

Experimental Parameters of Fine Particle Plasmas Explicitly Expressed by Dimensionless Characteristic Parameters

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Experimental parameters of fine particle plasmas, densities and temperatures of components and the fine particle size, are explicitly expressed in terms of dimensionless characteristic parameters, such as Coulomb coupling and strength of screening, and typical examples are given. Though characteristic parameters are readily obtained from experimental parameters, the reverse is not trivial and it is shown that the charge neutrality condition limits the characteristic parameters realized by fine particle plasmas within the domain $(a/\lambda)^2/(\Gamma/A) \geq 3$. Here, a and λ are the mean distance between fine particles and the screening length, respectively, $\Gamma = (Qe)^2/ak_B T_p$ is the Coulomb coupling of fine particles, and $A = (n_e T_e + n_i T_i)/n_p T_p$ is the ratio of the ideal gas pressures of ambient plasma (of ions and electrons) and fine particles.

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1. Introduction

Fine particle plasmas are mixtures of fine particles (particulates) and ambient plasma of ions and electrons. Fine particle plasmas provide us with a stage for various interesting phenomena especially in the domain of strong coupling such as formation of the Coulomb crystal and other structures. Statistical properties of fine particle plasmas are characterized by a set of several dimensionless characteristic parameters which properly describe the system. In order to observe some phenomenon expected to occur at some specific combination of characteristic parameters, it is necessary to interpret the combination into a set of experimental parameters which realizes the state with designated combination.

When characteristic parameters at a state of fine particle plasmas are obtained in experiment, one may want to modify their values by changing experimental parameters such as the size of fine particles and densities and temperatures of components. Since most of experimental parameters are related to not a single characteristic parameter in the set, it is not trivial to adjust experimental parameters appropriately. Characteristic parameters are readily calculated from experimental parameters by definitions but the reverse needs at least some manipulation and numerical solution of equations. We here give the reverse relations explicitly in a simple form including numerical results to facilitate theoretical and experimental investigations of fine particle plasmas.

In order to make a correspondence between theoretical and experimental parameters, it is necessary to char-

acterize statistical-mechanical properties of the system by dimensionless parameters. The simplest model of fine particle plasmas is the Yukawa one-component plasma (Yukawa OCP) model [1–4]. This has a sound basis in the domain of weak coupling and we expect the characterization of the system by its parameters to work at least as a reference frame also in the domain of strong coupling as follows.

There exist some aspects of fine particle plasmas which are not described by the Yukawa OCP model. One is the anisotropy of interaction between particles induced by the effect of gravity on particles. In experiments on the ground, fine particles are kept afloat in the domain of sheath by the electric field. The ion flow in the sheath then leads to anisotropic interactions between particles including (nonreciprocal) attractive part, through the dielectric response [5] or the focusing of ions due to deflections of their orbits by particles [6, 7], as observed in the formation of strings of particles along the ion flow. In the directions perpendicular to the flow, on the other hand, the measurements of the force between colliding particles have shown that the interaction is expressed by the Yukawa potential to a good accuracy at least within a few times the screening length [8–10]. The dispersion of oscillations in ordered monolayers of particles can also be fitted to those calculated by the Yukawa interaction [11].

In describing fine particle plasmas, we assume that they can be characterized by parameters for isotropic systems in the first approximation. This may be realized in microgravity experiments [12]. Under this condition, we need not to levitate particles by the sheath electric field which

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causes unidirectional near- or supersonic ion drift. We may also have approximately isotropic system balancing the gravity by, for example, the temperature gradient [13]. Since ions are not drifting in these cases, we expect the screening is mainly provided by ions. After the system is characterized by parameters for isotropic systems and the correspondence to experimental parameters is known, we can take the effects of anisotropy into account.

Apart from the anisotropy, there have also been some arguments that the (isotropic) precipitation of ions onto particles and other mechanisms lead to an additional attractive interaction between them [14]. These effects are expected to appear when some conditions are satisfied and the magnitude of the attraction can become larger than the Yukawa repulsion in the long range domain where the latter is strongly screened. There seem, however, to be no direct experimental observations hitherto. It is also possible to evaluate these effects once the correspondence within the Yukawa OCP characterization is known.

Since the magnitude of the charge on fine particles is typically 10^3 to 10^4 times the elementary charge, the screening around fine particles may not be properly described by the linear Debye-Hückel expression. The screening may also be modified by the collisions of ions forming screening cloud with neutral atoms and other effects, depending on the neutral atom density. In the domain less than a few times the screening length, however, the potential is approximately described by the Yukawa potential [8–10, 15] and the characterization within the Yukawa OCP model may work as a reference frame in considering these effects.

We give a description of fine particle plasmas within the Yukawa OCP model in Sec. 2 and relate characteristic parameters explicitly to experimental parameters in Sec. 3. Some examples and the discussion on the effect of ion-neutral collisions are given in Sec. 4. Concluding remarks are given in Sec. 5.

2. Characteristic Parameters of Fine Particle Plasmas in Yukawa OCP Model

We consider fine particle plasmas composed of fine particles (particulates) and ambient plasmas of ions and electrons. For simplicity, we assume fine particles are spheres of radius r_p and ions are of single species and with the positive charge e . We denote the density, the temperature, and the charge of fine particles by $(n_p, T_p, -Qe)$, assuming that their charge is negative. For ions and electrons, we denote the density and the temperature by (n_i, T_i) and (n_e, T_e) , respectively. Since it is believed that $T_e > T_i$ ($\sim T_p$) in most experiments, we assign different values for the temperatures of the components. We thus have eight parameters for charged components of the system, namely, $(r_p, n_p, T_p, Q, n_i, T_i, n_e, T_e)$.

