Shear Formation by a Poloidal Chain of Magnetic Islands

Vasyl I. MASLOV and Franco PORCELLI[1)]

NSC Kharkov Institute of Physics and Technology, Kharkov 61108, Ukraine
[1)]*Politecnico di Torino, Turin, Italy*

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We estimate the electron angular velocity shear $\partial_r \omega_{\theta o}$, which can be formed by plasma heating near the low-order rational surface with a poloidal chain of magnetic islands. We suppose that the plasma is heated sufficiently that its electrons start to miss the magnetic islands during their radial collisional shift and movement along the toroidal surface. This provides an ion volume charge in some regions of magnetic islands, which leads to shear formation. The time taken for shear formation is short. The conditions for magnetic island width leading to the shear are derived. It is shown that even narrow magnetic islands can lead to the shear. The shear can damp instabilities with a growth rate smaller than the ion cyclotron frequency. The spatial structures of convective vortical cells are described. We derive inverse dependences of the radial width of excited vortices on $\partial_r \omega_{\theta o}$ and radial gradient of plasma density $\partial_r n_{0e}$. Amplitude of electron radial oscillations is smaller for larger $\partial_r \omega_{\theta o}$ and $\partial_r n_{0e}$. These dependences promote a steep radial distribution of the plasma density and internal transport barrier.

Keywords: internal transport barrier, angular velocity shear, magnetic island, vortex, rational surface, radial electric field

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1. Introduction

Internal transport barrier (ITB) formation is being investigated widely [1–3]. Previously, the effect causing the shear of the electron angle velocity ($\partial_r \omega_{\theta o}$) to damp anomalous transport, thereby destroying coherent ordered motion due to the relative shift of layers, was investigated. ITB is formed for a shear above the threshold. We consider one more effect of anomalous transport suppression in plasma located in perpendicular magnetic ($\vec{H}_0$) and electric ($\vec{E}_{0r}$) fields. The shape of vortices is described. The nonlinear vector equation connecting the electron vorticity and density is derived. We derive inverse dependences of the radial width of excited vortices on the shear $\partial_r \omega_{\theta o}$ in perpendicular fields and the degree of steepness of the radial plasma density distribution, $\partial_r n_{0e}$. These dependences promote a steep radial distribution of the plasma density and ITB formation. The amplitude of the vortex saturation is inversely proportional to the shear. They also promote ITB formation, suppressing the transport, especially in the case of a small magnetic shear. It is determined that a small magnetic shear leads to a large spatial interval (Δ) between rational surfaces [4]. If radial correlation length of excited perturbations becomes less than Δ, the radial transport could be suppressed. A convective diffusion equation describing the transport of plasma particles in the field of a lattice of overlapped vortices is derived.

Currently, the formation [5, 6] and role [4, 7–9] of magnetic islands in nuclear fusion plasma are being investigated intensively. In particular, their effect on ITB formation is very important. In experiments [4] involving sufficient plasma heating, the plasma confinement was better in the presence of a poloidal chain of magnetic islands than in their absence. In some experiments [7, 8], the shear was formed by a poloidal chain of magnetic islands with sufficient plasma heating. This study shows that the electrons start to miss magnetic islands because of sufficient plasma electron heating near the rational surface with the poloidal chain of magnetic islands, and the shear is formed. The time of shear formation is derived and shown to be short.

The magnetic island width conditions lead to shear formation are derived. The magnetic island width conditions that do not lead to anomalous transport of trapped particles are known; the island should be narrow. It is shown here that even narrow islands can lead to shear. One can note that several poloidal chains of islands, observed in [7], are better for shear formation on longer radial interval.

It is shown that this formed shear can damp instabilities with a growth rate γ, which is smaller than the ion cyclotron frequency $\gamma < \omega_{ci}$.

