Trapping, Anomalous Transport, and Quasi-coherent Structures in Magnetically Confined Plasmas*)

Madalina VLAD and Florin SPINEANU

National Institute for Laser, Plasma and Radiation Physics, Association Euratom-MEdC, P.O.Box MG-36, Magurele, Bucharest, Romania

(Received 20 February 2009 / Accepted 27 August 2009)

Strong electrostatic turbulence in magnetically confined plasmas is characterized by trapping or eddying of particle trajectories produced by the $\boldsymbol{E} \times \boldsymbol{B}$ stochastic drift. Trapping is shown to produce strong effects on test particles and on test modes by causing nonstandard trajectory statistics: non-Gaussian distribution, memory effects, and coherence. Trapped trajectories form quasi-coherent structure. Trajectory trapping has strong nonlinear effects on the test modes on turbulent plasmas. We determine the growth rate of drift modes as function of the statistical characteristics of the background turbulence. We show that trapping provides the physical mechanism for the inverse cascade observed in drift turbulence and for the zonal flow generation.

Keywords: plasma turbulence, statistical approache, test-particle transport, Lagrangian method

DOI: 10.1585/pfr.4.053

1. Introduction

A component of particle motion in magnetized plasmas is the stochastic electric drift produced by the electric field of the turbulence and by the confining magnetic field. This drift causes a trapping effect or eddy motion in the turbulence with slow time variation [1]. Typical particle trajectories show sequences of trapping events (trajectory winding on almost closed paths) and long jumps. Numerical simulations have shown that the trapping process completely changes the statistical properties of the trajectories. Particle motion in a stochastic potential has been studied extensively (see the review papers [2–4] and references therein), but the process of trapping was not described until recently.

New statistical methods have been developed [5, 6] that permit determination of the effects of trapping. These are semi-analytical methods based on a set of deterministic trajectories obtained from the Eulerian correlation of the stochastic velocity. Trapping has been shown to cause memory effects, quasi-coherent behavior and non-Gaussian distribution [6]. Trapped trajectories exhibit quasi-coherent behavior and form structures similar to fluid vortices. Diffusion coefficients decrease due to trapping and their scaling in the stochastic-field parameters is modified. We have shown that anomalous diffusion is caused by collisions and average flows. A review of the effects of trapping on test particle statistics and on turbulent transport is presented in the first part of this paper.

The effects of trajectory trapping on the nonlinear dynamics of the test modes for the drift turbulence are presented in the second part of the paper. The semi-analytical methods developed for test particles are extended to test modes in a turbulent magnetized plasmas. Test modes are usually studied for modeling wave-wave interaction in turbulent plasmas [7]. A different perspective is developed here by considering test modes on turbulent plasmas. They are described by gyrokinetic equations with the advection term containing the stochastic $\boldsymbol{E} \times \boldsymbol{B}$ drift whose statistical characteristics are considered known. Test-mode growth rate is determined as a function of these statistical parameters. We develop a Lagrangian approach of the type of that introduced by Dupree [8, 9]. The difference is that the stochastic trajectory trapping is neglected in Dupree's method and consequently the results can be applied only to quasilinear turbulence. Our method takes into account trapping and the consequent nonstandard trajectory statistics, and thus can describe nonlinear effects that appear in strong turbulence. We use results obtained for test particles to determine the average propagator for ion response to a potential perturbation.

The paper is organized as follows. Section 2 presents the test-particle model. Section 3 describes the statistical methods. Section 4 describes the nonlinear effects of trajectory trapping on test-particle statistics and transport. Section 5 describes the problem of test modes in turbulent plasmas for drift turbulence where growth rate and frequency are determined as functions of the statistical characteristics of turbulence. Section 6 analyzes the complex effects of trajectory trapping on drift modes. Section 7 summarizes our conclusions.

author's e-mail: madi@ifin.nipne.ro

*) This article is based on the invited talk at the 14th International Congress on Plasma Physics (ICPP2008).

2. Test Particle Model

Test particle studies rely on known statistical characteristics of the stochastic field determined from experimental studies or numerical simulations. The main goal of these studies is to determine the diffusion coefficients. The statistics of test-particle trajectories provide information on the transport coefficients in turbulent plasmas without the need to address the very complicated problem of self-consistent turbulence that explains the detailed mechanism of generation and saturation of the turbulent potential. The possible diffusion regimes can be obtained by considering various models for the statistics of the stochastic field.

We consider in slab geometry electrostatic turbulence represented by electrostatic potential $\phi^{e}(\boldsymbol{x}, t)$, where $\boldsymbol{x} \equiv (x_1, x_2)$ are the Cartesian coordinates in the plane perpendicular to the confining magnetic field directed along the z axis, $\boldsymbol{B} = B\boldsymbol{e}_z$. The test-particle motion in the guiding center approximation is determined by

$$\frac{d\boldsymbol{x}(t)}{dt} = \boldsymbol{v}(\boldsymbol{x}, t) \equiv -\nabla\phi(\boldsymbol{x}, t) \times \boldsymbol{e}_z, \tag{1}$$

where $\boldsymbol{x}(t)$ is the trajectory of the particle guiding center, ∇ is the gradient in the (x_1, x_2) plane and $\phi(\boldsymbol{x}, t) = \phi^{e}(\boldsymbol{x}, t)/B$. The electrostatic potential $\phi(\boldsymbol{x}, t)$ is considered to be a stationary and homogeneous Gaussian stochastic field, with zero average. It is completely determined by the two-point Eulerian correlation function (EC), $E(\boldsymbol{x}, t)$, defined by

$$E(\boldsymbol{x}, t) \equiv \langle \phi(\boldsymbol{x}', t')\phi(\boldsymbol{x}' + \boldsymbol{x}, t' + t)\rangle . \tag{2}$$

