Mechanism of Structure-Driven Nonlinear Instability of Double Tearing Mode in Reversed Magnetic Shear Plasmas

Miho JANVIER, Yasuaki KISHIMOTO and Jiquan LI

Department of Fundamental Energy Science, Graduate School of Energy Science, Kyoto University, Gokasho, Uji, Kyoto 611-0011, Japan

(Received 7 June 2010 / Accepted 3 September 2010)

The trigger for the nonlinear destabilization of the double tearing mode (DTM), referred to as a structure-driven instability leading to explosive growth and subsequent collapse, is investigated. We use the reduced MHD equations that solve the evolution of perturbations from an equilibrium deformed by two-dimensional magnetic islands during the slow evolution of the quasi-steady nonlinear regime. By examining conditions near marginal stability (under which the explosive growth is not triggered), we have identified a new secondary instability that starts growing when the magnetic energy of the primary fluctuations associated with the islands reaches a critical level. The energy source of this instability is different from that of the linear DTM; it originates in the spatial deformation due to the DTM-driven magnetic islands and is responsible for the subsequent nonlinear destabilization. The growth rate of this secondary instability is found to be proportional to the magnetic energy, suggesting that it exhibits modulational characteristics.

Keywords: double tearing mode, secondary instability, modulational instability, reduced MHD, magnetic island

DOI: 10.1585/pfr.5.038

Among various resistive modes, the double tearing mode (DTM) induced by two current layers has received considerable attention in magnetically confined fusion plasmas as well as space and astrophysical plasmas owing to its prominent dynamics. For instance, internal transport barriers (ITBs) form in reversed magnetic shear configurations; however, the appearance of DTMs has been observed to restrict such high-performance plasmas, leading to a collapse or disruption [1]. In the solar corona, a similar magnetic configuration can be established in which a shear reversal in the magnetic field triggers the flare dynamics [2]. An interesting phenomenon is the nonlinear destabilization of the DTM, which causes sudden growth of the potential flows and subsequent rapid magnetic reconnection. A flux-structure-driven nonlinear instability [3, 4] as well as a fast reconnection driven by intrinsic flows [5, 6] have been proposed as possible causes. However, the underlying physical mechanisms, especially for the trigger, have not been fully understood. Though such an event was first observed in a cylindrical geometry for the $m/n = 3/1$ mode (where m/n are the poloidal/toroidal harmonics) [3, 4], the same nonlinear dynamics have been captured in a slab geometry, suggesting the universality of the event.

Recently, using the idea that magnetic islands deformation in the nonlinear evolution of the DTM can become a free energy source for a new secondary instability, we explored a methodology that consists in solving the linearized reduced magnetohydrodynamic (RMHD) equations in a slab geometry but under a two-dimensionally deformed equilibrium associated with DTM-driven magnetic islands [7]. We found that a new instability emerges once the magnetic energy exceeds a critical level. Its growth was proposed as the triggering for the explosive dynamics of the DTM. Here, to obtain further insights into the mechanism of this new instability, we first conduct a DTM simulation near the marginal stability at which the nonlinear destabilization occurs and then investigate its characteristics, specifically the triggering conditions and parametric dependence.

The two-field equations for the magnetic flux ψ and stream function ϕ are solved with no equilibrium flow ($\phi_0 = 0$), and the initial equilibrium field configuration is the same as in [8] (rational surfaces separated by a distance $2x_s = 1.6$, for which the resistivity dependence of the linear DTM growth rate is given as $\gamma_{\mathrm{lin}} \sim \eta^{0.57}$). The times are normalized to the Alfvén transit time, τ_A, and the lengths to a characteristic size of the system, a. The normalized resistivity is $\eta \sim 10^{-4}$ for a viscosity $\nu \sim 0$. Ten poloidal harmonics are employed in the simulation, which is sufficient to recover the fast nonlinear dynamics. Here, we control the strength of the instability by changing the system size in the y-direction L_y (equivalently, k_y or instability parameter Δ'). In [7], we have chosen $L_y = 1.2$, which corresponds to a strong nonlinear instability regime much above marginality.

In Fig. 1, we present two simulations with different L_y values, $L_y = 0.75$ (case A) and $L_y = 0.80$ (case B)

author's e-mail: janvier.miho@ay7.ecs.kyoto-u.ac.jp

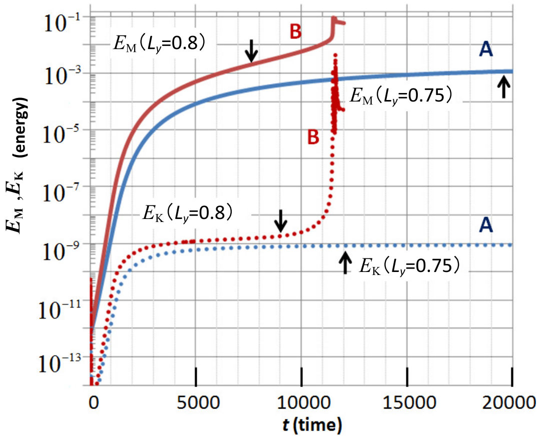

Fig. 1 Time history of the magnetic (solid line) and kinetic (dotted line) energies E_M and E_K with varying box size $2\pi\times L_y$ in the y-direction: L_y = 0.75 (in blue, case A) and L_y = 0.80 (in red, case B). Saturation is found for case A, whereas nonlinear destabilization is found for case B.