There are some effects connected with the shear. For strong anomalous transport determined by a streamer formed by a single wide vortex overlapping all inhomogeneous area, the shear destroys vortices. In this case, some anomalous transport remains after ITB formation. For weak anomalous transport determined by a lattice of vortices, the shear can spatially (radially) separate the vortices.

author's e-mail: vmaslov@kipt.kharkov.ua

2. Spatial Structure of Vortical Convective Cells

Let us describe a chain on azimuth θ of the vortices in the plasma, located in perpendicular magnetic $\vec{H}_0$ and radial electric $\vec{E}_{r0}$ fields in the cylindrical approximation. Neglecting nonstationary and nonlinear electric potential ϕ of members at the vortices, from electron equation of motion, one can obtain the following equation describing the oscillatory dynamics of electrons in the field of the perturbation, at small ($\delta r \equiv r - r_v$) deviations from radial position of vortices r_v:

$$(\delta r)^2 + 4(e\phi - \delta p_e/n_e(r_v))/r_v m_e \omega_{ce}(r_v)\partial_r \omega_{\theta o}\big|_{r=r_v} = C, \tag{1}$$

$$\omega_{\theta o} \equiv V_{\theta o}(r)/r,$$

$$\vec{V}_{\theta 0} = -e[\vec{e}_z,\ \vec{E}_{ro}]/m_e\omega_{ce} - [\vec{e}_z,\ \vec{\nabla} p_{0e}]/n_e m_e \omega_{ce}.$$

Here, r is the radial position of an electron; δp_e is the electron pressure perturbation; n_e is the electron density; e and m_e are the electron charge and mass; ω_{ce} is the electron cyclotron frequency; $\vec{E}_{ro}$ is the unperturbed radial electric field; p_{0e} is the unperturbed electron pressure; and $\vec{V}_{\theta 0}$ is the unperturbed electron drift velocity at azimuth θ. Eq. (1) describes the radial electron oscillations $\delta r \equiv r - r_v$ through dependences of $\phi(\theta,\ r)$ and electron density perturbations $\delta n_e(\theta,\ r)$ on θ and r. Let us connect $e\phi - \delta p_e/n_e(r_v)$ with a characteristic of electron vortical movement $\alpha \equiv \vec{e}_z \cdot \vec{\nabla} \times \vec{V}_e$. Here, $\vec{e}_z$ is the unit longitudinal vector and $\vec{V}_e$ is the electron velocity. From the electron equation of motion, we derive

$$\alpha/V_{thi}\rho_{ci} = e\Delta\phi/T_e - \Delta n_e/n_e. \tag{2}$$

Here, ρ_{ci} is the ion cyclotron radius, V_{thi} is the ion thermal velocity, and T_e is the electron temperature. For a vortex with the dimension ρ_{ci}, we have $\alpha < \omega_{ci}$ and $e\phi/T_e - \delta n_e/n_e \approx \alpha/\omega_{ci}$. One can see that the amplitude of the radial oscillations of electrons for the given amplitudes of the electric potential and density perturbation is smaller for a larger shear $\partial_r \omega_{\theta o}$. The larger shear helps the ITB formation.

In perpendicular magnetic and radial electric fields, the vortices can have a phase velocity $V_{ph} \approx V_{\theta 0}$, as well as slow vortices with $V_{ph} << V_{\theta 0}$, e.g., Rossby vortices. Let us derive the spatial structure of a slow vortex. For this purpose, one can derive a vector equation describing vortical electron dynamics, which is similar to the general nonlinear equation [10]:

$$d_t\left(\frac{\vec{\alpha} - \vec{\omega}_{ce}}{n_e}\right) = \frac{1}{n_e}((\vec{\alpha} - \vec{\omega}_{ce})\vec{\nabla})\vec{V}. \tag{3}$$

Here, $\vec{\alpha} = [\vec{\nabla} \times \vec{V}]$ and $d_t \equiv \partial_t + (\vec{V}\vec{\nabla})$. From eq. (3) one can obtain an equation describing a vortex with a small amplitude. Hence, we derive the following equation for the perturbation of electron trajectory δr in the field of the vortex

$$\delta r(\theta,\ r) = -\frac{1}{\omega_{ceo}\partial_r(n_{oe}/\omega_{ce})}\delta n_e(\theta,\ r). \tag{4}$$

From here, one can see that at the same amplitude of plasma density perturbation δn_e, the vortex is narrower in the radial direction for larger $\partial_r(n_{oe}/\omega_{ce})$, i.e., for a steeper radial distribution of the plasma density. The latter helps a steeper radial distribution of the plasma density and ITB formation.