The average $\langle \ldots \rangle$ is the statistical average over the realizations of $\phi(\boldsymbol{x}, t)$, or the space and time average over $\boldsymbol{x}'$ and t'. This function yields three parameters that characterize the (isotropic) stochastic field: the amplitude $\Phi = \sqrt{E(\mathbf{0}, 0)}$, the correlation time τ_c, which is the decay time of the Eulerian correlation, and the correlation length λ_c, which is the characteristic decay distance. These three parameters combine in a dimensionless Kubo number

$$K = \tau_c/\tau_{fl} \tag{3}$$

where $\tau_{fl} = \lambda_c/V$ is the time of flight of the particles over the correlation length and $V = \Phi/\lambda_c$ is the amplitude of the stochastic velocity.

The diffusion coefficient is determined by (see [10])

$$D_i(t) = \int_0^t d\tau\, L_{ii}(\tau) \tag{4}$$

where

$$L_{ij}(t;\, t_1) \equiv \left\langle v_i(\mathbf{0}, 0)\, v_j(\boldsymbol{x}(t), t)\right\rangle \tag{5}$$

is the Lagrangian velocity correlation (LVC). It is obtained using the decorrelation trajectory method, a semi-analytical approach presented below.

Equation (1) represents the nonlinear kernel of the test-particle problem. For simplicity, statistical methods are presented for Eq. (1). The methods can be applied to complex models with other components of motion (particle collisions, average flows, motion along the confining magnetic field, and so on). The effects of these components on transport are discussed in Sec. 4.

3. Nested Subensemble Approach

Trajectory trapping is essentially related to the invariance of the Lagrangian potential. Thus, a statistical method is suitable for the study of this process if it is compatible with the invariance of the potential. Analytical methods with this property were developed only in the last decade. They are known as the decorrelation trajectory method (DTM) [5] and the nested subensemble approach (NSA) [6]. NSA is the development of DTM as a systematic expansion that validates DTM and obtains much more statistical information.

The main idea in NSA is to study the stochastic equation (1) in subensembles of realizations of the stochastic field. First the whole set of realizations R is separated into subensembles $(S1)$ that contain all realizations with given values of the potential and of the velocity in the starting point of the trajectories $\boldsymbol{x} = \mathbf{0}$, $t = 0$:

$$(S1): \quad \phi(\mathbf{0}, 0) = \phi^0, \quad \boldsymbol{v}(\mathbf{0}, 0) = \boldsymbol{v}^0. \tag{6}$$

Then each subensemble $(S1)$ is separated into subensembles $(S2)$ that correspond to fixed values of the second derivatives of the potential in $\boldsymbol{x} = \mathbf{0}$, $t = 0$

$$(S2): \quad \phi_{ij}(\mathbf{0}, 0) \equiv \left.\frac{\partial^2\phi(\boldsymbol{x}, t)}{\partial x_i \partial x_j}\right|_{\boldsymbol{x}=\mathbf{0}, t=0} = \phi^0_{ij} \tag{7}$$

where $ij = 11, 12, 22$. By continuing this procedure up to an order n, we construct a system of nested subensembles. The stochastic (Eulerian) potential and velocity in a subensemble are Gaussian fields but nonstationary and nonhomogeneous, with space- and time-dependent averages and correlations. The correlations are zero in $\boldsymbol{x} = \mathbf{0}$, $t = 0$ and increase with distance and time. The average potential and average velocity in a subensemble depend on the parameters of that subensemble and of the subensembles that include it. They are determined by the Eulerian correlation of the potential (see [6] for details).

The stochastic equation (1) is studied in each highest-order subensemble (Sn). The average Eulerian velocity determines an average motion in each (Sn). Neglecting trajectory fluctuations, the average trajectory in (Sn), $\boldsymbol{X}(t; Sn)$, is obtained from

$$\frac{d\boldsymbol{X}(t; Sn)}{dt} = -\varepsilon_{ij}\frac{\partial\Phi(\boldsymbol{X}, t; Sn)}{\partial X_j}. \tag{8}$$

where ε_{ij} is the anti-symmetric tensor and $\Phi(\boldsymbol{x}, t; Sn)$ is the average potential in the subensemble (Sn), $\Phi(\boldsymbol{x}, t; Sn) = \langle\phi(\boldsymbol{x}, t)\rangle_{Sn}$. This approximation is acceptable because it is performed in the subensemble (Sn) where trajectories are similar as they are super-determined. In addition to the necessary and sufficient initial condition $\boldsymbol{x}(0) = \boldsymbol{0}$, supplementary initial conditions are determined by the subensemble definitions in Eqs. (6-7). The strongest condition is the initial potential $\phi(\boldsymbol{0}, 0) = \phi^0$, which is a conserved quantity in the static case and determines comparable trajectory sizes in a subensemble. Moreover, the amplitude of the velocity fluctuations in $(S\,n)$, the source of the trajectory fluctuations, is zero in the starting point of the trajectories and reaches the value corresponding to the whole set of realizations only asymptotically reducing the differences among the trajectories in $(S\,n)$ and thus their fluctuations.

