Modeling Solar Wind Turbulence: The Kolmogorov-like Way[*)]

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The spectral energy distributions of the magnetohydrodynamic (MHD) fluctuations of the solar wind turbulence are derived using the dimensional arguments a la Kolmogorov within the framework of the Hall magnetohydrodynamics. While the velocity and the magnetic field fluctuations are dynamically related, the density fluctuations could behave as a passive scalar and be simply convected by the velocity or the magnetic field fluctuations. The Hall effect removes the degeneracy of the ideal Alfvénic spectra of the velocity and the magnetic fluctuations, at spatial scales shorter than or equal to the ion- inertial scale, adding steeper branches to the ideal MHD spectra. Which spectrum would the density fluctuations, behaving as a passive scalar, follow in such a case? The answer leads to the interesting consequence that the electron density fluctuations should follow the magnetic spectra since the electrons are frozen to the magnetic field and the ion density fluctuations should follow the kinetic energy spectra as ions carry the inertia. Thus the electron and the ion density would have different spectra at spatial scales equal to and smaller than the ion-inertial scale. However this raises the issue of the quasineutrality that must be maintained at each scale within the Hall-MHD. One way to accommodate both the quasineutrality as well as the electron- magnetic freezing in the Hall MHD is to discard the passive nature of the density fluctuations; they must be dynamically active in the turbulence. The quasineutrality could also be restored by a third species of particles providing a stationary background.

Keywords: solar wind, turbulent spectra, Hall effect

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1. Introduction

That the solar wind is turbulent has been known for a long time. Fluctuations in the density, the velocity and the magnetic fields exist on three major scales: (1) 11 year solar cycle related variations and fast stream - slow stream interactions due to solar rotation; (2) transient disturbances originating on the sun and propagating out such as those associated with solar flare caused blast wave with energy $\sim 10^{32}$ erg and (3) on hours or less scales are the waves and turbulence in the plasma. For example the Alfvén waves fluctuations have power law spectra k^{-p}, $p \sim 5/3$ along with other values of p. Key observations of the solar wind turbulence are: 1. velocity, density, magnetic field and temperature vary in time; 2. magnetohydrodynamics accounted for fluctuations reasonably well, particularly the shear Alfvén waves, indicating that the magnetosonic waves are damped by kinetic effects; 3. Alfvén waves are found always propagating outwards from the sun; 4. similarity between the power spectra of the magnetic fluctuations and the velocity fluctuations for an isotropic magnetofluid as well as fluid turbulence; 5. turbulence is driven by stream - shear instabilities. The Alfvén waves being the exact solutions lack evolution. However, the solar wind turbulence shows evolution. Perhaps it is not all Alfvénic; 6. Voyager and Helios missions provided observations from 0.3 to > 30 AU; 7. Reduced spectra are obtained by averaging over the two directions perpendicular to the solar wind velocity (V_s). The study of the solar wind turbulence is of immense importance on several counts. The solar wind is a distant plasma system that is accessible to direct observations of nonlinear processes such as the wave-particle and wave-wave interactions which have essential bearing on the propagation of the cosmic rays. The coupling of the solar wind and the magnetosphere defines the solar-terrestrial relationship. The turbulence modifies the transport processes with consequences for the space weather.

The reduced spectra of the fluctuations are obtained by averaging over the two directions perpendicular to the solar wind velocity V_s. The spectra are a function of the wavenumber along V_s. The spectral energy distributions of the velocity and the magnetic field fluctuations in the solar wind are now known in a wide frequency range, starting from much below the proton cyclotron frequency (0.1 - 1 Hz) to hundreds of Hz. The inferred power spectrum [1] of magnetic fluctuations (Fig. 1) consists of multiple segments- a Kolmogorov like branch ($\propto k^{-5/3}$) flanked,

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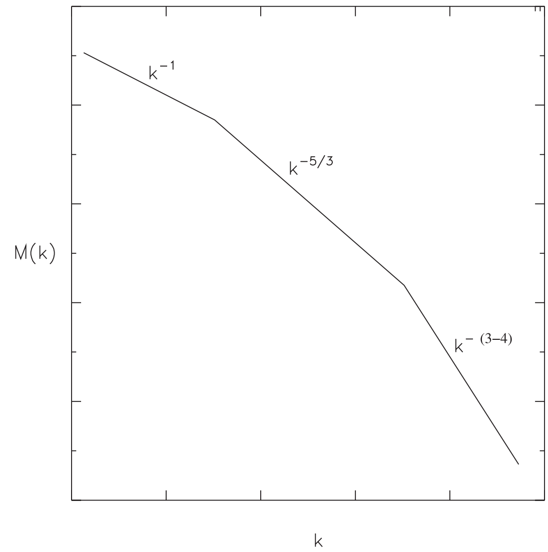

Fig. 1 Schematic representation of the observed magnetic energy spectrum in the solar wind on a log-log scale.

on the low frequency end by a flatter branch($\propto k^{-1}$) and, on the high frequency end, by a much steeper branch ($\propto k^{-\alpha_1}, \alpha_1 \simeq$ 3-4). Attributing the Kolmogorov branch ($\propto k^{-5/3}$) to the standard inertial range cascade, initial explanations invoked dissipation [2] processes, in particular, the collisionless damping of Alfvén and magnetosonic waves , to explain the steeper branch ($\propto f^{-\alpha_1}$, $\alpha_1 \simeq$ 3-4). However, a recent critical study has concluded that damping of the linear Alfvén waves via the proton cyclotron resonance and of the magnetosonic waves by the Landau resonance, being strongly k (wave vector) dependent, is quite incapable of producing a power-law spectral distribution of magnetic fluctuations, the damping mechanisms lead, instead, to a sharp cutoff in the power spectrum [3]. Cranmer and Ballogoeijen [4] have however, demonstrated a weaker than an exponential dependence of damping on the wave vector by including kinetic effects. However it is still steeper than that required for explaining the observed spectrum.

