# Plasma Shape Reconstruction of Spherical Tokamak using CCS Method

F. WANG, K. NAKAMURA[1)], O. MITARAI[2)], K. KURIHARA[3)], Y. KAWAMATA[3)], M. SUEOKA[3)], K.N. SATO[1)], H. ZUSHI[1)], K. HANADA[1)], M. SAKAMOTO[1)], H. IDEI[1)], M. HASEGAWA[1)], S. KAWASAKI[1)], H. NAKASHIMA[1)] and A. HIGASHIJIMA[1)]

*IGSES, Kyushu University*

[1)]*RIAM, Kyushu University,* [2)]*Kyushu Tokai University,* [3)]*Japan Atomic Energy Agency*

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Plasma shape reconstruction is important for plasma control of tokamak. Cauchy-Condition Surface (CCS) method is a numerical approach to reproduce plasma shape which has good precision in conventional tokamak. In order to apply it in the plasma shape reproduction of Compact PWI experimental Device (CPD) which is a new spherical tokamak in Kyushu University, the calculation precision of CCS method in CPD is analyzed in the paper.



Keywords: plasma, spherical tokamak, Cauchy-Condition surface, shape reproduction

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## 1. Introduction

From the viewpoint of plasma control of tokamak, plasma shape reproduction is very important, especially for the non-circular and triangular plasmas. Compared with the conventional tokamak, spherical tokamak has high natural elongation and natural triangularity due to smaller aspect ratio, and the equilibrium and stability properties are different.

There are some kinds of numerical methods to reproduce plasma shape such as filament current method. The CCS method is a kind of numerical approach to reproduce plasma shape by using magnetic measurement, and shows good results in comparison filament current method and equilibrium method. It can reproduce plasma shape with precision corresponding to the number and types of available sensors, and it can be used in the real-time plasma shape control.

In order to apply it in the plasma shape control of CPD which is a spherical tokamak, the calculation precision and flux loops dependence is checked. Since various plasma shapes appear in the real plasma discharge experiment, shape reconstruction precision in case of different kind of plasma shape is also analyzed.

## 2. CCS Method Outline

The Cauchy-Condition Surface method is a kind of exact numerical method which is based on the boundary integral equation. The Cauchy-Condition surface is defined as a hypothetical plasma surface, where both the Dirichlet ($\phi$) and Neumann ($Bt$) conditions are unknown, as shown in Fig. 1. This surface is located inside the real plasma region. It is assumed that CCS encloses all the plasmas and there are no plasmas outside the CCS [1, 2].

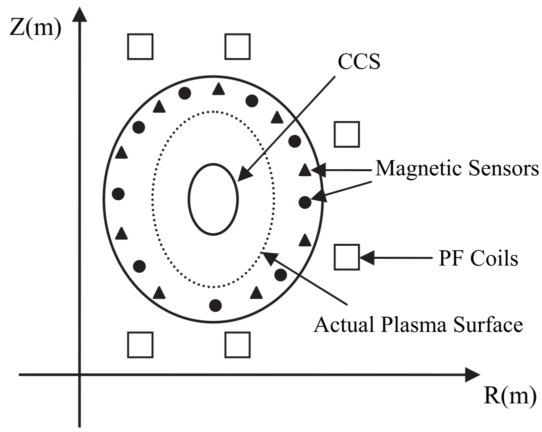



Fig. 1 Outline of Cauchy-Condition Surface method.

According to the static Maxwell's equation, three types of boundary integral equations can be given by using the magnetic sensors signals and poloidal coils current data. The discretized formulas are shown as follows [3].

(1) For flux loop signal

$$\phi(x_f) = \sum_{i=1}^{M} W_{F1}(x_f, z_i)\phi(z_i) + \sum_{i=1}^{M} W_{B1}(x_f, z_i)Bt(z_i) + W_{C1}(x_f)I_{PF} \quad (1)$$

(2) For magnetic field $Bt$

$$Bt(x_B) = \sum_{i=1}^{M} W_{F2}(x_B, z_i)\phi(z_i)$$

*author's e-mail: maple@aees.kyushu-u.ac.jp*

$$+\sum_{i=1}^{M} W_{B2}(x_B, z_i)Bt(z_i) + W_{C2}(x_B)I_{PF} \quad (2)$$

(3) For discretized points along CCS

$$\frac{1}{2}\phi(x) = \sum_{i=1}^{M} W_{F3}(x, z_i)\phi(z_i) + \sum_{i=1}^{M} W_{B3}(x, z_i)Bt(z_i) + W_{C3}(x)I_{PF} \quad (3)$$