Trajectory statistics for the whole set of realizations (in particular the LVC) are obtained as weighted averages of these trajectories $\boldsymbol{X}(t; S\,n)$. The weighting factor is the probability that a realization belongs to the subensemble $(S\,n)$, and is analytically determined.

NSA essentially reduces the problem of determining the statistical behavior of stochastic trajectories to calculation of weighted averages of some smooth, deterministic trajectories obtained from the stochastic potential EC. This semi-analytical statistical approach is a systematic expansion that satisfies at each order $n > 1$ all statistical conditions required by the invariance of the Lagrangian potential in the static case. The order $n = 1$ corresponds to the decorrelation trajectory method introduced in [5], for which only the average potential is conserved.

NSA is quickly convergent because the mixing of periodic trajectories, which characterizes this nonlinear stochastic process, is directly described at each order. Results obtained in the first order (the decorrelation trajectory method) for $D(t)$ are essentially not modified in the second order [6]. Thus, the decorrelation trajectory method is a good approximation for determining diffusion coefficients. Second-order NSA is important because it provides detailed statistical information on trajectories, which contributes to the understanding of the trapping process.

4. Trapping Effects on Test Particles

4.1 Trajectory structures

Detailed statistical information about particle trajectories has been obtained by NSA [6]. This method determines the statistics of trajectories that start in points with given values of the potential, and demonstrates the high degree of coherence of the trapped trajectories.

Trapped trajectories correspond to large absolute values of the initial potential. In contrast trajectories starting from points with the potential close to zero have long displacements before decorrelation. These two types of trajectories have completely different statistical characteristics [6].

Trapped trajectories exhibit quasi-coherent behavior. Their average displacement, dispersion and probability distribution function saturate in a time τ_s. Time evolution of the square distance between two trajectories is very slow, showing that neighboring particles have coherent

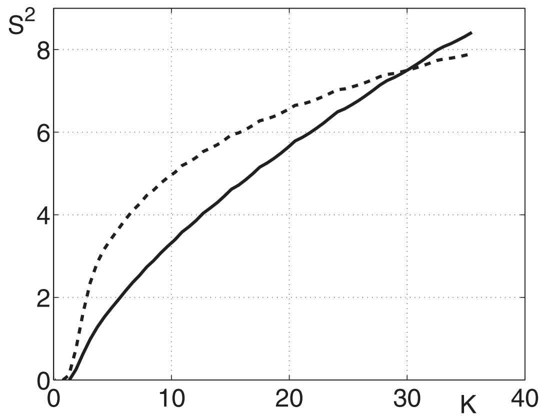

Fig. 1 Average size of the trajectory structures for a Gaussian EC (dashed line) and for an EC that decays as $1/r^2$ (solid line).

motion for a time much longer than τ_s. These trajectories are characterized by a strong clump effect: increase in average square distance is slower than suggested by the Richardson law. The trajectories form structures that are similar to fluid vortices and represent eddying regions. Structure size and buildup time depend on the value of the initial potential. Trajectory structures appear with all sizes, but their characteristic formation time increases with the size. The structures or eddying regions are permanent in static stochastic potentials. The saturation time τ_s is the average time necessary for the formation of the structure. In time-dependent potentials, structures with $\tau_s > \tau_c$ are destroyed and the corresponding trajectories contribute to the diffusion process.

Free trajectories have a continuously growing average displacement and dispersion. They exhibit incoherent behavior and no clump effect.

Figure 1 shows a plot of trajectory-structure average size $S(K)$ in a time-dependent potential. At $K < 1$ structures are absent ($S \cong 0$). At $K > 1$ they appear and continue to increase with K. Dependence on K at large K is a power law with the exponent dependent on the EC of the potential. For Gaussian EC, the exponent is 0.19; for an EC that decays as $1/r^2$ it is 0.35.

4.2 Anomalous diffusion regimes

Test-particle studies connected with experimental measurements of the statistical properties of the turbulence provide the transport coefficients with the condition that there is space-time scale separation between the fluctuations and the average quantities. Particle density advected by the stochastic $\boldsymbol{E} \times \boldsymbol{B}$ drift in turbulent plasmas leads in these conditions to a diffusion equation for the average density with the diffusion coefficient given by the asymptotic value of Eq. (4). Recent numerical simulations [11] confirm a close agreement between the diffusion coefficient obtained from the density flux and the test-particle diffusion coefficient. Experimental based studies of test-particle transport permit us to simplify the complicated self-consistent problem of turbulence and to model the transport coefficients by means of test-particle stochastic advection. The running diffusion coefficient $D(t)$ is defined as the time derivative of the mean square displacement of test particles and is determined according to Eq. (4) as the time integral of the Lagrangian velocity correlation (LVC). Thus, the test-particle approach is based on the evaluation of the LVC for a given fluctuating potential EC.