We characterize fine particle plasmas by the parameters of the Yukawa OCP model or the one-component Yukawa system embedded in polarizable and neutralizing background plasma of ions and electrons [1–4]. In this model, the potential around a point particle with the charge $-Qe$ is given by the Yukawa potential

$$\frac{-Qe}{r} \exp(-r/\lambda), \quad (1)$$

where λ is the screening length given by

$$\frac{1}{\lambda} = \left(\frac{4\pi n_i e^2}{k_B T_i} + \frac{4\pi n_e e^2}{k_B T_e} \right)^{1/2}. \quad (2)$$

In experiments on Earth, the ion flow gives the anisotropy of interaction between fine particles which is neglected in the above model. There exist, however, many aspects of fine particle plasmas within the isotropic interaction and we assume that our system can be regarded as isotropic at least in the first approximation. In recent experiments under microgravity, we may expect much reduced ion flow and anisotropy.

In the Yukawa OCP model, the statistical properties of fine particles are characterized by the strength of the Coulomb coupling

$$\Gamma = \frac{(Qe)^2}{ak_B T_p} = \left(\frac{4\pi n_p}{3} \right)^{1/3} \frac{(Qe)^2}{k_B T_p} \quad (3)$$

and the strength of the screening

$$\xi = \frac{a}{\lambda}. \quad (4)$$

Here a is the mean distance between fine particles defined by

$$a = \left(\frac{3}{4\pi n_p} \right)^{1/3}. \quad (5)$$

When the particle have a spherical core of radius r_p and the charge $-Qe$ is uniformly distributed on the surface, the potential at $r > r_p$ is modified into the form [16, 17]

$$\begin{aligned} & \frac{(-Qe)}{1 + r_p/\lambda} \frac{\exp[-(r - r_p)/\lambda]}{r} \\ &= \frac{(-\tilde{Q}e)}{r} \exp(-r/\lambda) \quad \text{for } r > r_p, \end{aligned} \quad (6)$$

where

$$\tilde{Q} = Q \frac{\exp(\tilde{r}_p)}{1 + \tilde{r}_p} \quad (7)$$

and

$$\tilde{r}_p = \frac{r_p}{\lambda}. \quad (8)$$

This potential field is equivalent to that of a point particle with the renormalized charge $-\tilde{Q}e$. The interaction between fine particles is given by the Deryagin-Landau-Verwey-Overbeek (DLVO) potential which is still of the Yukawa type

$$\frac{(-\tilde{Q}e)^2}{r} \exp(-r/\lambda) \quad \text{for } r > 2r_p, \quad (9)$$

r being the distance between centers [16, 17].

3. Experimental Parameters in Terms of Characteristic Parameters

We now consider the correspondence between the characteristic parameters of fine particle plasmas and experimental parameters of densities and temperatures within the characterization by the Yukawa OCP model parameters. As the latter, we have eight parameters and we need eight conditions to determine them.

3.1 Conditions

We first assume that the charge neutrality condition is satisfied:

$$n_i = n_e + Qn_p. \quad (10)$$

When characteristic parameters (Γ , ξ) are specified, we have two conditions:

$$\left(\frac{4\pi n_p}{3}\right)^{1/3} \frac{(Qe)^2}{k_B T_p} = \Gamma \quad (11)$$

and

$$\left(\frac{3}{4\pi n_p}\right)^{1/3} (4\pi e^2)^{1/2} \left(\frac{n_i}{k_B T_i} + \frac{n_e}{k_B T_e}\right)^{1/2} = \xi. \quad (12)$$

The charge on a fine particle $-Qe$ is determined by the balance between the fluxes of ions and electrons onto the surface. When we write Q in the form

$$Q = f_Q \frac{k_B T_e}{e^2/r_p} \quad (13)$$

introducing f_Q , the condition is given by

$$n_e \left(\frac{k_B T_e}{m_e}\right)^{1/2} \exp\left(-\frac{f_Q}{1 + \tilde{r}_p}\right) - n_i \left(\frac{k_B T_i}{m_i}\right)^{1/2} \left(1 + \frac{f_Q}{1 + \tilde{r}_p} \frac{T_e}{T_i}\right) = 0 \quad (14)$$

in the orbital-motion-limited (OML) theory [18]. Here the effect of reduction of the electron density [19] is reflected through (10). We take into account that the surface potential of the fine particles is given by

$$\left[\frac{(-\tilde{Q}e)}{r} \exp(-r/\lambda)\right]_{r=r_p} = \frac{(-\tilde{Q}e)}{r_p} \exp(-r_p/\lambda) = \frac{(-Qe)}{r_p(1 + \tilde{r}_p)}. \quad (15)$$

We discuss the applicability of the OML theory when examples of experimental parameters are given.

Since we have imposed four conditions, (10), (11), (12), and (14) for eight experimental parameters, we are left with four degrees of freedom. As these four, we take the radius r_p ,

$$\Gamma_0 = \frac{(Qe)^2}{r_p k_B T_p}, \quad (16)$$

$$A = \frac{n_e T_e + n_i T_i}{n_p T_p}, \quad (17)$$

and the ratio of the ion and fine particle temperatures

$$\tau_{ip} = \frac{T_i}{T_p}. \quad (18)$$

The value of A is the ratio of the ideal gas pressure of ambient plasma (ions and electrons) to that of fine particles. It is to be noted that

$$\eta = \left(\frac{\Gamma}{\Gamma_0}\right)^3 = \left(\frac{r_p}{a}\right)^3 = \frac{4\pi}{3} n_p r_p^3 \quad (19)$$

is the packing fraction of fine particles, which we denote by η , and

$$\tilde{r}_p = \frac{\Gamma}{\Gamma_0} \xi. \quad (20)$$

In principle, the ratio τ_{ip} is determined by other parameters through the energy relaxation processes which include neutral atoms. In most experiments, however, $\tau_{ip} \sim 1$ is implicitly assumed without detailed analysis of the latter processes or temperature measurements. In this paper we treat this ratio as an externally determined parameter expecting to be around unity, instead of giving other conditions to determine τ_{ip} .

