Interferometry for Weakly Relativistic Plasmas

Hitoshi HOJO, Atsushi MASE[1)] and Kazuo KAWAHATA[2)]

Plasma Research Center, University of Tsukuba, Tsukuba 305-8577, Japan
[1)]Art, Science and Technology Center for Cooperative Research, Kyushu University, Kasuga 816-8580, Japan
[2)]National Institute for Fusion Science, Toki 509-5292, Japan

(Received 16 December 2008 / Accepted 23 January 2009)

Interferometry for weakly relativistic plasmas is studied in this paper. It is shown that the phase difference in plasma and vacuum propagation becomes small due to the relativistic mass correction of electrons. The axisymmetric density profile obtained from the usual Abel inversion equation also becomes small by the correction factor, which is $1 \sim 1.3$ for $T_\mathrm{e} = 0 \sim 60\,\mathrm{keV}$. It is shown that we have to use the Abel inversion equation to take into account the relativistic mass correction of electrons in order to reconstruct the correct density profile.

Keywords: interferometry, relativistic plasma, relativistic mass correction, Abel inversion

DOI: 10.1585/pfr.4.010

Burning plasma physics is currently receiving growing attention in fusion research. For burning plasmas such as ITER, the electron temperature is expected to be several ten's of keV, and thus relativistic electron effect becomes important in fusion researches. In microwave plasma diagnostics, the relativistic effect of electrons has been studied [1–5].

In this paper, we study the relativistic correction of electron mass in microwave interferometry. The dispersion relation of the ordinary mode (O-mode) for relativistic Maxwellian plasma is given by

$$\frac{kc}{\omega} = N = \left[1 - \frac{1}{A}\left(\frac{\omega_\mathrm{pe}}{\omega}\right)^2\right]^{1/2}, \tag{1}$$

where A denotes the relativistic correction of electron mass for the O-mode cutoff, and is given by

$$A = \frac{3K_2(\rho)}{\rho^2 \int_0^\infty \mathrm{d}p\left(p^4/\gamma^2\right)\mathrm{e}^{-\rho\gamma}}, \tag{2}$$

where $\rho = m_\mathrm{e}c^2/T_\mathrm{e}$, $\gamma = \left(1 + p^2\right)^{1/2}$, $p = |\boldsymbol{p}|/(m_\mathrm{e}c)$, m_e is the electron mass, c the light speed, T_e the electron temperature, and $K_2(\rho)$ the modified Bessel function. We here estimate the relativistic correction of electron mass A for weakly relativistic plasmas, where $\mathrm{e}^{-\rho\gamma}$ is approximated by

$$\mathrm{e}^{-\rho\gamma} \simeq \mathrm{e}^{-\rho}\mathrm{e}^{-\rho\frac{p^2}{2}}\left(1 + \rho\frac{p^4}{8}\right).$$

In this case, the relativistic correction of electron mass A is

$$\begin{aligned} A &= \frac{3K_2(\rho)}{\rho^2 \int_0^\infty \mathrm{d}p\left(p^4/\gamma^2\right)\mathrm{e}^{-\rho\gamma}} \\ &\simeq \frac{3K_2(\rho)}{\rho^2\mathrm{e}^{-\rho}\int_0^\infty \mathrm{d}p p^4 \mathrm{e}^{-\rho\frac{p^2}{2}}\left(1 - p^2 + \rho\frac{p^4}{8}\right)} \\ &\simeq \frac{3\mathrm{e}^{-\rho}\sqrt{\frac{\pi}{2\rho}}\left(1 + \frac{15}{8\rho}\right)}{\rho^2\mathrm{e}^{-\rho}\left(\frac{2}{\rho}\right)^{5/2}\frac{3\sqrt{\pi}}{8}\left(1 - \frac{5}{8\rho}\right)} \simeq 1 + \frac{5}{2\rho}. \end{aligned} \tag{3}$$

In Ref. [4], the relativistic correction $A = (1 + 5/\rho)^{1/2}$ is proposed, which is approximately equal to Eq. (3) for $\rho \gg 1$. We see that $A \to 1$ as $T_\mathrm{e} \to 0$. We show A (solid line) as a function of T_e in Fig. 1, where the approximated expressions $1 + 5/2\rho$ (dashed line), and $(1 + 5/\rho)^{1/2}$ (dot-dashed line) are also shown. We see that the expression $1 + 5/2\rho$ is in good agreement with the exact form of A up to $T_\mathrm{e} = 60\,\mathrm{keV}$.

In interferometry, the phase difference in plasma and

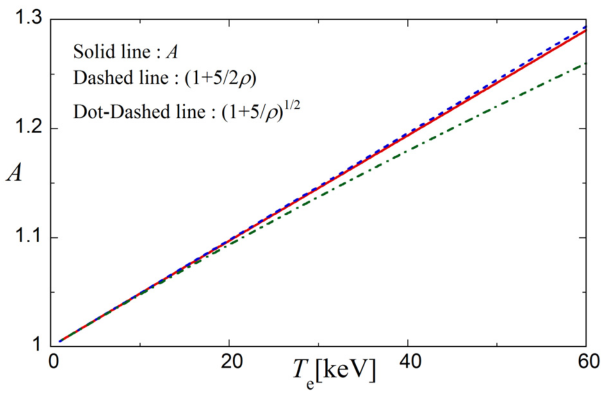

Fig. 1 The relativistic correction factor A (solid line) as a function of T_e; $1 + 5/2\rho$ (dashed line), and $(1 + 5/\rho)^{1/2}$ (dot-dashed line) are also shown.

author's e-mail: hojo@prc.tsukuba.ac.jp

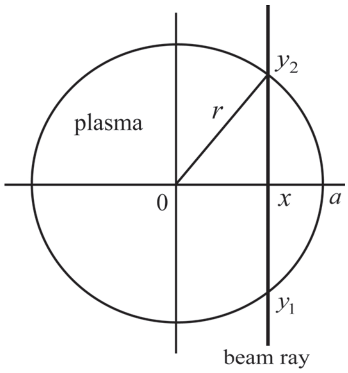

Fig. 2 Interferometer configuration, where a is the plasma-vacuum boundary.