The vortex is excited up to its maximum amplitude, at which the layers trapped by it during the excitation time γ^{-1} are shifted relative to each other due to the shear with an angle $\leq 2\pi/\ell_\theta$, i.e., $\delta r_v(\ell_\theta/2\pi)\partial_r\omega_{\theta 0}\big|_{r=r_v} \leq \gamma$. Here, ℓ_θ is the azimuthal wave number of excited vortices. From this expression, we derive

$$|e\phi_o - \delta p_{e0}/n_e(r_v)| = (\gamma\pi/\ell_\theta)^2 r_v m_e \omega_{ce}/2\left|\partial_r\omega_{\theta o}\right|_{r=r_v}. \tag{5}$$

Here, ϕ_0 and δp_e are the saturation amplitudes of electric potential and electron pressure perturbation in the vortex, respectively. The amplitude of vortex saturation is inversely proportional to $\partial_r\omega_{\theta 0}$. The decrease in the level of fluctuations upon ITB formation has been observed in [3], which promotes ITB formation.

3. Convective Diffusion Equation

Let us consider the finite amplitudes of excited vortices, when the frequency Ω_r of the electron oscillations forming the vortex becomes greater than the growth rate γ of the vortex excitation, i.e., $\Omega_r > \gamma$. Then in vicinity of vortex borders, the radial distribution of electron density $n_e(r)$ jumps. On these n_e jumps, new cells with the largest growth rates are excited. This causes an ordering of convective cells. Therefore, after achieving large amplitudes, an instability is developed in the ordering of cells. Similar instability has been investigated in [11]. Inside the borders of the vortex, an ordered convective movement of the electrons occurs. However, it is influenced by background fluctuations and vortex fields. Further, it is important that the amplitudes of vortexes are not stationary. Instead of an average radial distribution of electron density $n_{oe}(t, r)$, which does not consider correlations, we use four electron densities $n_{ke}(t, r)$ averaged for small-scale oscillations: $n_{1e}(t, r)$ and $n_{2e}(t, r)$ are the average electron densities in region 1 in the middle of a cell for $r > r_v$ and in region 2 in the middle of a cell for $r < r_v$, (see Fig. 1); $n_{3e}(t, r)$ and $n_{4e}(t, r)$ are the electron average densities in region 3 near the border of a cell for $r > r_v$ and in region 4 near border of a cell for $r < r_v$. The importance of using different $n_{ke}(t, r)$ is also determined; the angular speeds of electron rotation inside a cell vary with distance from its axis. In addition, in central area of the convective cell, the following processes are still realized: 1) $n_e(r)$ plateau is formed due to the difference in angular speeds of electron rotations; and 2) $n_e(r)$ jump

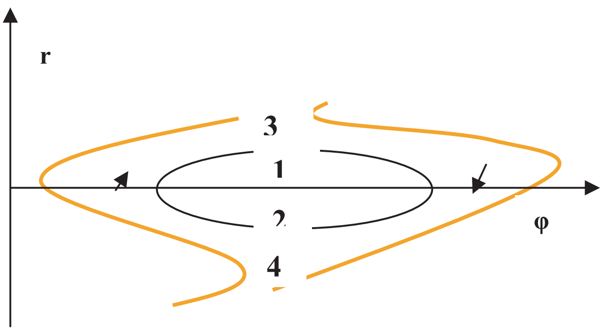

Fig. 1 Single convective cell. r is the radius and φ is the poloidal angle.

formation at certain times in regions 1 and 2 causes accelerated diffusion and exchange of electrons between regions 1 and 3 (factor α), and between regions 2 and 4. α is the factor of mixing that depends on the fluctuations, growth of amplitudes, and differences in characteristic times of the electrons.

But after ordering, adjacent cells form an integrated border. The particles in the space between individual cell borders and the integrated border move in the radial direction from cell to cell, with a distance $\min\{\ell_{\rm cor}, \delta r_{\rm v}\tau_{\rm cor}\Omega_r/\pi\}$. $\ell_{\rm cor}$ and $\tau_{\rm cor}$ are the correlation length and time of vortical convective cell turbulence, respectively.