3.2 Solutions

We first note that, from (10), (11), (12), (14), (19), (17), and (18), n_p , $n_i/(A/\tau_{ip})$, $n_e/(A/\tau_{ip})$, $T_i/(A/\tau_{ip}) = \tau_{ip} T_p/(A/\tau_{ip})$, and $T_e/(A/\tau_{ip})$ are explicitly expressed in terms of r_p , Γ/A , ξ , Γ/Γ_0 , and f_Q (see Appendix) as

$$n_p = \frac{3}{4\pi r_p^3} \left(\frac{\Gamma}{\Gamma_0}\right)^3, \quad (21)$$

$$\frac{n_i/(A/\tau_{ip})}{n_p} = \frac{f_Q^2}{\Gamma_0/A} \times \frac{[1 + (\Gamma_0/A)/f_Q]^2}{\{[f_Q^2/(\Gamma_0/A)][1 + (\Gamma_0/A)/f_Q] + (1/3)\xi^2(\Gamma_0/\Gamma) + f_Q\}}, \quad (22)$$

$$\frac{n_e/(A/\tau_{ip})}{n_p} = f_Q^2 \frac{[1 + (\Gamma_0/A)/f_Q]}{[(1/3)\xi^2(\Gamma_0/\Gamma) + f_Q]} \times \frac{[(1/3)\xi^2/(\Gamma/A) - 1]}{\{[f_Q^2/(\Gamma_0/A)][1 + (\Gamma_0/A)/f_Q] + (1/3)\xi^2(\Gamma_0/\Gamma) + f_Q\}}, \quad (23)$$

$$\frac{T_i}{(A/\tau_{ip})} = \tau_{ip} \frac{T_p}{(A/\tau_{ip})} = \frac{f_Q^2}{\Gamma_0/A} \left[\frac{1 + (\Gamma_0/A)/f_Q}{[(1/3)\xi^2(\Gamma_0/\Gamma) + f_Q]} \right]^2 \frac{e^2}{r_p}, \quad (24)$$

and

$$\frac{T_e}{(A/\tau_{ip})} = \left[\frac{1 + (\Gamma_0/A)/f_Q}{[(1/3)\xi^2(\Gamma_0/\Gamma) + f_Q]} \right] \frac{e^2}{r_p}. \quad (25)$$

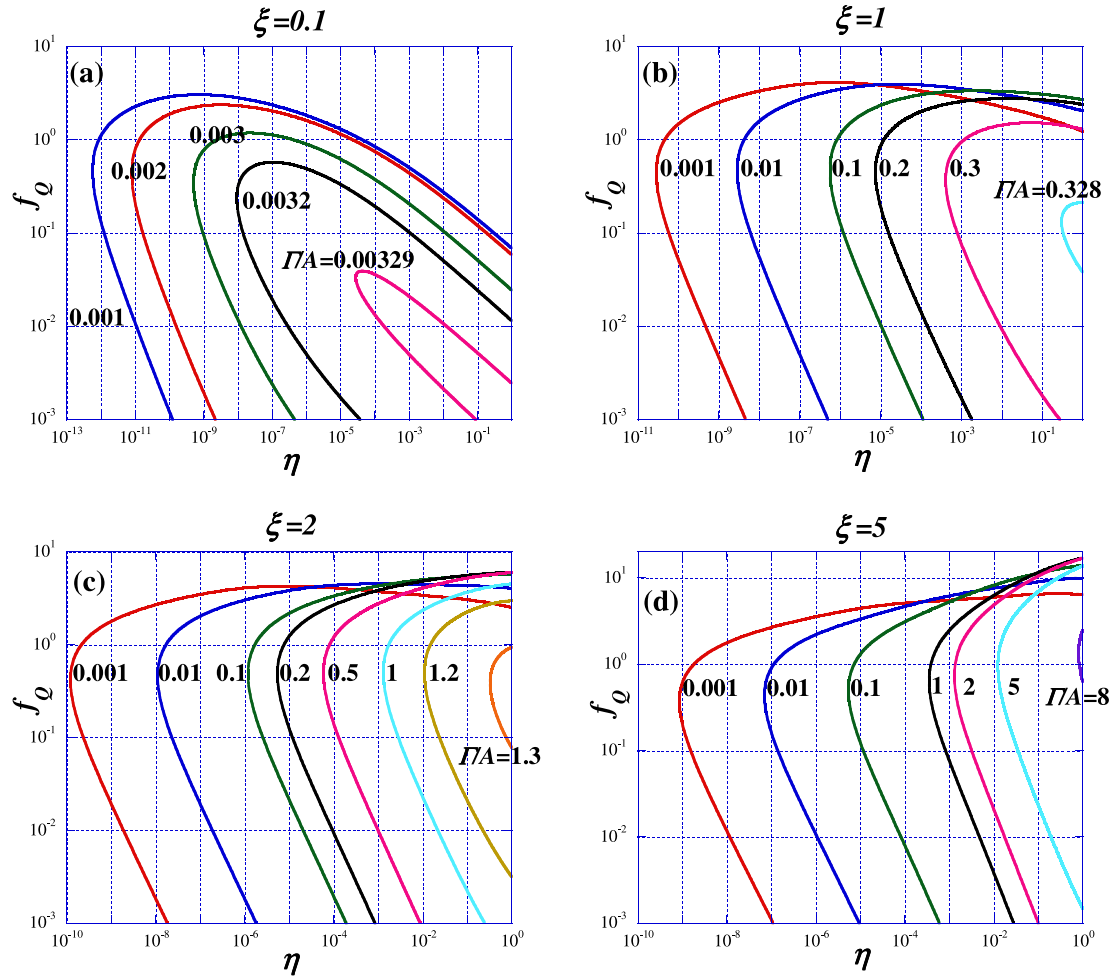


Fig. 1 Values of f_Q as function of packing fraction $\eta = (\Gamma/\Gamma_0)^3$ for some values of ξ and Γ/A . (a) $\xi = 0.1$ and $0.001 \leq \Gamma/A \leq 0.00329$, (b) $\xi = 1$ and $0.001 \leq \Gamma/A \leq 0.328$, (c) $\xi = 2$ and $0.001 \leq \Gamma/A \leq 1.3$, and (d) $\xi = 5$ and $0.001 \leq \Gamma/A \leq 8$.