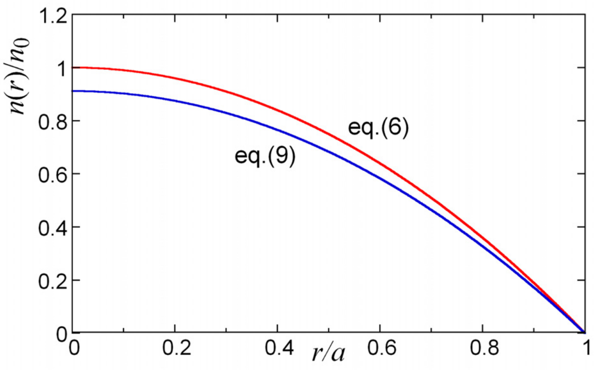

Fig. 3 Reconstructed density profiles Eq. (9) (blue) from Eq. (8) and T_e = 20 keV, and Eq. (6) (red) from Eq. (8-1) with the relativistic mass correction of electrons.

vacuum propagation is important for density profile reconstruction, and is given by

$$\varphi = \int_{y_1}^{y_2} (k_0 - k)\mathrm{d}y = \frac{2\pi}{\lambda}\int_{y_1}^{y_2} (1 - N)\mathrm{d}y, \tag{4}$$

where $k_0 = \omega/c = 2\pi/\lambda$ is the wavenumber in vacuum. When $\omega \gg \omega_{\mathrm{pe}}$, Eq. (4) is approximated by

$$\varphi(x) \simeq \frac{\pi}{\lambda n_c}\int_{y_1}^{y_2} \frac{n(r)}{A}\mathrm{d}y = \frac{\pi}{\lambda n_c}\int_x^a \frac{n(r)}{A}\frac{r\mathrm{d}r}{\sqrt{r^2 - x^2}},\ r > x, \tag{5}$$

where $n_c = \omega^2 m_e \varepsilon_0/\mathrm{e}^2$, and a is the plasma–vacuum boundary (see Fig. 2). If we assume a parabolic (axi-symmetric) density profile given by

$$n(r) = n_0\left[1 - \left(\frac{r}{a}\right)^2\right], \tag{6}$$

the phase difference $\varphi(x)$ is reduced to

$$\varphi(x) = \frac{4\pi a}{3\lambda}\frac{n_0}{An_c}\left[1 - \left(\frac{x}{a}\right)^2\right]^{3/2}. \tag{7}$$

We see that the phase difference becomes small due to relativistic mass correction of electrons. From the assumption of an axi-symmetric density profile, we can reconstruct the density profile using the Abel inversion equation, which is given by

$$n(r) = -\frac{\lambda n_c}{\pi^2}\int_r^a \frac{\mathrm{d}\varphi}{\mathrm{d}x}\frac{\mathrm{d}x}{\sqrt{x^2 - r^2}},\ x > r. \tag{8}$$

In this case, if we substitute Eq. (7) into Eq. (8), we obtain

$$n(r) = \frac{n_0}{A}\left[1 - \left(\frac{r}{a}\right)^2\right]. \tag{9}$$

We see that the reconstructed density profile of Eq. (9) is small compared with that of Eq. (6) due to the relativistic mass correction of electrons by a factor A (see also Fig. 3). The reason for this discrepancy in the density profile is clear. That is, we did not take into account the relativistic mass correction of electrons in the Abel inversion of Eq. (8), while we did include it in the phase difference φ calculation with Eq. (4) and Eq. (5). In fact, from Eq. (5), we have to take into account the relativistic mass correction of electrons in the Abel inversion equation. The Abel inversion equation with this correction is given by

$$n(r) = -A\frac{\lambda n_c}{\pi^2}\int_r^a \frac{\mathrm{d}\varphi}{\mathrm{d}x}\frac{\mathrm{d}x}{\sqrt{x^2 - r^2}},\ x > r, \tag{8-1}$$

and, in this case we obtain Eq. (6) in place of Eq. (9) as the reconstructed density profile.

The relativistic mass correction factor A of Eq. (2) was derived under the assumption of a relativistic Maxwellian equilibrium distribution with uniform T_e in Ref. [1]. However, we think that the present result remains valid too for a *local* relativistic Maxwellian equilibrium distribution with a space-dependent T_e.

We here discussed interferometry; however, the situation is the same in reflectometry [5]. If we use the usual Abel inversion equation without the relativistic mass correction of electrons for reconstructing the density profile, the reconstructed density profile is found to be small compared to the original profile by the factor A [5]. Therefore, we have to use the Abel inversion equation with this correction to obtain a correct density profile.

This work was partly supported by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports, and Culture ("Advanced Diagnostics for Burning Plasma" No. 20026003), and also was performed in part as a collaborating program at the National Institute for Fusion Science.

[1] D.B. Batchelor, R.C. Goldfinger and H. Weitzner, Phys. Fluids **27**, 2835 (1984).
[2] H. Bindslev, Plasma Phys. Control. Fusion **33**, 1775 (1991).
[3] H. Bindslev, Plasma Phys. Control. Fusion **35**, 1093 (1993).
[4] E. Mazzucato, Phys. Fluids **B4**, 3460 (1992).
[5] H. Hojo, A. Mase and K. Kawahata, Plasma Fusion Res. **2**, S1022 (2006).