Where, M is the discretized number of CCS. $\phi(x_f)$ is the flux loop measurement. $Bt(x_B)$ is the magnetic sensor measurement. $\phi(z_i)$ and $Bt(z_i)$ is the flux and Bt value at discretized points on CCS. $I_{PF}$ is the poloidal field coils current included in the calculation region. $W_{F1}(x_f, z_i)$, $W_{B1}(x_f, z_i)$, $W_{C1}(x_f)$, $W_{F2}(x_B, z_i)$, $W_{B2}(x_B, z_i)$, $W_{C2}(x_B)$, $W_{F3}(x, z_i)$, $W_{B3}(x, z_i)$, $W_{C3}(x)$ are coefficient matrix which can be calculated beforehand.

Equations (1), (2) and (3) are coupled and can be expressed in matrix form, and then $\phi(z_i)$ and $Bt(z_i)$ at several discretized points along CCS can be evaluated by using the least square method.

Then the flux distribution can be calculated using equation (4), and the outmost magnetic flux surface or plasma shape can be found by plotting the contour.

$$\phi(x) = \sum_{i=1}^{M} W_{F4}(x, z_i)\phi(z_i) + \sum_{i=1}^{M} W_{B4}(x, z_i)Bt(z_i) + W_{C4}(x)I_{PF} \quad (4)$$

Where, $\phi(x)$ is the flux value at any position. $W_{F4}(x, z_i)$, $W_{B4}(x, z_i)$ and $W_{C4}(x)$ are coefficient matrix.

# 3. Application to CPD

The Compact PWI experimental device is a new experimental spherical tokamak device whose main parameters are as follow:

Plasma major radius $R = 0.3$ m
Plasma minor radius $a = 0.2$ m
Toroidal field $B_t = 0.3$ T @ $R = 0.25$ m
Operation period $\tau_d = 1.00$ sec for $B_t = 0.3$ T
Plasma current $I_p = 150$ kA

The calculation configuration for CPD is shown in Fig. 2. There are 7 poloidal field coils including CS and 45 flux loops, and the CCS is an ellipse with minor/major axis 0.06 m/0.10 m located at plasma center. The ellipse is discretized into 6 points for numerical calculation. The mesh size for calculation is $100 \times 200$ and mesh precision is 1 cm.

In order to check the calculation precision, an equilibrium code [4] based on Green's function is used to make ideal flux signal whose calculation error is about 0.5%. The main steps of comparison are as follows:

(1) Ideal flux surfaces are made by equilibrium code.

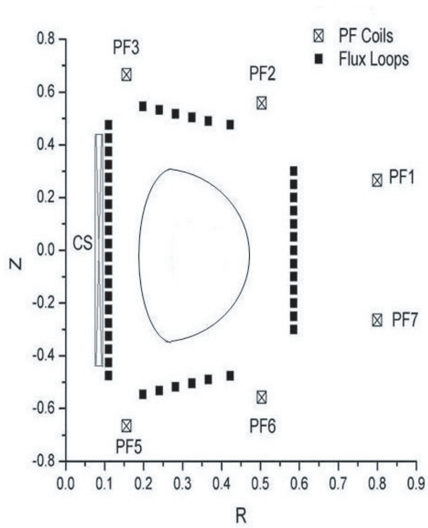



Fig. 2 Magnetic sensor positions of CPD.

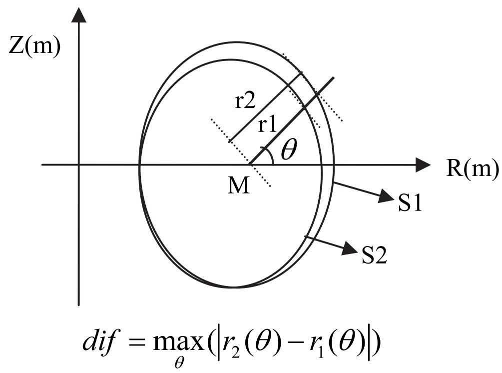



Fig. 3 Plasma shape difference.
S1 is original shape, S2 is reproduced shape
M is the center of vacuum chamber

(2) These plasma shapes are reproduced by using CCS method.