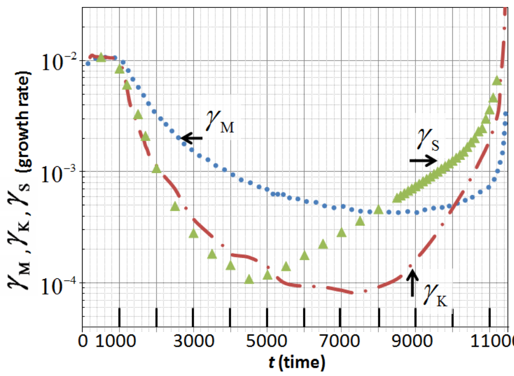

Fig. 2 Comparison of the magnetic and kinetic instantaneous growth rates γ_M and γ_K from case B (L_y = 0.80) and the linear growth rate of the secondary instability γ_S at various equilibria with preexisting magnetic islands.

and the time histories of the total magnetic (solid line) and potential (dotted line) energies E_M and E_K of the fluctuation are shown. The energies (both $\langle|\psi|^2\rangle/2$ and $\langle|\phi|^2\rangle/2$) first evolve exponentially from $t \sim 0$ to $t \sim 1000$ with a slightly higher growth rate in case B than in case A and then enter the nonlinear regime as the growth rate gradually decreases. However, the energies in case A saturate, whereas those in case B exhibit an explosive growth before saturating, suggesting that the critical system size L_y in triggering this abrupt evolution, i.e., L_{yc}, is in the range $0.75 < L_{yc} < 0.8$. Therefore, case B constitutes the reference case (primary simulation) for the nonlinear destabilization of the DTM, and case A is slightly below marginal stability at which the DTM does not resume with an abrupt nonlinear growth. Note that the time of the trigger for the potential in case B is $t \sim 9000$, which is much longer than that observed in the case of $L_y = 1.2$ ($t \sim 1200$) [7].

In typical tearing instabilities, small islands are expected to saturate after a nonlinear stage (Rutherford stage) during which the linear growth rate is reduced [9], as in case A. In case B, however, observations suggest that the islands on each tearing layer continue to grow, and when the magnetic energy exceeds a certain level, enhanced dynamics, i.e., the secondary instability and the subsequent explosive growth, are triggered. From Fig. 1, the energy of the magnetic fluctuations necessary to trigger the instability is roughly estimated as $E_\psi \sim 10^{-3}$.

To understand the trigger dynamics, the definition of the linear growth rate is extended as in [10] to specify an instantaneous growth rate $\gamma_{\rm inst} = \partial_t(\ln E)$ even in the nonlinear regime, which has been plotted for case B in Fig. 2, for both the total magnetic energy (blue dotted line) and the kinetic energy (red dot/dashed line). Note that the y-axis is on a logarithm scale. The growth rates decrease from their linear value $\gamma_{\rm lin} = 10^{-2}$ to their lowest values, at which they remain almost constant before increasing again. The small value reached suggests that the nonlinear regime is near marginal stability, validating the hypothesis of a quasi-steady state and making possible a secondary instability analysis such as that in [7].

Here, the configuration of case B with deformed magnetic islands, taken at different times, constitutes the instantaneous quasi-steady state equilibrium. Numerically, the secondary analysis consists in linearly solving the RMHD equations with this two-dimensional new equilibrium (x and y directions with magnetic islands).

$$\partial_t\tilde{\psi} = -\left[\tilde{\phi}, \psi_E(x,y)\right] - \left[\phi_E(x,y), \tilde{\psi}\right] + \eta\nabla^2\tilde{\psi}, \quad (1)$$

$$\partial_t\nabla^2\tilde{\phi} = -\left[\tilde{\phi}, \nabla^2\phi_E(x,y)\right] - \left[\phi_E(x,y), \nabla^2\tilde{\phi}\right] + \left[\tilde{\psi}, \nabla^2\psi_E(x,y)\right] + \left[\psi_E(x,y), \nabla^2\tilde{\psi}\right]. \quad (2)$$