From the above, we have approximately

$$\begin{aligned} n_1(t+\tau,\ r) &= (1-\alpha)n_2(t,\ r) + \alpha\beta n_3(t,\ r),\\ n_2(t+\tau,\ r) &= (1-\alpha)n_1(t,\ r) + \alpha\beta n_4(t,\ r). \qquad (6)\\ n_3(t+\tau,\ r) &= \alpha n_1(t,\ r) + \beta(1-\alpha)n_3(t,\ r-\delta r_{\rm v})\\ &\quad +0.5(1-\beta)[n_3+n_4],\\ n_4(t+\tau,\ r) &= \alpha n_2(t,\ r) + \beta(1-\alpha)n_4(t,\ r+\delta r_{\rm v})\\ &\quad +0.5(1-\beta)[n_3+n_4], \end{aligned}$$

β is the factor of the convective exchange of particles of cells. The value of β is determined by the ratio of the area with convective electron dynamics, located between individual cell borders and the integrated border to the entire area, located between individual cell borders and the integrated borders of adjacent cells. From these equations, using $\bar{n} = (n_3+n_4)/2$, $\delta n = n_3 - n_4$, $\bar{N} = (n_1+n_2)/2$, and $\delta N = n_1 - n_2$, we derive

$$\begin{aligned} &\tau\partial_t\bar{n} = \alpha(\bar{N}-\beta\bar{n}) - (\beta/2)(1-\alpha)\delta r_{\rm v}\partial_r\delta n\\ &\tau\partial_t\delta n + [1-\beta(1-\alpha)]\delta n = \alpha\delta N - 2\beta(1-\alpha)\delta r_{\rm v}\partial_r\bar{n}, \quad (7)\\ &\tau\partial_t\bar{N} = \alpha(\beta\bar{n}-\bar{N}), \quad \tau\partial_t\delta N + (2-\alpha)\delta N = \alpha\beta\delta n. \end{aligned}$$

One can see that $\bar{n}$ is similar to the average $n_{\rm oe}(t, r)$ but includes correlations. From these equations, we have, similar to [11], the following convective diffusion equation

$$\begin{aligned} &\tau^2\partial_t^2\delta n + \tau\partial_t[(1-\beta(1-\alpha))\delta n - \alpha\delta N]\\ &\quad = -2\beta(1-\alpha)\delta r_{\rm v}\partial_r\\ &\quad\times\left[\alpha(\bar{N}-\beta\bar{n}) - \frac{\beta}{2}(1-\alpha)\delta r_{\rm v}\partial_r\delta n\right]. \qquad (8) \end{aligned}$$

As β is proportional to $(\delta r_{\rm v} - \Delta)/\delta r_{\rm v}$, at $\delta r_{\rm v} < \Delta$, we have $\beta = 0$, and there is no convective radial transport because the convective cell exchange of particles disappears.

4. Shear Formation by Magnetic Islands

Let us consider the shear formation near a poloidal chain of narrow magnetic islands. The electric field $E_{\rm r0}$ is approximately zero on the axis of a plasma column $r = 0$. It is maximum inside the plasma column at $r = r_m$. We suppose that on some radial interval $r_0 - r_{\rm f} < r < r_0 + \Delta r + r_{\rm f}$ of width $2r_{\rm f}$ around the chain of magnetic islands, the electric field $E_{\rm r0}$ in the absence of shear is proportional to r at $r_{\rm f} < (r_{\rm m} - \Delta r)/2$. The magnetic islands have a radial width Δr. The chain of islands is located at $r = r_0 + \Delta r/2$, where r_0 is the lower border of the islands. Then on this interval, one can present

$$\begin{aligned} &E_{\rm r0} = -2\pi e N_0 r, r_0 - r_{\rm f} < r < r_0 + \Delta r + r_{\rm f},\\ &\text{and} \quad N_0 \equiv n_{0\rm e} - n_{0\rm i}. \qquad (9) \end{aligned}$$

Here, $n_{0\rm e}$ and $n_{0\rm i}$ are the unperturbed plasma electron and ion densities, respectively. This suggests that there is no shear, i.e., $\omega_{\theta 0} \neq \omega_{\theta 0}(r)$. We include the effect of oscillations on electron transport using an effective collision frequency $\nu_{\rm ef}$ in the diffusion coefficient $D_\perp \propto (\nu_{\rm e} + \nu_{\rm ef})$, where $\nu_{\rm e}$ is the electron collision frequency.