We here point out that, since $n_e \geq 0$, we have to satisfy the condition $(1/3)\xi^2/(\Gamma/A) \geq 1$ or

$$\xi^2 = \left(\frac{a}{\lambda}\right)^2 \geq 3\left(\frac{\Gamma}{A}\right). \quad (26)$$

This comes from the condition of charge neutrality [19] but has not been explicitly shown in terms of characteristic parameters. Thus the lower limit of realizable ξ is determined by Γ . Since the value of A is usually many orders of magnitude larger than unity, this lower bound for ξ is not effective in most cases even in the domain of strong coupling where $A \gg \Gamma \gg 1$. In the case of very strong coupling where $\Gamma/A \sim 1$ [20–22], however, this bound does limit the space of characteristic parameters of Yukawa OCP realized by fine particle plasmas.

Substituting (22), (23), (24), (25), and (20) into the equation for f_Q or (14), we can determine the value of f_Q . We note that, since (14) includes only the ratios n_e/n_i and T_e/T_i which are independent of r_p or A/τ_{ip} , the value of f_Q is determined self-consistently when the set of values $(\Gamma/A, \xi, \Gamma/\Gamma_0)$ is specified. Let us now introduce n_0 and $E_0 = k_B T_0$ defined respectively by

$$n_0 \equiv \frac{3}{4\pi r_p^3} \quad (27)$$

and

$$E_0 = k_B T_0 \equiv \frac{e^2}{r_p}. \quad (28)$$

We also define A' by

$$A' = \frac{A}{\tau_{ip}}. \quad (29)$$

Then the values of $(f_Q, n_p/n_0, (n_i/n_0)/A', (n_e/n_0)/A', (T_i/T_0)/A' = \tau_{ip}(T_p/T_0)/A', (T_e/T_0)A')$ are determined by the set of values $(\Gamma/A, \xi, \Gamma/\Gamma_0)$ irrespective of the values of $(r_p, A/\tau_{ip})$:

$$\left(\frac{\Gamma}{A}, \xi, \frac{\Gamma}{\Gamma_0}\right) \text{ determines } \left(f_Q, \frac{n_p}{n_0}, \frac{n_i}{A'n_0}, \frac{n_e}{A'n_0}, \frac{T_i}{A'T_0} = \tau_{ip} \frac{T_p}{A'T_0}, \frac{T_e}{A'T_0}\right) \text{ irrespective of } (r_p, A/\tau_{ip}). \quad (30)$$

We may thus regard r_p , A , and τ_{ip} as a kind of *adjustable* parameters which can be chosen so as to satisfy the conditions for densities or temperatures.