An alternative possibility, suggested by Ghosh *et al.* [5], links the spectral break and subsequent steepening to a change in the controlling invariants of the system in the appropriate frequency range. Stawicki *et al.* [6] have invoked the short wavelength dispersive properties of the magnetosonic/whistler waves to account for the steepened spectrum and christened it as the spectrum in the dispersion range. Krishan and Mahajan [7, 8], however, invoke the Hall effect to model the steepened part of the spectrum, and this should be correct since the steepening begins at a scale close to the ion-inertial scale, a hallmark of the Hall effect. The incompressible turbulence has no associated density fluctuations. The density fluctuations, produced by an independent mechanism, could be convected by the velocity and the magnetic field fluctuations as a passive scalar or they may be concomitantly generated in compressible turbulence along with the other fluctuations. The relation between the velocity and the magnetic field fluctuations and the invariants of the Hall-MHD are derived in the next section. The power spectra of fluctuations within the Hall-MHD invoking the new invariant, the generalized helicity, are derived in Sec. 3. The dilemma of the density fluctuations, whether being convected by the velocity or the magnetic fluctuations at spatial scales near the ion-inertial scale, is discussed in Sec. 4 and the paper ends with a section on conclusion.

2. Hall-MHD and Invariants

In the HALL-MHD (HMHD) comprising of the two fluid Model, the electron fluid equation is given by

$$m_e n_e \left[\frac{\partial \underline{V}_e}{\partial t} + (\underline{V}_e.\nabla)\underline{V}_e\right] = -\nabla p_e - e n_e \left[\underline{E} + \frac{1}{c}\underline{V}_e \times \underline{B}\right]. \quad (1)$$

Assuming inertialess electrons ($m_e \to 0$), the electric field is found to be:

$$\underline{E} = -\frac{1}{c}\underline{V}_e \times \underline{B} - \frac{1}{n_e e}\nabla p_e \quad (2)$$

The ion fluid equation is :

$$m_i n_i \left[\frac{\partial \underline{V}_i}{\partial t} + (\underline{V}_i.\nabla)\underline{V}_i\right] = -\nabla p_i + e n_i \left[\underline{E} + \frac{1}{c}\underline{V}_i \times \underline{B}\right]. \quad (3)$$

Substitution for E from the inertialess electron Eq. begets:

$$m_i n_i \left[\frac{\partial \underline{V}_i}{\partial t} + (\underline{V}_i.\nabla)\underline{V}_i\right] = -\nabla(p_i + p_e) + \frac{1}{c}\underline{J} \times \underline{B}. \quad (4)$$

The magnetic induction equation becomes:

$$\frac{\partial \underline{B}}{\partial t} = -c\nabla \times \underline{E} = \nabla \times (\underline{V}_e \times \underline{B}), \quad (5)$$

where $\underline{B}$ is seen to be frozen to electrons. Substituting for $\underline{V}_e = \underline{V}_i - \underline{J}/en_e$, one gets :

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times \left[\left(\underline{V}_i - \frac{\underline{J}}{en_e}\right) \times \underline{B}\right]. \quad (6)$$

We see that $\underline{B}$ is not frozen to the ions. Here $n_e = n_i$.

The Hall term dominates for $(n_e ec)^{-1}\underline{J} \times \underline{B} \geq \underline{V}_i \times \underline{B}/c$ or the length scale $L \leq M_A c/\omega_{pi}$ and the time scale $T \geq \omega_{ci}^{-1}$ where M is the Alfvénic Mach number and ω_{pi} is the ion plasma frequency.

The Hall term decouples electron and ion motion on ion inertial length scales and ion cyclotron times. The importance of the nonlinear Alfvénic state for MHD prompts one to speculate if an analogous exact solution exists for the Hall MHD [10], a system which encompasses MHD. We write the equations in a dimensionless form. The magnetic and the velocity fields are respectively normalized by the uniform ambient field B_0 and the Alfvén speed $V_{\rm Ai} = B_0/\sqrt{4\pi\rho_{\rm i}}$, where $\rho_{\rm i}$ is the uniform ion mass density. The time and the space variables are normalized, respectively, with the Alfvén travel time $t_{\rm A} = L/V_{\rm Ai}$, and a scale length L. In these units the following dimensionless equations

