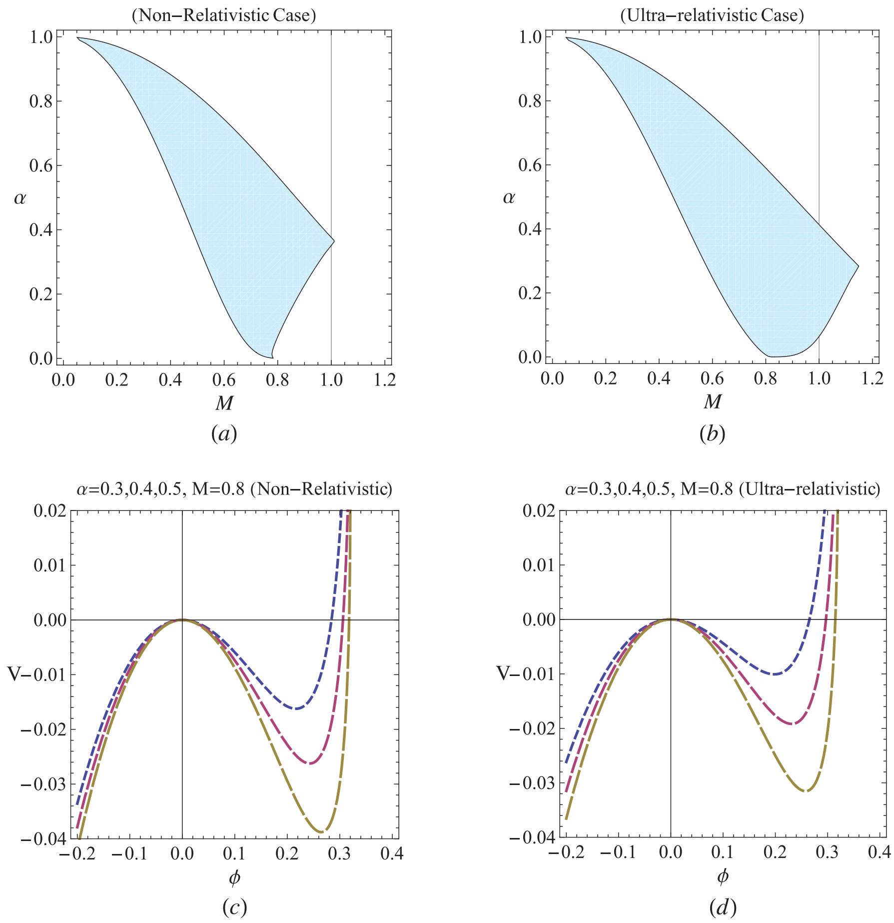

Fig. 1 The stability regions (dark) of arbitrary amplitude IASWs is shown in $\alpha - M$ plane for non-relativistic (Fig. 1 (a)) and ultra-relativistic (Fig. 1 (b)) electron/positron degeneracies. Figures 1 (c) and 1 (d) show the corresponding pseudo-potential dips for varied fractional positron to electron number-densities, $\alpha = 0.3$ (blue), $\alpha = 0.4$ (grey) and $\alpha = 0.5$ (red), and fixed Mach-number in non-relativistic (Fig. 1 (c)) and ultra-relativistic (Fig. 1 (d)) electron/positron degeneracies, respectively.

Noticing these conditions, Fig. 1 (a) and Fig. 1 (b) show the areas in M-α plane, where: namely, IA solitary waves can exist for non-relativistic and ultra-relativistic electron-positron degeneracy, respectively. It is remarked that, the minimum values of Mach number, M, decreases as the fractional positron to electron number-density ratio, α, increases for ultra-relativistic case (see Fig. 1 (b)). Nevertheless, the minimum values of M increases and reaches to a given maximum value then decreases as α increases (see Fig. 1 (a)). On the other hand, the maximum value of M increases as α increases up to the value $\alpha \simeq 0.37$ ($\alpha \simeq 0.28$) for non-relativistic (ultra-relativistic) case. The maximum value of M decreases as α increases in the range $1 > \alpha > 0.37$ ($1 > \alpha > 0.28$) for non-relativistic (ultra-relativistic) case.

Comparing Fig. 1 (a) and Fig. 1 (b) reveals that both supersonic and subsonic IASWs can propagate in ultra-relativistic case, whereas, only subsonic propagations can occur for non-relativistic case. Although both of supersonic and subsonic IASWs can propagate in ultra-relativistic degeneracy case, however, the former exist only for very small range of α, for approximately, $0.07 > \alpha > 0.4$. For $\alpha = 1$ i.e., in the absence of ions, $V(\varphi)$ does not depend on Mach number, M. This case corresponds to the solution of Poisson equation.