On the plasma cross-section $0 < r < a$, several chains of magnetic islands can exist [7]. However, we consider the influence of a poloidal chain of islands on shear formation for a simple case. We consider a low-order rational magnetic surface, because the condition of shear formation is satisfied easily for this surface for sufficient plasma electron heating. According to [4], we consider local plasma heating near this surface. This local heating leads to an important effect: the quick longitudinal electron dynamics necessary for shear formation is obtained near this low-order rational magnetic surface.

If the plasma is heated sufficiently, its electrons start to miss the island during their radial collisional shift. This is caused by the quicker movement of heated electrons along the toroidal surface. Some of the electrons located in the island escape upon reaching $E_{\rm r}$ to zero inside island, except for the layer of width

$$\begin{aligned} \delta r_{\rm sep} &\approx 2\pi n R\sqrt{D_\perp/D_\parallel} \qquad (10)\\ &\approx (n\rho_{\rm ce}/q)\sqrt{(\nu_{\rm e}+\nu_{\rm ef\perp})/(\nu_{\rm e}+\nu_{\rm ef\parallel})} \end{aligned}$$

near $r = r_0$. $D_\perp$ and $D_\parallel$ are the transversal and longitudinal diffusion coefficients respectively, $\nu_{\rm ef\perp}$ and $\nu_{\rm ef\parallel}$ are the transversal and longitudinal effective collision frequencies respectively, and q is the number of electron rotations around the torus during the time of free path motion. When some of the electrons escape the magnetic island, the plasma ion volume charge n_i is not compensated by the electron charge in the island; the difference in charges is

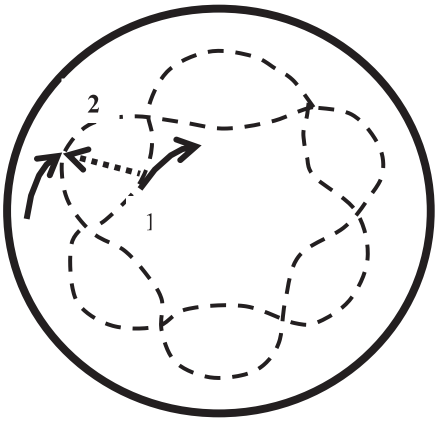

Fig. 2 Cross-section of magnetic islands. 1 and 2 are the 1st (lower) and 2nd (external) borders of the magnetic islands. Shaped arrow specifies radial collisional shift of electrons through the magnetic islands. Continuous arrows demonstrate directions of electron movements along the magnetic field lines.

δn. δn can be determined from experiments; it is established that $E_{r0} \approx 0$ in the island.

With sufficient plasma heating, the electron transport through magnetic islands changes from slow collisional to quick. Electrons miss the magnetic islands. Although the radial size of the magnetic islands is small, the quick electron transport leads to the appearance of uncompensated ion volume charge in the magnetic islands and in the shear $\partial_r \omega_{\theta \mathrm{o}}$.

It is necessary to note that the longitudinal inhomogeneity of the magnetic field results in longitudinal redistribution of charges upon shear formation. The consequences of this redistribution of charges will be considered in another paper.

For the uncompensated ion volume charge to appear in a magnetic island, it is necessary that its width Δr should be larger than the electron cyclotron radius, i.e., $\Delta r > \rho_{\mathrm{ce}}$.

Let us consider the radial electron dynamics in a small neighborhood of the chain of islands. The electrons move in the radial direction in perpendicular $\vec{E} \times \vec{H}$ fields with velocity $V_{0r} = -(eE_{\mathrm{o}r} + \partial_r p_{0\mathrm{e}}/n_{\mathrm{e}})(\nu_{\mathrm{ef}} + \nu_{\mathrm{e}})/m_{\mathrm{e}}\omega_{\mathrm{ce}}^2$ and diffusion coefficient $D_\perp$. When an electron reaches an island with small r, it propagates slow collisionally through island in the absence of shear. But in the case when shear can be formed, the electron quickly reaches the second (external) boundary of the island, in time $2\pi R/V_{\mathrm{the}}^{(\mathrm{hot})}$. Here, $V_{\mathrm{the}}^{(\mathrm{hot})}$ is the thermal velocity of heated electrons near a magnetic island. After this, the electron again propagates slowly in the direction of large r.