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times [(\underline{V}_{\rm i} - \epsilon \nabla \times \underline{B}) \times \underline{B}] \tag{7}$$

$$\frac{\partial (\nabla \times \underline{V}_{\rm i})}{\partial t} = \nabla \times [\underline{V}_{\rm i} \times (\nabla \times \underline{V}_{\rm i}) - \underline{B} \times (\nabla \times \underline{B})] \tag{8}$$

constitute the Hall-MHD in the incompressible limit. Here $\epsilon = \lambda_{\rm i}/L = c/\omega_{\rm pi} L = V_{\rm Ai}/L\omega_{\rm ci}$ and $\omega_{\rm pi} = (4\pi e^2 n_{\rm i}/m_{\rm i})^{1/2}$ is the ion plasma frequency, $\lambda_{\rm i}$ is the ion inertial length. Equation (8) has been obtained by taking the curl of the equation of motion of the ion fluid (Eq. (4)). We split the fields into their ambient and the fluctuating parts:

$$\underline{B} = \widehat{e_z} + \underline{b}, \qquad \underline{V}_{\rm i} = \underline{V}_0 + \underline{v}, \tag{9}$$

where $\underline{V}_0 = 0$ and substitute in Eqs. (7) and (8) to get :

$$\frac{\partial \underline{b}}{\partial t} = \nabla \times [(\underline{v} - \epsilon \nabla \times \underline{b}) \times \widehat{e_z} + (\underline{v} - \epsilon \nabla \times \underline{b}) \times \underline{b}], \tag{10}$$

$$\frac{\partial}{\partial t}(\nabla \times \underline{v}) = \nabla \times [\underline{v} \times (\nabla \times \underline{v}) + (\nabla \times \underline{b}) \times \widehat{e_z} + (\nabla \times \underline{b}) \times \underline{b}]. \tag{11}$$

Assuming a plane wave form $(\underline{V}_k, \underline{B}_k)\exp(ikz - i\omega t)$ we get the linear relations:

$$\underline{V}_k - \epsilon \nabla \times \underline{B}_k = -\frac{\omega}{k}\underline{B}_k, \tag{12}$$

$$\nabla \times \underline{B}_k = -\frac{\omega}{k}\nabla \times \underline{V}_k. \tag{13}$$

The solution of Eqs. (12) and (13) furnishes:

$$\nabla \times \underline{B}_k = \lambda \underline{B}_k, \quad \lambda^2 = k^2,$$
$$\underline{B}_k = \alpha(k)\underline{V}_k, \quad \alpha = -\frac{\omega}{k}, \tag{14}$$

and the dispersion relation is:

$$\alpha = -\frac{\epsilon k}{2} \pm \left(\frac{\epsilon^2 k^2}{4} + 1\right)^{\frac{1}{2}}. \tag{15}$$

One notices that the $\underline{V}_k$, $\underline{B}_k$ relation of the waves is now k dependent, the waves are dispersive and that they are nonlinear since for $\underline{b}$ given by Eqs. (12) and (13), the nonlinear terms in Eqs. (10) and (11) vanish. For $k \ll 1$,

$$\alpha \to \pm 1, \omega \to \mp k \tag{16}$$

reproducing the k independent MHD Alfvénic relationship for both the co- and the counter propagating waves. For $k \gg 1$, it is easy to recognize, in analogy with the linear theory, that the α_+ wave is the shear-cyclotron branch, while the α_- represents the compressional-whistler mode. The frequency of the α_+ wave approaches the ion gyro frequency. The $\underline{V}_k$, $\underline{B}_k$ relation would now give different spectral distributions for the kinetic and the magnetic energy. The Hall-MHD also supports an additional invariant called the generalized helicity. The additional invariant and its cascading characteristics along with the new spectral relations arising from the new $\underline{V}_k$, $\underline{B}_k$ relation are discussed in the next section.

The incompressible ideal and the Hall- MHD turbulence supports two global invariants: the total energy E and the magnetic helicity $H_{\rm M}$ defined as

$$\text{Total Energy } E = \frac{1}{2}\int (V^2 + B^2)\mathrm{d}^3x = \frac{1}{2}\sum_k |V_k|^2 + |B_k|^2, \tag{17}$$

$$\text{Magnetic Helicity } H_{\rm M} = \int \boldsymbol{A} \cdot \boldsymbol{B} \mathrm{d}^3x = \sum_k \frac{i}{k^2}(\boldsymbol{k} \times \boldsymbol{B}_k) \cdot \boldsymbol{B}_{-k}, \tag{18}$$

where $\boldsymbol{A}$ is the vector potential.

The additional invariant of the Hall-MHD system, the generalized helicity [11] is defined as:

$$H_{\rm G} = \int \mathrm{d}^3x\, (\underline{A} + \epsilon \underline{V}_{\rm i}) . (\underline{B} + \epsilon \nabla \times \underline{V}_{\rm i}) = \sum \left(i\frac{\underline{k} \times \underline{B}_k}{k^2} + \epsilon \underline{V}_k\right) . (\underline{B}_k + i\epsilon \underline{k} \times \underline{V}_k), \tag{19}$$

where $\underline{A}$ is the vector potential. Notice that $H_{\rm G} - H_{\rm M}$ is a combination of the kinetic and the cross helicities.