Turbulent transport in magnetized plasmas is strongly nonlinear. It is characterized by the trapping of trajectories, which strongly influences the transport coefficient and the statistical characteristics of the trajectories. Transport induced by the $\boldsymbol{E}\times\boldsymbol{B}$ stochastic drift in electrostatic turbulence [12] (including the effects of collisions [13], average flows [14], motion along magnetic field [15], and the effect of magnetic shear [16]) and transport in magnetic turbulence [17, 18] have been studied by the decorrelation trajectory method. It was also shown that direct transport (an average velocity) appears in turbulent magnetized plasmas due to the inhomogeneity of the magnetic field [19–21]. This statistical method was developed for the study of complex processes such as zonal flow generation [22, 23].

The results of all these studies are rather unexpected when the nonlinear effects are strong. The diffusion coefficients differ completely from those obtained in quasilinear conditions. A rich class of anomalous diffusion regimes is obtained for which the dependence on the parameters is completely different compared to the scaling obtained in quasilinear turbulence. All the components of particle motion such as parallel motion, collisions, and average flows strongly influence the diffusion coefficients in nonlinear regimes characterized by trajectory trapping.

The reason for these anomalous transport regimes can be understood by analyzing the shape of the correlation of the Lagrangian velocity for particles moving by the $\boldsymbol{E}\times\boldsymbol{B}$ drift in a static potential [24]. In the absence of trapping, the typical LVC for a static field is a function that decays to zero in a time of the order $\tau_{\mathrm{fl}} = \lambda_c/V$. This leads to Bohm-type asymptotic diffusion coefficients $D_B = cV^2\tau_{\mathrm{fl}} = cV\lambda_c$. Only the constant c is influenced by the shape of the stochastic field EC. For $\boldsymbol{E}\times\boldsymbol{B}$ drift, a completely different LVC shape is obtained for static potentials. Figure 2 shows a typical example of the LVC. This function decays to zero in a time of the order τ_{fl}, becomes negative, reaches a minimum, and then decays to zero with a long negative tail. The tail exhibits power-law decay with an exponent that depends on the potential EC [12]. Positive and negative parts compensate such that the integral of $L(t)$, the running diffusion coefficient $D(t)$, decays to zero. Transport in static potential is thus subdiffusive. The long LVC tail shows that stochastic trajectories in static potential have a long time memory. This LVC shape is a consequence of trajectory trapping.

The stochastic process that has the LVC of the type

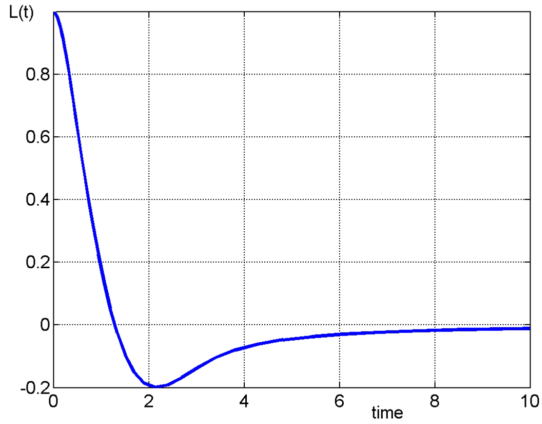

Fig. 2 Typical Lagrangian velocity correlation in static potential.

shown in Fig. 2 is unstable, and any weak perturbation strongly influences transport. A perturbation represents a decorrelation mechanism. Its strength is characterized by a decorrelation time τ_d. Weak perturbations correspond to long decorrelation times, $\tau_d > \tau_{\mathrm{fl}}$. In the absence of trapping, such weak perturbations do not modify the diffusion coefficient because the LVC is zero at $t > \tau_{\mathrm{fl}}$. In the presence of trapping, which is characterized by a long time LVC as in Fig. 2, such perturbation influences the tail of the LVC and destroys the equilibrium between positive and negative parts. Consequently, the diffusion coefficient is a *decreasing function of* τ_d. This means that, when the decorrelation mechanism becomes stronger (τ_d decreases), transport increases because the long time LVC is negative. This behavior differs completely from that obtained in stochastic fields that do not produce trapping. In the latter case, transport is stable to weak perturbations. Decorrelation influences appear only when decorrelation is strong such that $\tau_d < \tau_{\mathrm{fl}}$ and determine the increase of the diffusion coefficient with the increase of τ_d.

This inverse response to perturbations in the presence of trapping is due to the fact that stronger perturbations (with smaller τ_d) release larger numbers of trajectories, which contribute to diffusion.

5. Test Modes on Drift Turbulence

Test-particle trajectories are strongly related to plasma turbulence. Plasma dynamics basically result from the Vlasov-Maxwell system of equations representing conservation laws along particle trajectories for the distribution functions. Studies of plasma turbulence based on trajectories were initiated by Dupree [8, 9] and developed especially in the 1970s (see review paper [7] and references therein). These methods do not account for trajectory trapping and thus apply to the quasilinear regime or to unmagnetized plasmas. A very important problem that has yet to be understood is the effect of the nonstandard statistical characteristics of test-particle trajectories on the evolution of instabilities and turbulence in magnetized plasmas.