Thus with sufficient plasma heating near the island, a shear is formed. Using approximation of a poloidal chain of narrow magnetic islands as an azimuthal symmetrical narrow layer, we have the following equation for the radial distribution of electric field

$$E_{\mathrm{r}0} \approx -2\pi e \begin{cases} N_{\mathrm{a}} r, & 0 \le r \le r_0 \\ N_{\mathrm{a}} r_0^2/r - \delta n(r - r_0^2/r), & \\ & r_0 \le r \le r_0 + \delta r_{\mathrm{sep}} \\ 0, & r \ge r_0 + \delta r_{\mathrm{sep}} \end{cases} \tag{11}$$

If the field is small, $E_{r0} \approx 0$, at $r = r_0 + \delta r_{\mathrm{sep}}$, we have $N_{\mathrm{a}} \approx 2\delta n \delta r_{\mathrm{sep}}/r_0$. One can see that the density of the uncompensated ion volume charge is relatively, large in the island with $r_0 \gg \Delta r$, if $E_r \approx 0$ at $r = r_0 + \delta r_{\mathrm{sep}}$.

One can use the estimation $N_{\mathrm{a}} \approx 4n_0(e\Delta\varphi/T_{\mathrm{i}})(r_{\mathrm{di}}/L)^2$. L is the width of the region with $E_r \neq 0$ and $\Delta\varphi$ is the potential for L. One can conclude that the island can be sufficiently narrow for shear formation if

$$(e\Delta\varphi/T_{\mathrm{i}})(r_{\mathrm{di}}^2/L\delta r_{\mathrm{sep}}) < 1. \tag{12}$$

This inequality suggests that in a real island, the small uncompensated ion volume charge, $\delta n << n_0$, is sufficient for shear formation.

Let us calculate the shear of the electric field and normalize it for the electric field $E_r = -2\pi e N_0 r$ in the absence of shear: $S \equiv (E_r|_{\text{no TB}})^{-1} r^2 \partial_r (E_r/r)$. If $E_r\big|_{r=r_0+\delta r_{\mathrm{sep}}} \approx 0$, we have

$$S = \begin{cases} (N_{\mathrm{a}}/N_0)(r_0^2/r^2)(2 + r_0/\delta r_{\mathrm{sep}}), & \\ & r_0 \le r \le r_0 + \delta r_{\mathrm{sep}} \\ 0, & r \ge r_0 + \delta r_{\mathrm{sep}} \end{cases} . \tag{13}$$

The shear is large for a region of narrow magnetic islands $r_0 >> \Delta r > \delta r_{\mathrm{sep}}$,

$$S \approx (N_{\mathrm{a}}/N_0)(r_0/\delta r_{\mathrm{sep}}), \quad r_0 \le r \le r_0 + \delta r_{\mathrm{sep}}. \\ |S| >> 1 \tag{14}$$

Let us consider the shear of $\omega_{\theta 0} = V_{\theta 0}/r$.

$$V_{\theta 0} = (m_{\mathrm{e}}\omega_{\mathrm{He}})^{-1}(-eE_{r0} - \partial_r p_{0\mathrm{e}}/n_{0\mathrm{e}}). \tag{15}$$

For ITB formation, this shear $\partial_r \omega_{\theta 0}$ is more important. We determine the angular velocity shear

$$S_\omega \equiv (\omega_{\theta 0}|_{\text{no TB}})^{-1} r \partial_r \omega_{\theta 0},$$

$$\omega_{\theta 0}|_{\text{no TB}} = (\omega_{\mathrm{pe}}^2/\omega_{\mathrm{He}})(N_0/2n_0).$$

Then we derive

$$S_\omega = -\begin{cases} (N_{\mathrm{a}}/N_0)(r_0^2/r^2)(2 + r_0/\delta r_{\mathrm{sep}}), & \\ & r_0 \le r \le r_0 + \delta r_{\mathrm{sep}} \\ 0, & r \ge r_0 + \delta r_{\mathrm{sep}} \end{cases} .$$