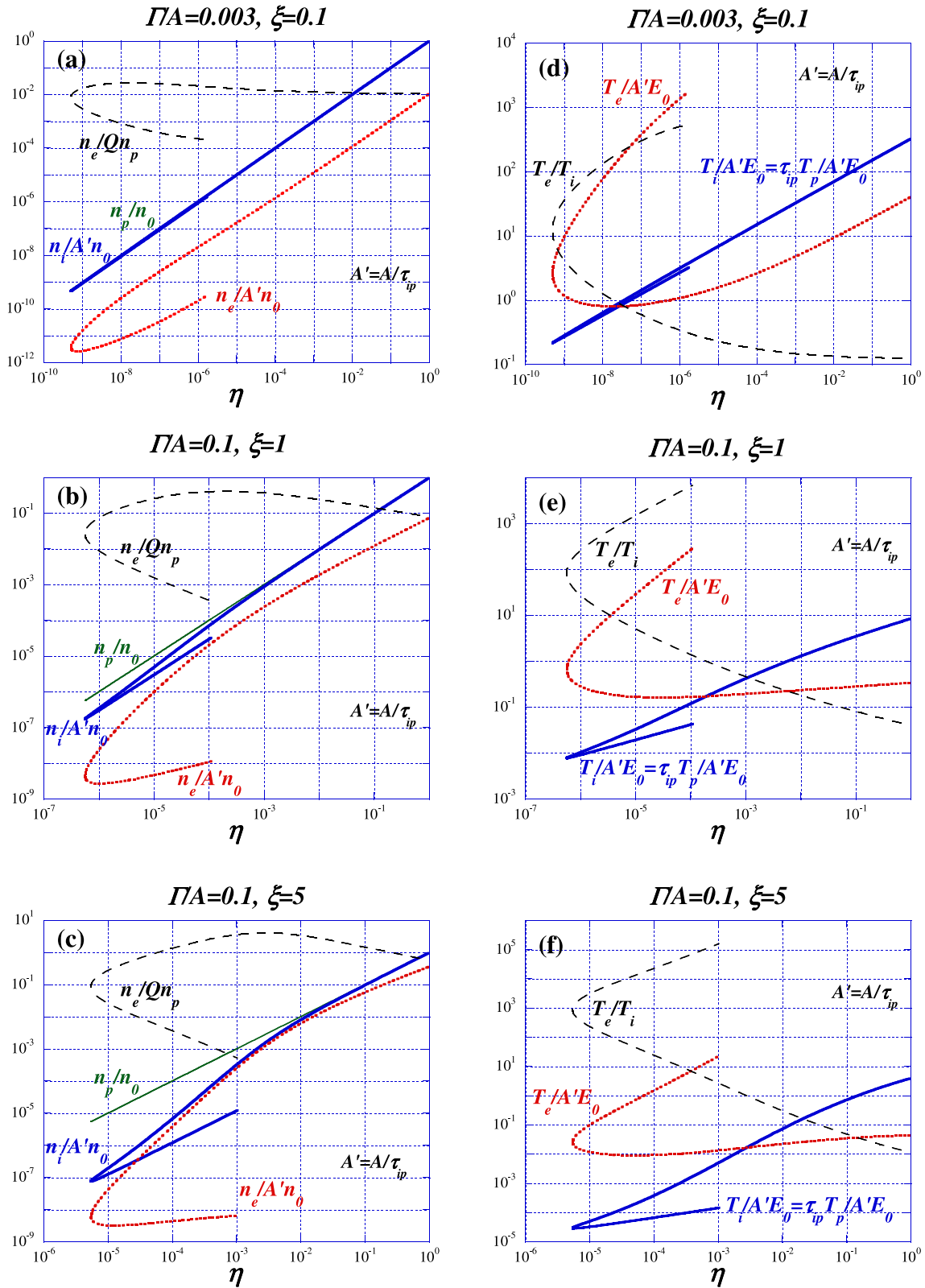


Fig. 2 (a-c) Values of $(n_i/n_0)/A'$ (thick solid line), $(n_e/n_0)/A'$ (dotted line), n_p/n_0 (thin solid line) and n_e/Qn_p (broken line). (d-f) Values of $(T_i/T_0)/A' (= \tau_{ip}(T_p/T_0)/A')$ (solid line), $(T_e/T_0)/A'$ (dotted line), and T_e/T_i (broken line).

4. Examples

We may assume that the experiment is done in inert gases. The results are not restricted within the usual range

of the fine particle radius which is typically $1 \mu\text{m} \leq r_p \leq 10 \mu\text{m}$. In examples shown in Sec.4.2, however, we assume that $r_p = 1 \mu\text{m}$. As for τ_{ip} , we leave its value un-

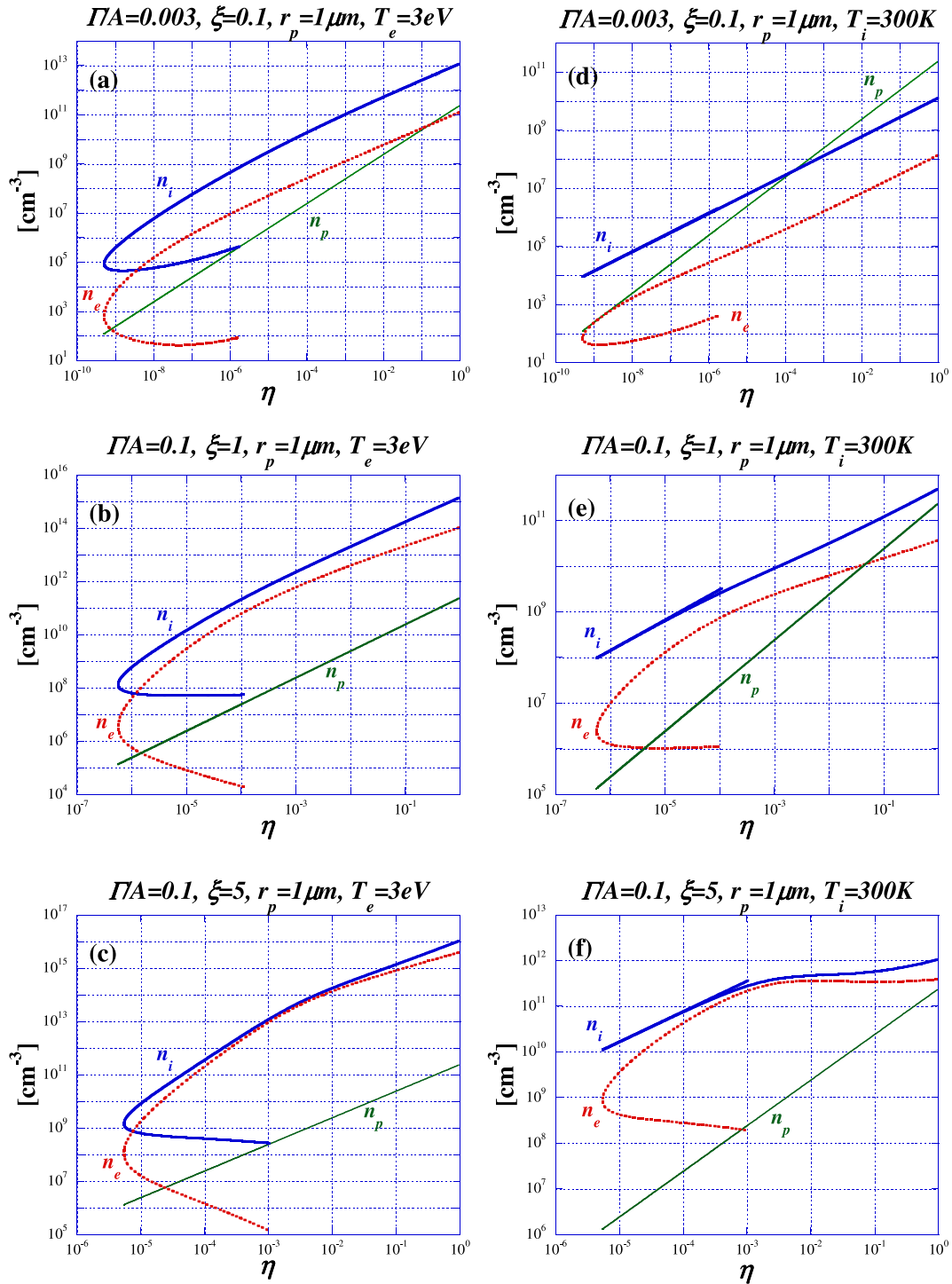


Fig. 3 Values of n_i [cm^{-3}] and n_e [cm^{-3}] in the case of (a-c) $T_e = 3 \text{ eV}$ and (d-f) $T_i = 300 \text{ K}$.

specified as far as possible in order to include the cases where T_p and T_i seem to be not equal against the natural expectation $\tau_{ip} = 1$.