In Fig. 1 (c) and Fig. 1 (d), we have numerically analyzed the Sagdeev potential (Eq. (3)) and investigated the effects of allowed values of M and α on the profile of the potential-well for both cases of non-relativistic and ultra-relativistic degeneracy. It is remarked that for fixed α (M) value, the increase of Much number M (α values) gives rise to an increase of both the potential depth and amplitude. The energy equation Eq. (9) has been numerically solved for some values of α and M. The potential pulse profiles has been depicted in Figs. 2 (a)-(c). It is obvious that the potential profile becomes taller and narrower by increasing α and M for both types of non-relativistic and ultra-relativistic degeneracy, which is in agreement with above

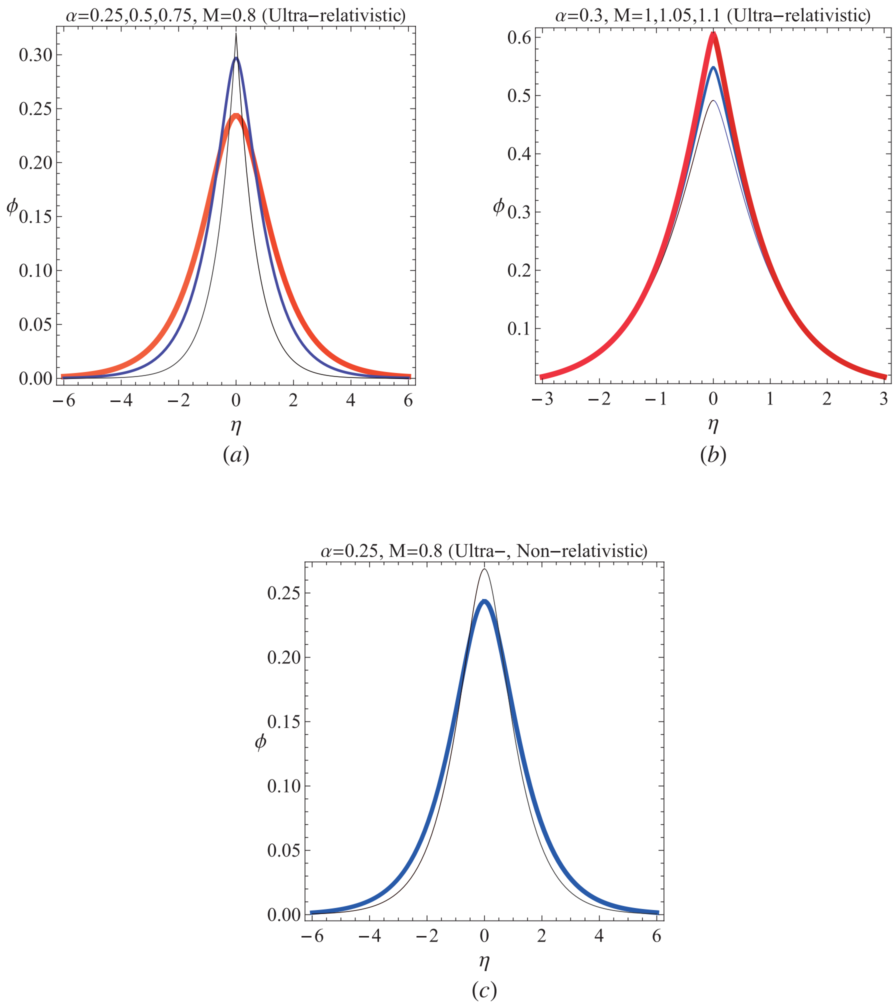

Fig. 2 (a) Ion acoustic solitary wave profiles obtained by numerical solutions of Eq. (9) corresponding to pseudo-potentials shown in Fig. 1 (d) which correspond to the values $\alpha = 0.25$ (black), $\alpha = 0.5$ (blue) and $\alpha = 0.75$ (red). (b) Profiles for different supersonic values of Mach-number $M = 1$ (black), $M = 1.05$ (blue) and $M = 1.1$ (red). (c) Comparison between non-relativistic (thin-line in black) and ultra-relativistic (thick-line in blue) electron/positron degeneracies for similar values of α and M.

result. Also, we note that higher pulses are narrower, while shorter are wider, in agreement with the existing soliton phenomenology. Another important result is that the amplitude of pulse in non-relativistic case is higher than ultra-relativistic one for fixed values of α and M (Fig. 2 (c)). Finally, we note that rarefactive IASWs do not exist for both of non-relativistic and ultra-relativistic degeneracy cases.

6. Conclusion

The Sagdeev-Potential approach was used to investigate the propagation of ion-acoustic solitary waves (IASWs) in electron-positron-ion plasma with ultra-relativistic or non-relativistic degenerate electrons and positrons. The matching criteria of existence of such solitary wave were numerically investigated for both cases of ultra-relativistic or non-relativistic degenerate electrons and positrons. It is remarked that the characteristics of nonlinear IASW propagation differ in the mentioned cases. Both supersonic and subsonic IASWs can propagate in ultra-relativistic case, whereas, only subsonic propagations can occur for non-relativistic case. Only compressive IASWs can propagate in both of desired non-relativistic and ultra-relativistic degenerate plasmas. Also, it was concluded that, the differences tend to amplify by moving towards smaller values of fractional positron-to-electron number-density α. In this work we consider cold ions and the effects of warm ions on IASWs in such plasmas can be investigated in future.

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