The equations above, solved as an initial value problem, describe the linear evolution of the new infinitesimal perturbations $\tilde{\psi}$, $\tilde{\phi}$ evolving under the new equilibrium ψ_E, ϕ_E. These equilibrium functions contain all 10 poloidal harmonics from the nonlinear DTM of case B. This result is plotted in Fig. 2 (triangles) for different equilibria in terms of the time of the reference (case B) simulation. Later times therefore correspond to larger islands and higher magnetic energy. Note that because of this specific equilibrium, perturbations are linearly coupled with each other via the Poisson brackets. Therefore, all newly generated harmonics evolve with the same linear growth rate, leading to a global mode. Equations (1) and (2) recover the linear growth rate of the DTM when the magnetic islands are small (up to $t \sim 1000$ in Fig. 2).

In Fig. 2, up to $t \sim 4500$, the linear growth rate decreases as the growing magnetic islands slightly modify the current sheets. This is because less free energy is available owing to the quasi-linear current diffusion in the Rutherford regime. However, this growth rate rapidly and discontinuously increases again from $t \sim 4500$, and around $t \sim 8000$ reaches a level higher than that of the primary magnetic energy, which is defined as $E_M = \langle|\psi|^2\rangle/2$, where $\langle\ \rangle$ represents spatial averaging. This feature clearly displays the characteristics of a *secondary instability* with an energy source different from that of the linear instability, i.e., a current gradient. Interestingly, the growth rate of this instability is found to evolve exponentially in time as $\gamma_s \sim \exp(\alpha t)$ during $5000 \leq t \leq 10000$ (note that the y-axis is on a logarithm scale). During this interval, the instantaneous growth rate of the primary magnetic energy reaches its lowest level and is almost constant ($\gamma_\psi \sim 5 \times 10^{-4}$), as seen in Fig. 2, suggesting that $E_M \sim \langle|\psi|^2\rangle \sim \exp(2\gamma_\psi t)$. These results show that the free energy of the secondary instability is related to the two-dimensional deformation of the magnetic islands due to the growth of the DTM fluctuations, and its growth rate is related to the magnetic energy as $\gamma_s \sim \langle|\psi|\rangle \sim \sqrt{E_M}$. This dependence has not been detected in previous simulations far above marginal stability [7] but is determined for the first time in the present simulation near marginality. We also found that a significant proportion of the magnetic energy originates from the $m = 1$ component.

The instantaneous growth rate for the potential flows in the nonlinear simulation remains at a lower value than that of the secondary instability, but it starts to increase around $t \sim 8000$ and comes to a value similar to that of the new instability around $t \sim 10000$. This suggests that when the secondary instability is weak ($6000 \leq t \leq 7500$), it is masked in the nonlinear simulation due to other nonlinear terms that may appear as dissipation.

Note that the characteristics of the present secondary instability, which has a growth rate proportional to the magnetic energy of the fluctuations associated with the islands, are similar to that of a modulational instability that results from a positive feedback among the pump and the seed fields and the corresponding side bands. Macro-scale zonal flow nonlinearly produced from micro-scale turbulence is a typical example. For modulational instability, the growth rate depends on the pump field energy, which produces a proportional relationship with the pump amplitude.

In conclusion, to investigate the origin of structure-driven nonlinear instability in the DTM, simulations were conducted for a secondary instability analysis by choosing parameters near marginal stability. Thus, we have identified a new instability, the free energy of which originates from two-dimensional deformation due to DTM-driven magnetic islands. Its trigger corresponds to a critical value of the magnetic fluctuation energy and leads to the nonlinear destabilization of the observed DTM dynamics. A noteworthy characteristic is that its growth rate is proportional to the magnetic energy, as in modulational instability. Finally, the slight delay in the triggering time of the nonlinear growth of the magnetic flux after that of the potential flow will be investigated in greater details in a later paper.

[1] Z. Chang *et al.*, Phys. Rev. Lett. **77**, 3553 (1996).
[2] K. Kusano *et al.*, Astrophys. J. **610**, 537 (2004).
[3] Y. Ishii *et al.*, Phys. Plasmas **7**, 4477 (2000).
[4] Y. Ishii *et al.*, Phys. Rev. Lett. **89**, 205002 (2002).
[5] Z.X. Wang *et al.*, Phys. Rev. Lett. **99**, 185004 (2007).
[6] Z.X. Wang *et al.*, Phys. Plasmas **15**, 082109 (2008).
[7] M. Janvier *et al.*, to be published in Plasma Fusion Res. **5** Special Issue, ITC-19 (2010).
[8] P.L. Pritchett *et al.*, Phys. Fluids **23**, 1368 (1980).
[9] P.H. Rutherford, Phys. Fluids **16**, 1903 (1973).
[10] N.F. Loureiro *et al.*, Phys. Rev. Lett. **95**, 235003 (2005).