When r_p is measured in μm , we have for n_0 and E_0

$$n_0 = 2.39 \times 10^{11} \frac{1}{(r_p[\mu\text{m}])^3} \text{ cm}^{-3} \quad (31)$$

and

$$E_0 = 1.44 \times 10^{-3} \frac{1}{r_p[\mu\text{m}]} \text{ eV}. \quad (32)$$

4.1 Determination of f_Q

The value of f_Q depends on the species of the neutral gas through the mass of an ion. We here consider the case of Ar. The values of f_Q are shown in Figs. 1 (a)-1 (d) as functions of $\eta = (\Gamma/\Gamma_0)^3$ for typical values of Γ/A and ξ : We have solutions for f_Q when Γ/A is less than the critical value determined by ξ . There generally exist two possibilities when the solution exists for (14). It is to be noted, however, that these two solutions give possible combina-

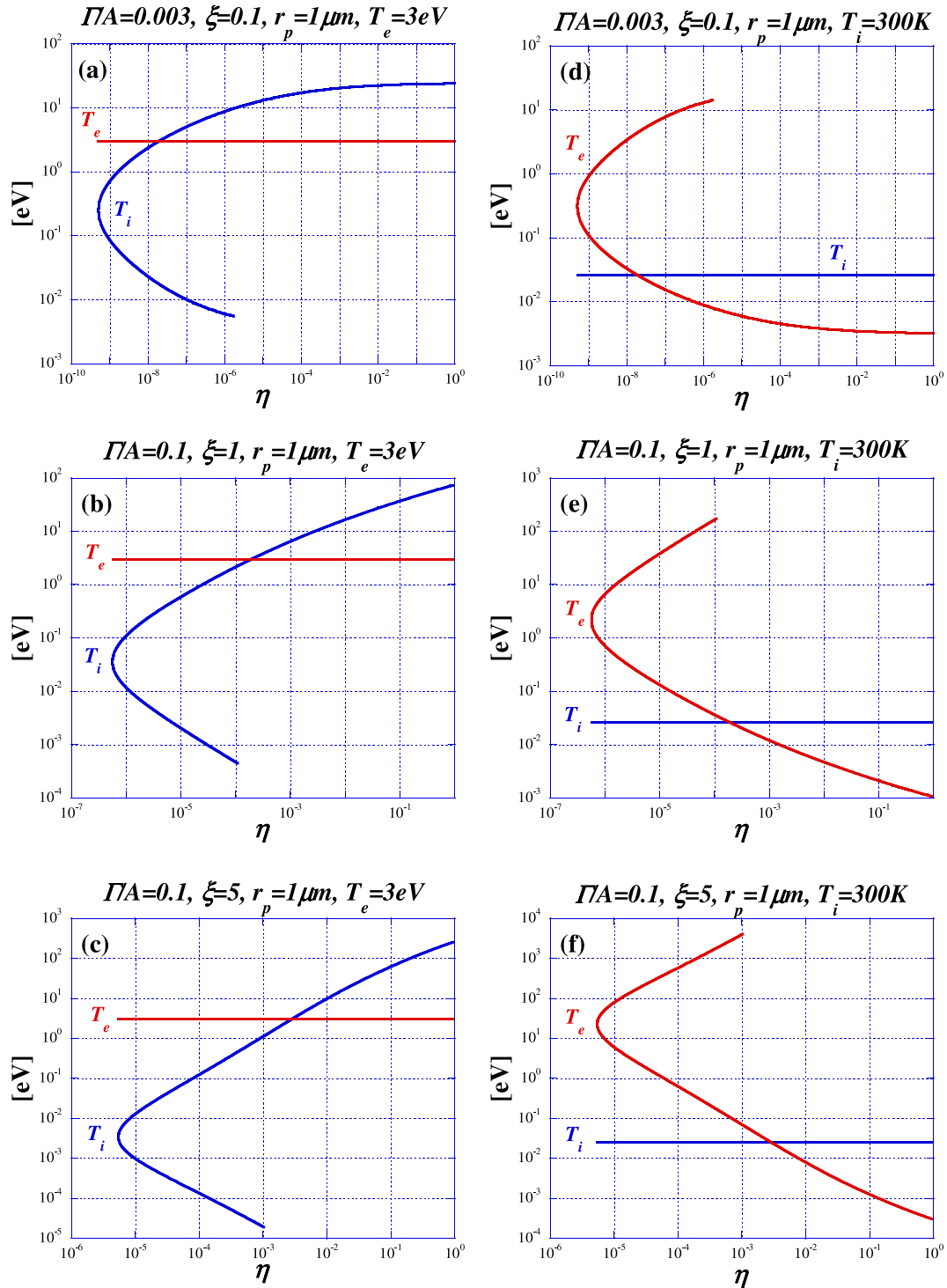


Fig. 4 Values of T_i [eV] and T_e [eV] in the case of (a-c) $T_e = 3$ eV and (d-f) $T_i = 300$ K.

tions of parameters and, when n_i , n_e , T_i , T_e , and \tilde{r}_p are specified as is in its usual application, we have only one solution for (14) if it exists.

Substituting the values of f_Q into (22), (23), (24), and (25), values of $(n_i/n_0)/A'$, $(n_e/n_0)/A'$, $(T_i/T_0)/A' = \tau_{ip}(T_p/T_0)/A'$, and $(T_e/T_0)/A'$ are obtained for given values of $(\Gamma/A, \xi, \Gamma/\Gamma_0)$. Examples of $(n_i/n_0)/A'$ and $(n_e/n_0)/A'$ are shown in Fig. 2 (a) together with n_p/n_0 and

n_e/Qn_p . In some cases, n_e/n_i and n_e/Qn_p are significantly smaller than unity indicating that, in these cases, the overall charge neutrality is maintained mostly by ions and negatively charged particulates.