Absolute value of the relative angular velocity shear is of the order of $S_\omega = -(N_{\mathrm{a}}/N_0) r_0/\delta r_{\mathrm{sep}}$. But the absolute shear can be increased. In several experiments, strong localization of the region with $V_{0\theta} \neq 0$ has been observed. Since the radial width of area $V_{0\theta} \neq 0$ localization $\Delta r_{\mathrm{sh}} = 1$ has been observed experimenally ($\Delta r_{\mathrm{sh}} = 1$ cm), the shear $(\partial_r V_{0\theta})_{\mathrm{apr}}$ can be increased in existing nuclear fusion installations in comparison with the smooth case $(\partial_r V_{0\theta})_{\mathrm{smooth}} \approx V_{0\theta}/a$ strongly $(\partial_r V_{0\theta})_{\mathrm{apr}} \approx V_{0\theta}/\Delta r_{\mathrm{sh}} \approx (\partial_r V_{0\theta})_{\mathrm{smooth}} a/\Delta r_{\mathrm{sh}}$, where a is the poloidal radius.

The shear is formed during

$$\tau_{TB} \approx (\nu_{\mathrm{ef}} + \nu_{\mathrm{e}})^{-1}(\omega_{\mathrm{ce}}^2/2\omega_{\mathrm{pe}}^2)(r_0/\Delta r). \tag{16}$$

The time is short for magnetic islands, which are not very narrow.

Although narrow islands provide fast electron transport through the island dimension Δr, they strongly suppress transport in their neighborhood. Actually in an experiment [4], the plasma confinement is better in the presence of islands and $\Delta r \ll a$.

In an island placed on a rational surface with numbers n and m, the uncompensated ion volume charge appears at $(\Delta r)^2/D_\perp > (2\pi nR)^2/D_\|$, where R is the toroidal radius. Hence the island should be wider.

$$\max\{\rho_{\mathrm{ce}},\ 2\pi nR\sqrt{D_\perp/D_\|}\} < \Delta r. \tag{17}$$

$D_\perp/D_\| = (\nu_{\mathrm{ei}} + \nu_{\mathrm{ef}\perp})(\nu_{\mathrm{ei}} + \nu_{\mathrm{ef}\|})/\omega_{\mathrm{ce}}^2 \ll 1$. But for ITB formation, the island should be narrow [12], i.e., $\Delta r \ll a$.

Eq. (17) can be presented as follows:

$$\max\{\rho_{\mathrm{ce}},\ (n\rho_{\mathrm{ce}}/q)\sqrt{(1+\nu_{\mathrm{ef}\perp}/\nu_{\mathrm{ei}})(1+\nu_{\mathrm{ef}\|}/\nu_{\mathrm{ei}})}\} < \Delta r. \tag{18}$$

From the obtained expressions and from the condition [1–3]

$$L\partial_r\omega_{\theta 0} > \gamma \tag{19}$$

using

$$\Delta\varphi \approx 2\pi e N_{\mathrm{a}} L^2, \tag{20}$$

one can show

$$(\Delta\varphi e/T_{\mathrm{i}})(\rho_{\mathrm{ci}}^2/L\delta r_{\mathrm{sep}}) > \gamma/\omega_{\mathrm{ci}}. \tag{21}$$

Thus, the shear can damp low-frequency instabilities with a growth rate $\gamma < \omega_{\mathrm{ci}}$.

5. Conclusions

It is shown that the amplitude of the radial electron oscillations is smaller or the vortex is narrower in the radial direction for a larger shear $\partial_r\omega_{\theta\mathrm{o}}$ and a steeper radial distribution of the plasma density in nuclear fusion plasma. The latter helps ITB formation. The amplitude of vortex saturation is inversely proportional to $\partial_r\omega_{\theta\mathrm{o}}$. It also promotes ITB formation. The convective diffusion equation for electron transport has been derived.

It is shown that the value of the shear can be large. The shear is formed by sufficient plasma heating near the low-order rational surface with a poloidal chain of narrow magnetic islands. The time of shear formation is short. The conditions for the island width that lead to shear formation are derived. Even narrow islands can lead to shear formation. The condition for island width that do not lead to anomalous transport are known. This shear can damp instabilities with a growth rate smaller than the ion cyclotron frequency $\gamma < \omega_{\mathrm{ci}}$.

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