3. Power Spectra of Solar Wind Fluctuations

The solar wind turbulence is modeled in terms of the magnetohydrodynamic fluctuations. An assumed input spectrum k^{-1} of the Alfvén waves is believed to decay to generate the Kolmogorov $k^{-5/3}$ spectrum. since the Alfvénic fluctuations are characterized by velocity fluctuation $\underline{V} = \pm \underline{B}$, the velocity and the magnetic fluctuations have identical spectra. The question of the origin of the k^{-1} spectrum has as yet no satisfactory answer although such a spectrum is observed under different and disparate circumstances related to self-organized criticality. One alternative is to derive the spectra using the dimensional arguments of the Kolomogorov hypotheses according to which the spectral cascades proceed at a constant rate governed by the eddy turn over time $(k\boldsymbol{V}_k)^{-1}$. The single power law $(k^{-5/3})$ Kolmogorov spectrum is derived invoking the cascade of a single invariant, the total energy. The interaction between near neighbour wave vectors such that $K_3 = K_1 \sim K_2$, K_1 K_2 and $K_3 = 2K_1$ leads to cascade of energy to large K or small spatial scales. In the presence of two invariants as in 2D turbulence with total energy and the enstrophy as the two invariants there is a dual cascade as discussed by Hasegawa (1985) [9] and a fraction of the energy cascades to large spatial scales as verified by numerical simulations. The dual cascade originates at the merging point of the two spectral branches which is also identified as the energy injection scale. Thus the region around the merging point is not in the inertial range. The regions away from this point are in the inertial range and the locality is well satisfied there. The ideal MHD supports two invariants, the total energy and the magnetic helicity, and a dual cascade is expected as discussed later in the paper. This is also supported by the observed solar wind spectra. Again the spectral break region, identified as the energy injection region, is not in the inertial region.

The case of the Hall- MHD brings in a third invariant the generalized helicity. Thus it is evident while the Kolmogorov conditions are applicable individually to each spectral branch, the merging region of the two branches is not strictly in the inertial range. As discussed by Hasegawa, the energy injected in this region undergoes a dual cascade and this region may possess additional structure. In conclusion the locality may not be satisfied in the immediate neighbourhood of these regions but the rest of the spectral branch well satisfies the locality condition. Now about the cascade rate: The Iroshnikov- Kraichnan phenomenology uses rate of cascade as $(kV_k)(V_k/V_A)$ and obtain a $k^{-3/2}$ spectrum instead of the $k^{-5/3}$ obtained by using the cascade rate as (kV_k). Now (kV_k) is greater than or equal to $(kV_k)(V_k/V_A)$ for V_k less than or equal to V_A, the Alfve'n velocity. The condition $V_k \sim B_k \sim B_0$ may exist sometime in the solar wind but $B_k \gg B_0$ must be extremely rare. Thus we have decided to use the hydrodynamic rate kV_k. Besides the solar wind experts insist that they observe $(-5/3)$ spectrum and not $(-3/2)$! (Goldstein *et al.* 1995). With these qualifying remarks we can begin deriving the spectra for each invariant.

For ε_E denoting the constant cascading rate of the total energy E, we get the dimensional equality

$$(kV_k)\frac{(V_k^2 + B_k^2)}{2} = \varepsilon_{\rm E}. \tag{20}$$

where the wavevector k has been taken to be parallel to the local ambient magnetic field. The omnidirectional spectral distribution function $W_E(k)$ (kinetic energy per gram per unit wave vector, $\frac{V_k^2}{2k}$), then, takes the form

$$W_{\rm E}(k) = 2^{-1/3}\,(\varepsilon_{\rm E})^{\frac{2}{3}}\,[1 + (\alpha)^2]^{-\frac{2}{3}} k^{-\frac{5}{3}} \tag{21}$$

and

$$M_E(k) = (\alpha)^2 W_E(k), \tag{22}$$

where $M_E(k) = (B_k^2/2k)$ is the similarly defined omnidirectional spectral distribution function of the magnetic energy density. The identical spectra ($k^{-5/3}$)of the magnetic and the velocity fluctuations in the absence of the Hall effect are recovered for $\alpha = 1,\ \epsilon = 0$.

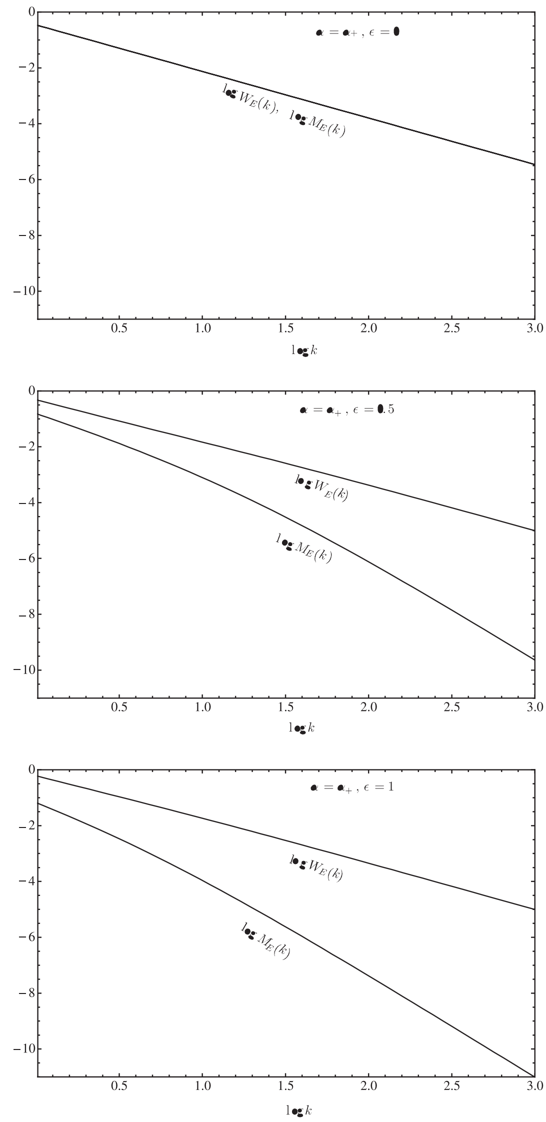

Fig. 2 Power spectra (log-log scale) of the kinetic energy density $W_E(k)$ and the magnetic energy density $M_E(k)$ derived from the total energy invariant E with Hall effect for the α_+ root.