We now extend the Lagrangian methods of the type described in [9, 25, 26] to the nonlinear regime characterized by trapping. We study linear modes on turbulent plasma with know statistical characteristics. The dispersion relation for such test modes is determined as a function of the turbulence characteristics. We consider the drift instability in slab geometry with constant magnetic field. The combined effect of the parallel motion of electrons (nonadiabatic response) and the finite Larmor radius of the ions destabilizes the drift waves.

The gyrokinetic equations are linearized not around the unperturbed state as in the linear theory but rather around a turbulent state with known spectrum. The perturbations of the electron and ion distribution functions are obtained from the gyrokinetic equation by the method of characteristics as integrals along test-particle trajectories of the source terms determined by the average density gradient.

Background turbulence produces two modifications to the mode equation one for stochastic $\boldsymbol{E}\times\boldsymbol{B}$ drift that appears in the trajectories and the other for fluctuations in diamagnetic velocity. Both effects are important for ions and depend on turbulence parameters. Electron response is approximately the same as in quiescent plasma.

5.1 Statistics of the characteristics

We start from the basic gyrokinetic equation for constant magnetic field in slab geometry

$$\partial_t f^\alpha - \nabla\phi\times\boldsymbol{e}_z\cdot\nabla f^\alpha + v_z\partial_z f^\alpha - \frac{e_\alpha}{m_\alpha}\left(\partial_z\phi\right)\partial_{v_z} f^\alpha = 0 \quad (9)$$

where α represents the species ($\alpha = \mathrm{e,i}$). Temperatures are constant and $T_\mathrm{e} = T_\mathrm{i}$. A density gradient is taken along the x direction. The solution for the potential in the zero Larmor radius limit is

$$\phi(\boldsymbol{x}, z, t) = \phi_0(\boldsymbol{x} - \boldsymbol{V}_* t, z), \quad (10)$$

where ϕ_0 is the initial condition and $\boldsymbol{V}_*$ is the diamagnetic velocity produced by the density gradient. This shows that the potential is not changed but rather is displaced by the diamagnetic velocity. The wave-type solution corresponds to drift waves that have $\omega = k_y V_*$ and are stable in this limit. The finite Larmor radius effects combined with the nonadiabatic response of the electrons destabilizes the drift waves. Consequently, the amplitude and the shape of the potential are modified, but on a much slower time scale.

Equation (9) is linearized around the turbulent state with potential $\phi(\boldsymbol{x}, t)$. The latter is considered to be Gaussian with known EC. A wave-type perturbation of the potential that is small enough to have negligible influence on the particle trajectories is introduced. Solutions for the electron and ion density perturbations are obtained by the method of characteristics as integrals along particle trajectories in the background potential. The characteristic times for drift turbulence are in the order

$$\tau_\parallel^\mathrm{e} \ll \tau_* \ll \tau_\mathrm{c} \ll \tau_\parallel^\mathrm{i}, \quad (11)$$

where $\tau_\parallel^\mathrm{e}$, $\tau_\parallel^\mathrm{i}$ are the parallel decorrelation times for electrons and ions, respectively ($\tau_\parallel^\mathrm{e,i} = \lambda_\parallel / v_\mathrm{th}^\mathrm{e,i}$ where $\lambda_\parallel$ is the parallel correlation length and $v_\mathrm{th}^\mathrm{e,i}$ the thermal velocity), $\tau_* = \lambda_\mathrm{c}/V_*$ is the characteristic time for the potential drift, and τ_c is the correlation time of the potential. The linear and nonlinear regimes are determined by the position of the time of flight in this ordering. It is much smaller than τ_c and much larger than $\tau_\parallel^\mathrm{e}$ and thus the statistical characteristics of the trajectories essentially depend on the ratio τ_*/τ_fl.

The quasilinear case corresponds to $\tau_*/\tau_\mathrm{fl} \ll 1$, which means turbulence with the amplitude of the $\boldsymbol{E}\times\boldsymbol{B}$ drift smaller than the diamagnetic velocity ($V/V_* \ll 1$). Motion of the potential produces fast decorrelation, and trapping does not appear. The displacement probability is Gaussian and the diffusion coefficient is $D_\mathrm{ql} = V^2\tau_*$.

The nonlinear case corresponds to $\tau_*/\tau_\mathrm{fl} > 1$ ($V/V_* > 1$). Motion of the potential is slow and trajectory structures produced by trapping appear.

Test-particle motion in a drifting potential is obtained by a Galilean transformation from the motion produced by a stochastic $\boldsymbol{E}\times\boldsymbol{B}$ drift and the average velocity V_d. This process was studied in [14]. It was shown that strips of opened contour lines of the effective potential $\phi + xV_\mathrm{d}$ appear due to average velocity V_d, the strips increase in width with V_d until at $V_\mathrm{d} > V$ they completely eliminate the closed contour lines. The Lagrangian correlation of velocity in the presence of an average velocity $V_\mathrm{d} < V$ does not decay to zero as in Fig. 2 for a static potential, but rather has positive asymptotic value at $t \to \infty$. Consequently, transport along the average velocity is superdiffusive in the static potential and diffusive with a large diffusion coefficient (proportional to the average velocity) in the time-dependent potential. Some of the particles are trapped and the rest move on the strips of opened contour lines of the effective potential. The invariance of the Lagrangian velocity distribution shows that the average velocity of the free particles V'_fr fulfills the condition

$$n_\mathrm{fr} V'_\mathrm{fr} = V_\mathrm{d}, \quad (12)$$

where n_fr is the fraction of free trajectories (related to n_tr, the fraction of trapped trajectories, by $n_\mathrm{tr} + n_\mathrm{fr} = 1$). This shows that the free trajectories move with an average velocity that is larger than the Eulerian average velocity ($V'_\mathrm{fr} > V_\mathrm{d}$).