Examples of $(T_i/T_0)/A' = \tau_{ip}(T_p/T_0)/A'$ and $(T_e/T_0)/A'$ are shown in Fig. 2 (b) together with T_e/T_i . We note that the result includes the cases where T_e/T_i is unusual. The domain of parameters realized in experiments

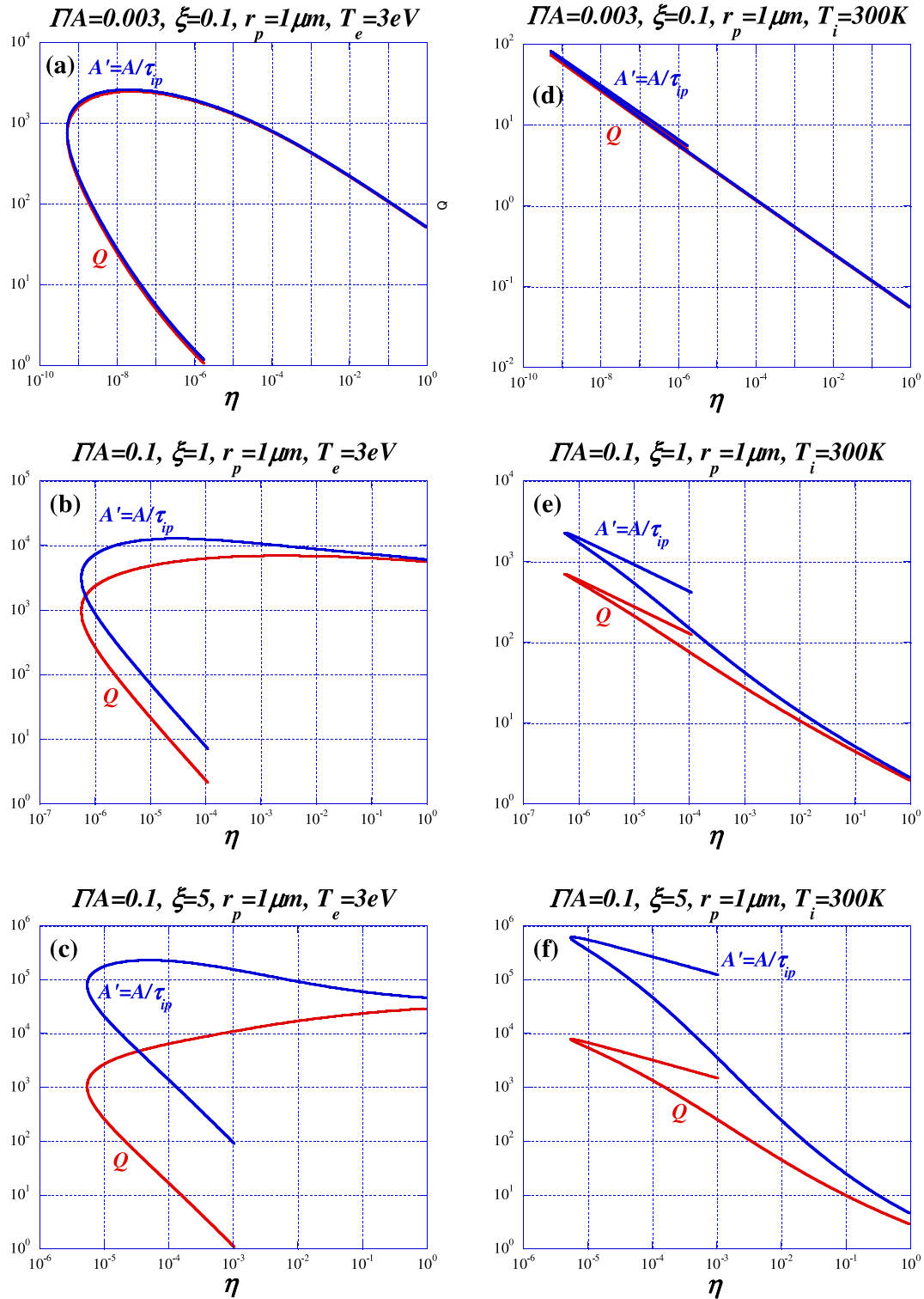


Fig. 5 Values of A' and Q in the case of (a-c) $T_e = 3 \text{ eV}$ and (d-f) $T_i = 300 \text{ K}$.

is limited when we impose some condition on this ratio corresponding to realistic situations in experiments.

In these and following examples, we include the cases with a wide range of densities and temperatures. They may be helpful to explore the possibility to extend usual limit of characteristic parameters of fine particle plasmas.

4.2 Determination of r_p , A , and τ_{ip}

Up to this point, we have not specified the particulate radius r_p , A , and τ_{ip} which determine the actual (not normalized) values of densities and temperatures. For selected radius of fine particles r_p , we can determine the value of $A' = A/\tau_{ip}$ so as to adjust values of T_i or T_e . According to the value of τ_{ip} , all parameters are determined.

We here show two sets of examples of values corresponding to typical values of temperatures, one with $T_e = 3$ eV and another with $T_i = 300$ K, both for the case of $r_p = 1$ μm . Resultant values of densities, temperatures, $A' = A/\tau_{ip}$, and Q for these two sets are shown in Figs. 3 (a-c), Figs. 4 (a-c), and Figs. 5 (a-c) and Figs. 3 (d-f), Figs. 4 (d-f), and Figs. 5 (d-f), respectively.

We observe that, in the case of ($\Gamma/A = 0.1$, $\xi = 1$), both conditions on the temperatures are satisfied when $\eta \sim 5.9 \times 10^{-7}$ and $\Gamma/\Gamma_0 = \eta^{1/3} \sim 8.4 \times 10^{-3}$. In this case, $A \sim 2.3 \times 10^3 \tau_{ip}$, $f_Q \sim 0.34$, $Q \sim 7.0 \times 10^2$, $n_p \sim 1.4 \times 10^5 \text{cm}^{-3}$, $n_i \sim 1.0 \times 10^8 \text{cm}^{-3}$, and $n_e \sim 1.9 \times 10^6 \text{cm}^{-3}$. Realized combination of characteristic parameters is $\xi \sim 1$, $\Gamma \sim 2.3 \times 10^2 \tau_{ip}$, and $\Gamma_0 \sim 2.7 \times 10^4 \tau_{ip}$. We observe that these results include experimental parameters which are often realized in dusty plasma experiments.

The above values are for the case of $r_p = 1$ μm . As for the r_p -dependence, it should be noted that, when (Γ/A , ξ , Γ/Γ_0) and the temperatures are given, $A/\tau_{ip} \propto r_p$, $n_p \propto r_p^{-3}$, $n_i, n_e \propto r_p^{-2}$, and $\Gamma/\tau_{ip}, \Gamma_0/\tau_{ip} \propto r_p$.