The spectral distributions $W_E(k)$ and $M_E(k)$ are shown in Figs. 2 and 3 respectively, for the two roots α_+ and α_- for different values of the Hall parameter ϵ. The steepening of the spectra towards large values of k is evident for nonzero values of ϵ.

The spectral distributions obtained from the cascading of the magnetic helicity are found to be:

$$W_H(k) = 2^{-1}\,(\varepsilon_H)^{\frac{2}{3}}\ (\alpha)^{\frac{-4}{3}} k^{-1}, \tag{23}$$

and

$$M_H(k) = (\alpha)^2 W_H(k). \tag{24}$$

Again the identical spectra (k^{-1})of the magnetic and the

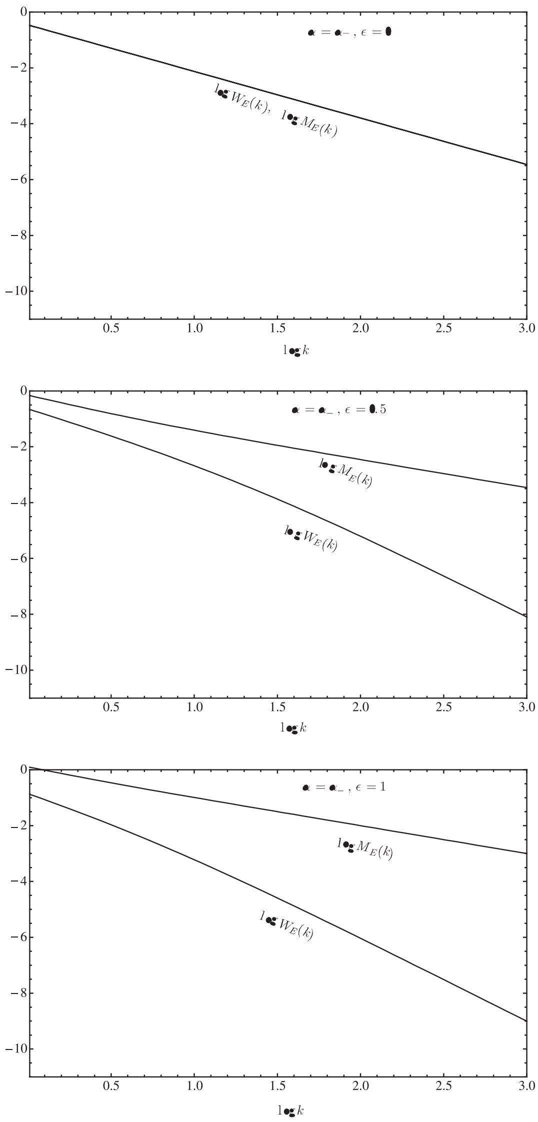

Fig. 3 Power spectra (log-log scale) of the kinetic energy density $W_{\rm E}(k)$ and the magnetic energy density $M_{\rm E}(k)$ derived from the total energy invariant E with Hall effect for the α_- root.

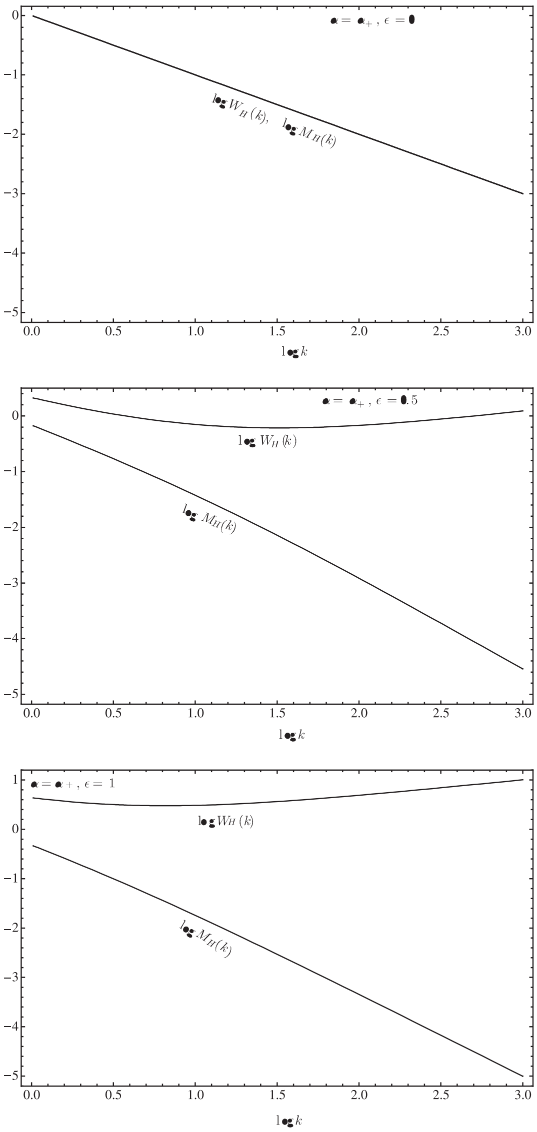

Fig. 4 Power spectra (log-log scale) of the kinetic energy $W_{\rm H}(k)$ and the magnetic energy $M_{\rm H}(k)$ derived from the magnetic helicity invariant $H_{\rm M}$ with Hall effect for α_+ root.

velocity fluctuations in the absence of the Hall effect are recovered for $\alpha = 1,\ \epsilon = 0$.