The characteristics of drift turbulence are obtained from the above problem by changing the reference frame to one that moves with the velocity $-V_\mathrm{d}$ (such that the average velocity becomes zero) and taking $V_* = -V_\mathrm{d}$. Accordingly, Eq. (12) leads to

$$n_\mathrm{fr} V_\mathrm{fr} + n_\mathrm{tr} V_* = 0, \quad (13)$$

where $V_{\rm fr} = V'_{\rm fr} - V_{\rm d}$. Thus, trapped particles (structures) are advected by the moving potential while the other particles have an average motion in the opposite direction with a velocity $V_{\rm fr} = -V_* n_{\rm tr}/n_{\rm fr}$. This shows that there are particle flows in opposite directions, induced by the drifting potential if the amplitude of the stochastic $\boldsymbol{E}\times\boldsymbol{B}$ velocity is larger than the velocity of the potential. Displacement probability is split into two parts: the probability of trapped particles and the probability of free particles. The first is a picked function that has constant width and moves with velocity V_*. The second, is a Gaussian-like function with average displacement $\langle y\rangle_{\rm fr} = V_{\rm fr}t = -V_* t\; n_{\rm tr}/n_{\rm fr}$. The displacement probability at $t < \tau_{\rm c}$ is modeled by

$$P(x,y,t) = n_{\rm tr} G(x, y - V_* t; S_x, S_y) + n_{\rm fr} G(x, y - V_{\rm fr} t; S'_x, S'_y) \quad (14)$$

where $G(x,y;S_x,S_y)$ is the two-dimensional Gaussian distribution with dispersion S_x, S_y. For simplicity, we consider the distribution of trapped particles to be a Gaussian function but with small (fixed) dispersion that represents the average size of the structures. The shape of this function does not change much these estimations. Dispersion for the free trajectories grows linearly in time: $S'_x = S_x + 2D_x t$, $S'_y = S_y + 2D_y t$.

5.2 Growth rate of drift modes in turbulent plasma

The average propagator of a mode with frequency ω and wave number $\boldsymbol{k} = \left(k_x, k_y\right)$

$$\Pi = \int_{-\infty}^{t} \mathrm{d}\tau \left\langle \exp\left(-i\boldsymbol{k}\cdot\boldsymbol{x}^{\alpha}(\tau)\right)\right\rangle \exp\left(i\omega(t-\tau)\right) \quad (15)$$

is evaluated using the above results for trajectory statistics. Here, $\boldsymbol{x}^{\alpha}(\tau)$ is the trajectory of the particle of type α in the moving potential integrated backward in time with the condition $\boldsymbol{x}$ at time t. Using Eq. (14) to determine the average in this equation, we obtain for the ion propagator

$$\Pi = -i\mathcal{F}\left[\frac{n_{\rm tr}}{\omega + k_y V_*} + \frac{n_{\rm fr}}{\omega + k_y V_{\rm fr} + i k_i^2 D_i}\right] \quad (16)$$

where

$$\mathcal{F} = \exp\left(-\frac{1}{2}k_i^2 S^2\right). \quad (17)$$

The average ion propagator is thus a function of the size $S(K)$ of the structures and of the fractions of trapped and free particles. The propagator for the electrons is not changed.

The dispersion relation (the quasineutrality condition) is obtained as

$$2 + i\sqrt{\pi}\frac{\omega - k_y V_*}{|k_z|\, v_{\rm Te}} = i\Pi\Gamma_0\left[\omega + V_*\left(k_y + i k_i k_j R_{ij}\right)\right] \quad (18)$$

where $\Gamma_0 = exp(-b)I_0(b)$, $b = k_\perp^2\rho_{\rm L}^2/2$ and $\rho_{\rm L}$ is the ion Larmor radius. The tensor R_{ij} has the dimension of a length and is defined by

$$R_{ji}(\tau,t) \equiv \int_{\tau}^{t}\mathrm{d}\theta' \int_{-\infty}^{\tau-\theta'} \mathrm{d}\theta M_{ji}(|\theta|) \quad (19)$$

where M_{ij} is the Lagrangian correlation

$$M_{ji}(|\theta'-\theta|) \equiv \left\langle v_j\left(\boldsymbol{x}^i(\theta'), z, \theta'\right)\ \partial_2 v_i\left(\boldsymbol{x}^i(\theta), z, \theta\right)\right\rangle, \quad (20)$$

and v_j is the $\boldsymbol{E}\times\boldsymbol{B}$ drift velocity component. The summation rule over the repeated indices is used in Eq. (18).