4.3 Applicability of OML theory

It has been argued [23] that the expression of the OML theory is modified by the effect of ion-neutral collisions when the ion-neutral collision mean free path is smaller than the screening length λ . For λ , we have

$$\frac{\lambda}{r_p} = \frac{\Gamma_0/\Gamma}{\xi} = \frac{1}{\eta^{1/3}\xi}. \quad (33)$$

On the other hand, at the pressure p_n , the number density of neutral atoms (of inert gases) n_n is given by

$$n_n = 2.41 \times 10^{14} p_n [\text{Pa}] \frac{300}{T_n [\text{K}]} \text{cm}^{-3}. \quad (34)$$

Here the pressure is measured in Pa and the neutral gas temperature T_n is measured in K. Since the ion-neutral collision cross section σ_{in} is $\sim 10^{-14} \text{cm}^2$, the ion-neutral collision mean free path is estimated as $1/\sigma_{in} n_n \sim 5 \times 10^3 (p_n [\text{Pa}])^{-1} \mu\text{m}$ when $T_n \sim 300$ K. The ratio $\lambda/(1/\sigma_{in} n_n)$ is given by

$$\frac{\lambda}{1/\sigma_{in} n_n} \sim \frac{r_p [\mu\text{m}] p_n [\text{Pa}]}{5 \times 10^3 \eta^{1/3} \xi} \frac{300 [\text{K}]}{T_n}. \quad (35)$$

We may expect the simple OML theory is applicable as a first approximation when this ratio is much smaller than unity. Rewriting the above ratio into

$$\frac{\lambda}{1/\sigma_{in} n_n} \sim \frac{r_p [\mu\text{m}]}{1} \frac{p_n [\text{Pa}]}{50} \left(\frac{10^{-6}}{\eta} \right)^{1/3} \frac{1}{\xi} \frac{300 [\text{K}]}{T_n}, \quad (36)$$

we see that the above condition may be satisfied in relatively high density experiments with appreciable screening ($\xi > 1$) where, for example, $\eta \xi^3 / (r_p [\mu\text{m}])^3 > 10^{-6}$ in the neutral gas of relatively low pressure $p_n < 50$ Pa at the room temperature $T_n \sim 300$ K. Our results with the simple OML theory are thus applicable in a wide domain of parameters.

In the case of low density experiments with weak screening or with high neutral gas pressures where the ratio (36) is comparable with or larger than unity, we may need to take into account the effect of ion-neutral collisions in (14). In the case of high densities, the effect of existence of other fine particles may also need consideration. In this paper, we limit ourselves within the simple OML theory leaving such cases to separate analyses.

5. Conclusion

Experimental parameters (the size of fine particles and densities and temperatures of fine particles, ions, and electrons) and the charge of a fine particle are explicitly expressed by characteristic parameters of fine particle plasmas modeled by the Yukawa system of finite-size particles embedded in the ambient plasma. Typical examples of the set of experimental parameters are given and the applicability of adopted charging theory is confirmed in the cases of fine particles of relatively high density in neutral gases of relatively low pressure. It is also shown for the first time that the charge neutrality condition limits the domain of fine particle plasmas such that $\xi^2/(\Gamma/A) \geq 3$ in terms of characteristic parameters. Though definitions of characteristic parameters are simple, the determination of experimental parameters in targeting a specific combination of characteristic parameters is not trivial. These results may be useful to design experiments in order to observe phenomena expected to occur at some specific values of the characteristic parameters [20–22].

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Appendix

Let us define normalized temperatures by

$$T'_i = \frac{k_B T_i}{e^2/r_p}, \quad T'_p = \frac{k_B T_p}{e^2/r_p}, \quad \text{and} \quad T'_e = \frac{k_B T_e}{e^2/r_p}. \quad (A1)$$

Then

$$Q = f_Q T'_e, \quad (A2)$$

$$\Gamma_0 = \frac{(f_Q T'_e)^2}{T'_p}, \quad (A3)$$

and the charge neutrality condition (10) is written as

$$\frac{n_e}{n_p} = \frac{n_i}{n_p} - f_Q T'_e. \quad (A4)$$

Eliminating n_e by (A4), we rewrite the definition of ξ , (12), into the form

$$\frac{n_i}{n_p} \left(\frac{1}{T'_i} + \frac{1}{T'_e} \right) = \frac{1}{3} \xi^2 \frac{\Gamma_0}{\Gamma} + f_Q. \quad (A5)$$

Also eliminating n_e from (17) and using (A3), we have

$$A + \frac{\Gamma_0}{f_Q} = \frac{n_i}{n_p} \frac{T'_e}{T'_p} \left(1 + \frac{T'_i}{T'_e} \right). \quad (\text{A6})$$

From (A5) and (A6), we have

$$T'_e = \frac{1}{\tau_{ip}} \frac{A - 1 + \Gamma_0/f_Q}{(1/3)\xi^2(\Gamma_0/\Gamma) + f_Q}. \quad (\text{A7})$$

Values of T'_p and T'_i are then given respectively by

$$T'_p = \frac{(f_Q T'_e)^2}{\Gamma_0} = \frac{f_Q^2}{\Gamma_0} \frac{1}{\tau_{ip}^2} \left[\frac{A - 1 + \Gamma_0/f_Q}{(1/3)\xi^2(\Gamma_0/\Gamma) + f_Q} \right]^2 \quad (\text{A8})$$

and

$$T'_i = \tau_{ip} T'_p = \frac{f_Q^2}{\Gamma_0} \frac{1}{\tau_{ip}} \left[\frac{A - 1 + \Gamma_0/f_Q}{(1/3)\xi^2(\Gamma_0/\Gamma) + f_Q} \right]^2. \quad (\text{A9})$$

Since we usually have $A \gg 1$, we rewrite $A - 1$ into A .

From these results for temperatures, (A4), and (A5), densities are given as (22) and (23).

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