The spectral distributions $W_{\rm H}(k)$ and $M_{\rm H}(k)$ are shown in Figs. 4 and 5 for the two roots α_+ and α_- for different values of the Hall parameter ϵ. The steepening of the magnetic spectra towards large values of k is evident for nonzero values of ϵ.

The cascading of the generalized helicity with a constant rate $\varepsilon_{\rm G}$, using the relation $B_k = \alpha(k)\underline{V}_k$ gives

$$(kV_k)\left[k^{-1}g(k)V_k^2\right] = \varepsilon_{\rm G}, \tag{25}$$

$$g(k) = (\alpha + \epsilon k)^2,$$

leading to the spectral energy distributions :

$$W_{\rm G}(k) = (\varepsilon_{\rm G})^{\frac{2}{3}}\,[g(k)]^{-\frac{2}{3}}k^{-1}, \tag{26}$$

and

$$M_{\rm G}(k) = (\alpha)^2 W_{\rm G}(k).$$

The spectral distributions $W_{\rm G}(k)$ and $M_{\rm G}(k)$ are shown in Figs. 6 and 7, respectively, for the two roots α_+ and α_- for different values of the Hall parameter ϵ. The generalized helicity reduces to the magnetic helicity for $\epsilon = 0$ and $\alpha_\pm = \pm 1$. The steepening of the spectra towards large values of k is evident for nonzero values of ϵ.

It is clear that the spectral distributions $(W_{\rm E}, M_{\rm E}, W_{\rm H}, M_{\rm H}, W_{\rm G}, M_{\rm G})$ reduce to the ones obtained in the MHD case for $\alpha = 1, \epsilon = 0$. For large k, α reduces to the two values: $\alpha_-\epsilon k$ and $\alpha_+ = \epsilon^{-1}k^{-1}$. For $\alpha = \epsilon^{-1}k^{-1}$, corresponding to the shear Alfvén wave, one can determine the kinetic and the magnetic energy spectra using again the receipe for stringing together

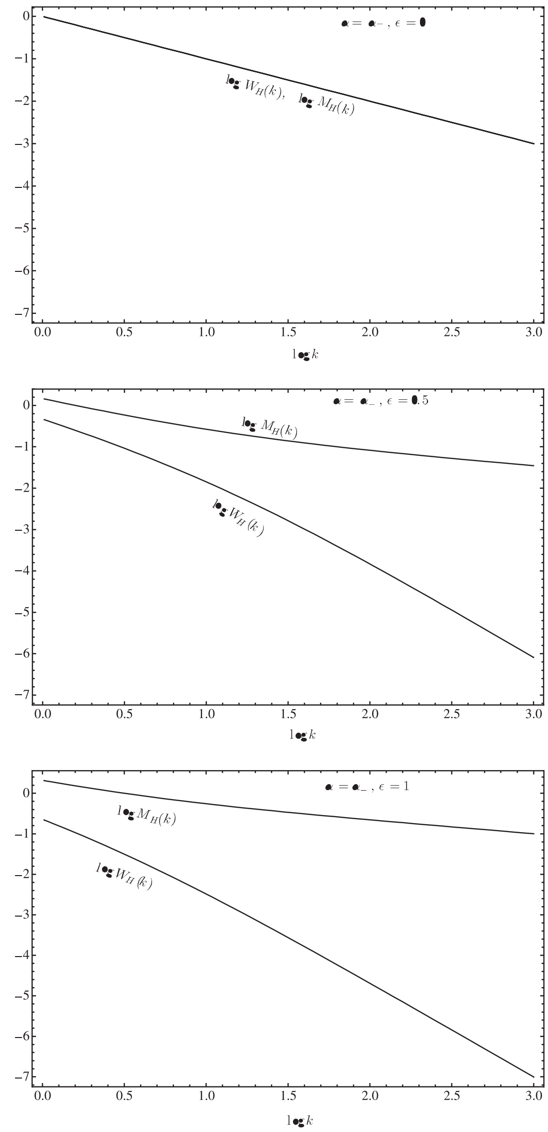

Fig. 5 Power spectra (log-log scale) of the kinetic energy $W_{\mathrm{H}}(k)$ and the magnetic energy $M_{\mathrm{H}}(k)$ derived from the magnetic helicity invariant H_{M} with Hall effect for α_- root.