The approximate solution of the dispersion relation (18) is

$$\omega = k_y V_*^{\rm eff}, \quad (21)$$

$$V_*^{\rm eff} = V_*\frac{\Gamma_0\mathcal{F}(1-n) + 2n}{2 - \Gamma_0\mathcal{F}}, \quad (22)$$

$$\begin{aligned}\gamma \;=\; & \frac{\sqrt{\pi}}{|k_z|\, v_{T\rm e}}\frac{k_y^2\left(V_* - V_*^{\rm eff}\right)\left(V_*^{\rm eff} - nV_*\right)}{2 - \Gamma_0\mathcal{F}} \\ & - n_{\rm fr} k_i^2 D_i \frac{\Gamma_0\mathcal{F}}{2}\left(\frac{2 - \Gamma_0\mathcal{F} n_{\rm tr}}{\Gamma_0\mathcal{F}(2n_{\rm fr} - n_{\rm tr})}\right)^2 \\ & + k_i k_j R_{ij} V_*^{\rm eff}, \end{aligned} \quad (23)$$

where $n = n_{\rm tr}/n_{\rm fr}$.

Several effects appear in the test-mode characteristics due to background turbulence. Ion-trajectory spreading produces diffusion D_i that influences the growth rate (23) in both linear and nonlinear conditions. The term proportional to D_i in Eq. (23) is essentially similar to the result of Dupree although there are influences due to trapping namely an attenuation factor and a change in D_i. Several additional influences appear only in the nonlinear regime. The first is the factor $\mathcal{F}$ given by Eq. (17), which is produced by the trajectory structures and modifies the mode frequency. The ion flows induced by the drifting potential are represented by the fractions $n_{\rm tr}$ and $n_{\rm fr}$. The tensor R_{ij} is determined by the fluctuations in diamagnetic velocity due to the background turbulence. We analyze these processes in the next section.

6. Trapping Effects on the Test Modes

Trajectory trapping has a complex influence on test mode. This can be understood by considering the evolution of drift turbulence starting from a stochastic potential with very small amplitude, which can be deduced from the growth rates of the test modes.

The trajectories are Gaussian, there is no trapping in such potential, and the only effect of background turbulence is ion-trajectory diffusion that produces resonance broadening. The well-known results of drift modes in quasilinear turbulence are obtained

$$\omega = k_y V_* \frac{\Gamma_0}{2 - \Gamma_0},$$

$$\gamma = \frac{\sqrt{\pi}}{|k_z|\, v_{T\rm e}}\frac{\left(k_y V_* - \omega\right)k_y V_*}{2 - \Gamma_0} - k_i^2 D_{\rm ql}, \quad (24)$$

where $D_x = D_y = D_{ql} = V^2\lambda_c/V_*$. This shows that modes with large k are damped due to ion-trajectory diffusion as the amplitude of the potential increases. The spectrum maximum is for $\omega = k_y V_*/2$ and corresponds to $k_\perp \rho_L \sim 1$.

When the nonlinear stage is attained for $V > V_*$, the first effect is produced by the quasi-coherent component of ion motion. The ion-trajectory structures determine the $\mathcal{F}$ factor (17), which modifies the effective diamagnetic velocity (22) and the frequency ω. At this stage, the flows can be neglected ($n_{tr} \cong 0$, $n_{fr} \cong 1$) and $R_{ji} \cong 0$ in Eqs. (21)-(23), so only the $\mathcal{F}$ factor is important. It is interesting to note that this factors appears in Eqs. (21)-(23) as a multiple of Γ_0, although it comes from a different source ($\mathcal{F}$ comes from the propagator, Γ_0 comes from the gyro-average of the mode potential). This shows that trapping or eddying motion has the same attenuation effect on potential as does the gyro-average. The spectrum maximum is obtained for smaller $k_\perp$, determined by the size of the trajectory structures from the condition $k_\perp S \sim 1$. This means that the unstable wave-number range is displaced toward small values. The maximum growth rate is not changed but rather displaced at values of the order $1/S$. Consequently, in this stage, both amplitude and correlation length of the turbulence increase.

At larger background-potential amplitudes, when the fractions of trapped and free ions become comparable, the ion flows induced by the moving potential become important. These flows determine the increase in effective diamagnetic velocity (22) toward the diamagnetic velocity and the modification of the drift-mode growth rate. The latter decreases and for $n_{tr} = n_{fr}$ is negative. The evolution of the amplitude becomes slower and eventually the growth rates vanishes and changes the sign. Thus, ion flows induced by the moving potential damp the drift modes.

Fluctuations in diamagnetic velocity due to background turbulence determine a direct contribution to the growth rate (the tensor R_{ij}). This term is zero for homogeneous and isotropic turbulence and depends strongly on the anisotropy parameters. The $i = j = 1$ component corresponds to zonal flows (modes with $k_y = 0$). Preliminary results show that it appears for trapped particles due to the anisotropy induced by the ion flows.