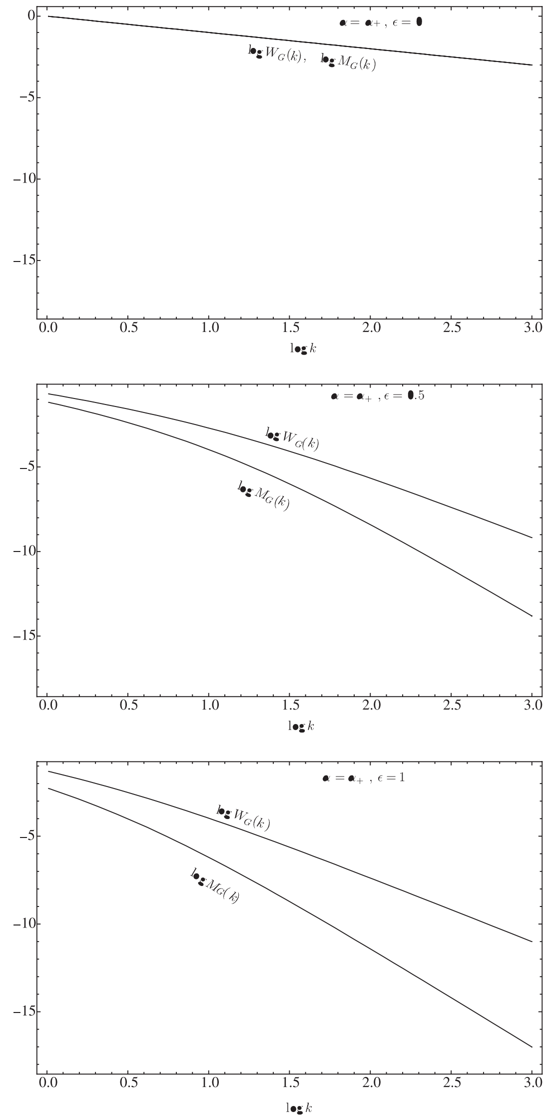

Fig. 6 Power spectra (log-log scale) of the kinetic energy density $W_{\mathrm{G}}(k)$ and the magnetic energy density $M_{\mathrm{G}}(k)$ derived from the generalized helicity invariant H_{G} with Hall effect for α_+ root.

the various spectral branches. This spectra is shown in Fig. 8 where the low k end (k^{-1}) and ($k^{-5/3}$) are derived in the ideal MHD regime and the high k end ($k^{-5/3}$) and ($k^{-7/3}$) for the kinetic energy and ($k^{-11/3}$) and ($k^{-13/3}$) for the magnetic energy) are derived in the HMHD regime. This is in accordance with the observed magnetic spectra with the steepened part, now, being proposed to be in the inertial range, in contrast to the other proposals which have unsuccessfully put it in the dissipation range. In the HMHD regime the kinetic and the magnetic energy spectra are different as the two fluids now have their own dynamics. There is a break in the spectrum at the ion inertial scale, a scale which is the hallmark of the Hall effect. We have built up the entire spectrum from the cascade of the three invariants. The low k end of the spectrum is governed by the ideal MHD and the high end by the Hall effect. In the Hall regime the size of the system is identified with the ion inertial scale, the largest spatial scale, and the fluctuations are at spatial scales smaller than the ion inertial scale. The scales are still related by $L_0 = V_0 t_0$ such that V_0 is the Alfvén speed and t_0 is the inverse of the ion cyclotron frequency or the Alfvén travel time of the ion inertial length. The system size changes from undetermined in the ideal MHD to a finite size (ion -inertial scale) in the Hall regime. One can consider this as two Kolmogorov systems with two inertial ranges joined by a break in the spectrum. The inertial range does shrink since it is already at high k and therefore close to the dissipation range. In fact for plasma beta greater than one the ion Larmor radius would wash out the ion inertial scale.

The spectra obtained from the second root ($\alpha = \epsilon k$)

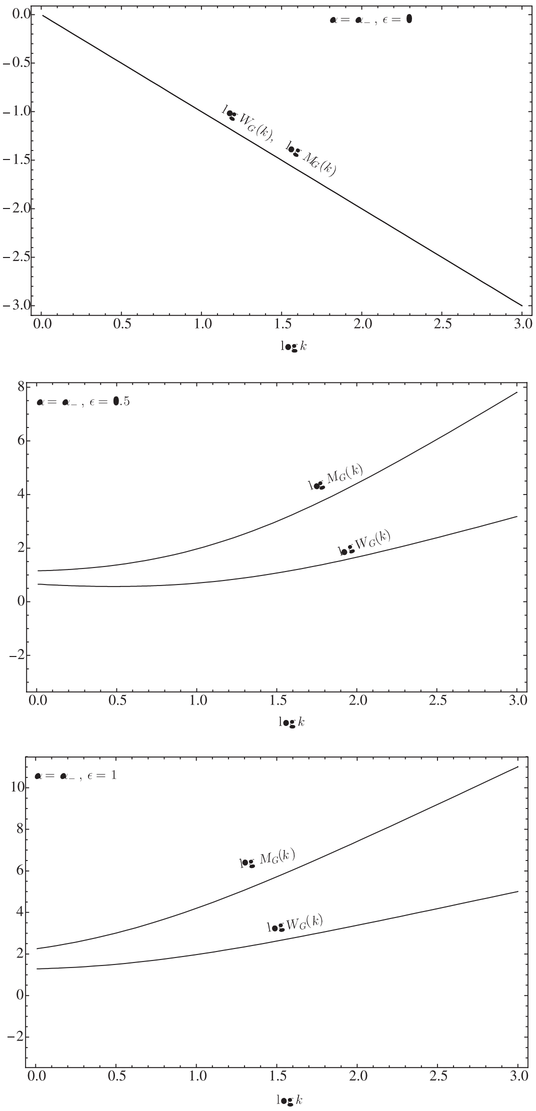

Fig. 7 Power spectra (log-log scale) of the kinetic energy density $W_G(k)$ and the magnetic energy density $M_G(k)$ derived from the generalized helicity invariant H_G with Hall effect for α_- root.

corresponding to the whistler mode would clearly be different. We find that the total energy invariant furnishes $W_E \propto k^{-3}, M_E \propto k^{-1}$ and the generalized helicity invariant furnishes $W_G \propto k^{-7/3}, M_G \propto k^{-1/3}$. Thus the magnetic spectrum corresponding to the whistler root has no steepend branch and thus cannot account for the observed magnetic spectrum. Besides the whistler waves undergo much stronger damping than the Alfvénic waves and may not be a major contributor to the solar wind turbulence. The whistler spectrum of $k^{-7/3}$ has been observed in the simulations of Dastgeer and Shukla [12]. This spectrum is not identified in the solar wind turbulent spectrum but it may contribute towards particle acceleration.

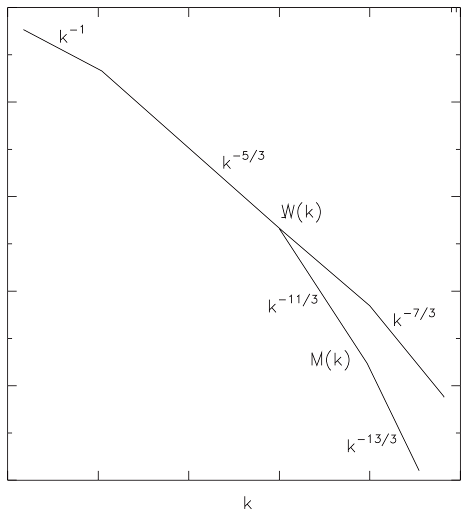

Fig. 8 Power spectra (log-log scale) of the kinetic and the magnetic energy density fluctuations including Hall effect.

4. Power Spectra of Density Fluctuations in Hall-MHD†

If the spectra of the kinetic and the magnetic fluctuations are different in HMHD then which would carry the passive density fluctuations? In order to answer this question let us look at the induction Eq. (5) which shows that the magnetic field is frozen to the electrons. Thus one would conclude that the electron density fluctuations would be frozen to the magnetic field fluctuations and would have the same spectrum as the magnetic field. An inspection of the induction Eq. (7) shows that the magnetic field is not frozen to the ions and the ions make the major contribution to inertia. Thus one could conclude that the ion density fluctuations would be carried by the velocity field fluctuations and would have the same spectrum as the velocity field. So, the electron and the ion density fluctuations have different spectral distributions. The entire picture is presented in Fig. 9. This raises the issue of the quasineutrality which should be maintained at each spatial scale. One may surmise that the different spectral distributions of the electron and the ion density may give rise to space charge effects and the ensuing ambipolar diffusion might wipe out the differences and the resultant distribution would be closer to that of the ions. However this would violate the frozen-in condition of the electrons and the magnetic field. One may bypass the issue by conclud-

† The tracking of the magnetic spectrum by the electron density spectrum is essentially a statistical form of pressure balance between the electron thermal and the magnetic pressure. Similarly the tracking of the kinetic energy spectrum by the ion density spectrum is essentially a statistical form of pressure balance between the ion thermal and the dynamical pressure. This furnishes, for equal electron and ion densities, equal electron and ion temperatures in the Alfvenic limit and unequal temperatures in the presence of the Hall effect.

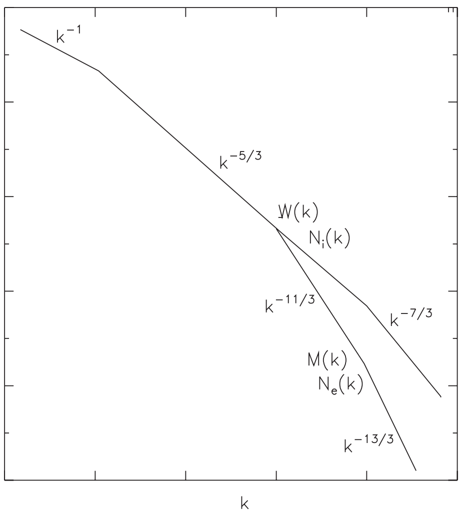

Fig. 9 Power spectra (log-log scale) of the electron and the ion density fluctuations including Hall effect.

ing that the plasma densities cannot be treated as passive scalars. The presence of a third species providing a neutralizing background can offer another way of permitting varied distributions of passive electron and the ion density fluctuations.

5. Conclusion

It has been shown that the observed spectral distributions of the velocity, the magnetic and the density fluctuations in the solar wind can be modeled within the framework of the Hall magnetohydrodynamics using the dimensional arguments of the Kolmogorov hypotheses. The Hall effect is particularly needed to account for the high k end of the spectra. The k dependent relation between the velocity fluctuations and the magnetic field fluctuations arising from the Hall-MHD waves results in different spectra for the fluctuations. Additionally, the spectra for the electron and the ion density fluctuations, treated as passive scalars, also differ at the high k end, again a consequence of the two fluid treatment. The spectrum of the electron density fluctuations steeper than the Kolmogorov spectrum at the ion inertial scale has been inferred from the interplanetary scintillation studies [13]. The spectrum of ion density fluctuations could be inferred from the plasma wave in situ measurements. The issue of the quasineutrality awaits a more detailed investigation. Several other consequences of the foray into the two-fluid treatment such as the inclusion of compressibility and the ensuing wave modes and anisotropies need to be investigated.

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