7. Summary and Conclusions

We investigated the problem of stochastic advection of test particles by the $\boldsymbol{E} \times \boldsymbol{B}$ drift in turbulent plasmas. We showed that trajectory trapping or eddying has complex nonlinear effects on the statistical characteristics of trajectories and transport. Nonlinear effects are very strong for static potentials. Trajectories are non-Gaussian, they possess statistical memory and coherence, and they form structures. These properties persist if the system is weakly perturbed by time variation of the potential or by other components of motion (collisions, poloidal rotation, parallel motion). The memory effect (long tail of the LVC) determines anomalous diffusion regimes.

Trajectory trapping also influences the evolution of turbulence. We presented recent results for test modes on turbulent plasmas. These results are based on a Lagrangian method that takes into account ion trapping or eddying. Drift-mode growth rates and frequencies in turbulent plasma are estimated as functions of the characteristics of turbulence. The effects of background turbulence appear in particle trajectories (characteristics of Vlasov equations) and in fluctuations in diamagnetic velocity produced by density fluctuations. We showed that the nonlinear process of trapping, which determines the nonstandard statistical properties of trajectories, has a very strong and complex influence on the evolution of turbulence that appears when the amplitude of the $\boldsymbol{E} \times \boldsymbol{B}$ drift becomes larger than the diamagnetic velocity.

This work presents a different physical perspective on the nonlinear evolution of drift waves. The main role is played by ion trapping in the stochastic potential that moves with diamagnetic velocity. We showed that the moving potential determines ion flows when the amplitude of the $\boldsymbol{E}\times\boldsymbol{B}$ velocity is greater than the diamagnetic velocity. Some of the ions are trapped and move with the potential while the rest of the ions drift in the opposite direction. These opposite (zonal) flows compensate such that the average velocity is zero. The evolution of turbulence toward large wavelengths (the inverse cascade) is due to ion trapping, which averages the potential and decreases the effective diamagnetic velocity. Ion flows produced by the moving potential determine the decay of the growth rate and eventually the damping of the drift modes. Ion flows also generate zonal flows due to nonlinear interaction with fluctuations in diamagnetic velocity.

Acknowledgements

We acknowledge financial support from the Romanian Ministry of Education and Research under Ideas-Exploratory Research Project No. 573 and Project LAPLAS 2 No. PN.09,39.03.01.

[1] R.H. Kraichnan, Phys. Fluids **19**, 22 (1970).
[2] W.D. McComb, *The Physics of Fluid Turbulence* (Clarendon, Oxford, 1990).
[3] G. Falkovich, K. Gawedzki and M. Vergassola, Rev. Mod. Phys. **73**, 913 (2001).
[4] R. Balescu, *Aspects of Anomalous Transport in Plasmas* (Institute of Physics Publishing (IoP), Bristol and Philadelphia, 2005).
[5] M. Vlad, F. Spineanu, J.H. Misguich and R. Balescu, Phys. Rev. E **58**, 7359 (1998).
[6] M. Vlad and F. Spineanu, Phys. Rev. E **70**, 056304 (2004).
[7] J.A. Krommes, Phys. Reports **360**, 1 (2002).
[8] T.H. Dupree, Phys. Fluids **9**, 1773 (1966).
[9] T.H. Dupree, Phys. Fluids **15**, 334 (1972).
[10] G.I. Taylor, Proc. London Math. Soc. **20**, 196 (1921).
[11] R. Basu, T. Jessen, V. Naulin and J.J. Rasmussen, Phys. Plasmas **10**, 2696 (2003).

[12] M. Vlad, F. Spineanu, J.H. Misguich, J.-D. Reusse, R. Balescu, K. Itoh and S.-I. Itoh, Plasma Phys. Control. Fusion **46**, 1051 (2004).
[13] M. Vlad, F. Spineanu, J.H. Misguich and R. Balescu, Phys. Rev. E **61**, 3023 (2000).
[14] M. Vlad, F. Spineanu, J.H. Misguich and R. Balescu, Phys. Rev. E **63**, 066304 (2001).
[15] M. Vlad, F. Spineanu, J.H. Misguich and R. Balescu, Nuclear Fusion **42**, 157 (2002).
[16] I. Petrisor, M. Negrea and B. Weyssow, Physica Scripta **75**, 1 (2007).
[17] M. Vlad, F. Spineanu, J.H. Misguich and R. Balescu, Phys. Rev. E **67**, 026406 (2003).
[18] M. Negrea, I. Petrisor and R. Balescu, Phys. Rev. E **70**, 046409 (2004).
[19] M. Vlad, F. Spineanu and S. Benkadda, Phys. Rev. Lett. **96**, 085001 (2006).
[20] M. Vlad, F. Spineanu and S. Benkadda, Phys. Plasmas **15**, 032306 (2008).
[21] M. Vlad, F. Spineanu and S. Benkadda, Plasma Phys. Control. Fusion **50**, 065007 (2008).
[22] R. Balescu, Phys. Rev. E **68**, 046409 (2003).
[23] R. Balescu, I. Petrisor and M. Negrea, Plasma Phys. Control. Fusion **47**, 2145 (2005).
[24] M. Vlad and F. Spineanu, Physica Scripta **T107**, 204 (2004).
[25] F. Spineanu and M. Vlad, Phys. Lett. A **133**, 319 (1988).
[26] M. Vlad, F. Spineanu and J.H. Misguich, Plasma Phys. Control. Fusion **36**, 95 (1994).