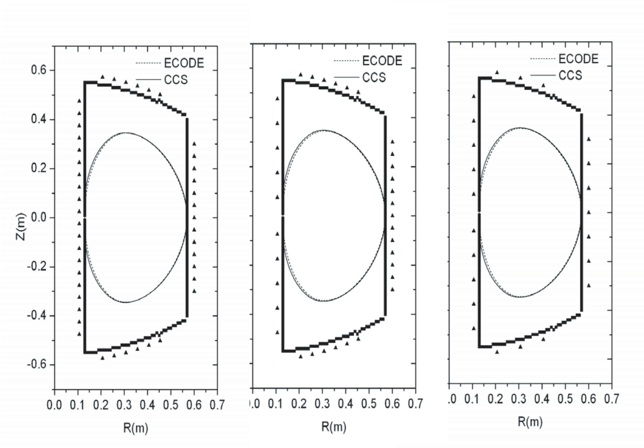
(3) The original and reproduced shapes are compared. The shape difference $dif$ is defined as shown in Fig. 3.

## 3.1 Flux loops dependence

In the present stage there are 45 flux loops measurements. In order to analyze the flux loops dependence, different numbers of flux loops (45, 35, 29 and 23) are used to reproduce the plasma shape. The typical results are shown in Fig. 4. When flux loops number is from 45 to 23, the shape difference is less than 1 cm.

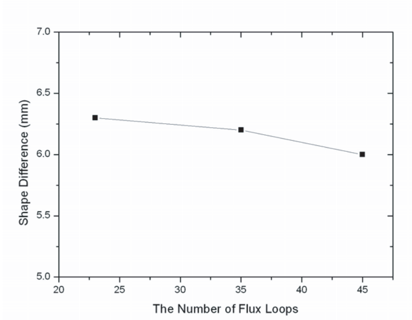
## 3.2 Cauchy-condition dependence

In case of much elongated and triangular plasmas in spherical tokamak, good precision can be achieved by increase in degrees of parametric freedom representing the Cauchy Condition (M) as shown in Fig. 5. The shape difference decreases from 8.5 mm to 2.5 mm, when M is increased from 6 to 10.



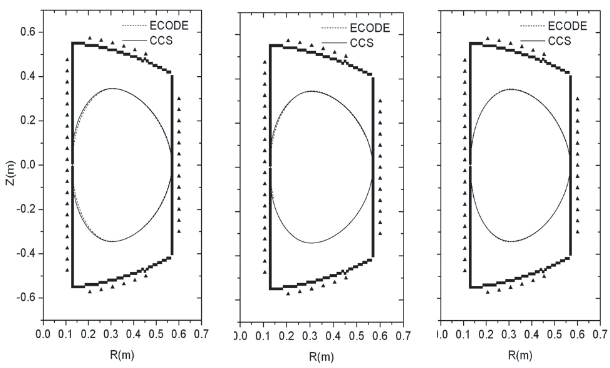

(45 flux loops) (35 flux loops) (23 flux loops)

(a) Reconstructed shape using different numbers of flux loops



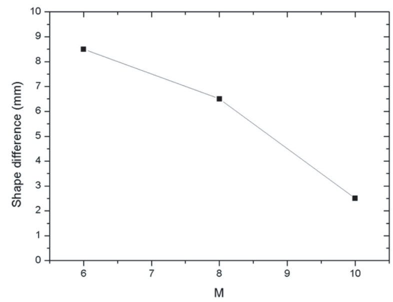

(b) Shape difference of different flux loops

Fig. 4 Flux loops dependence.




(M=6) (M=8) (M=10)

(a) Reconstructed shape using different M




(b) Shape difference of different M

Fig. 5 Cauchy-Condition dependence.

### 3.3 $\ell_i$ dependence

Various kinds of plasma current profiles may appear in the real discharge experiments. In order to control plasma shape precisely in real-time, $\ell_i$ dependence of CCS method to reproduce spherical tokamak plasma shape is studied. Two kinds of plasma shapes are considered.

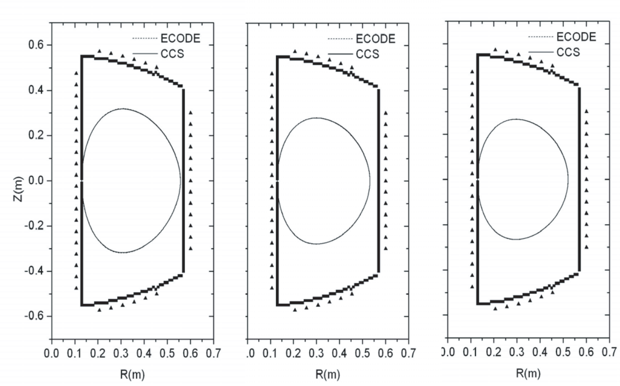
Fig. 6 shows a sample comparison of limiter plasma which is not much elongated. The shape difference of plasma shape is less than 1.4 mm, when $\ell_i$ is from 0.5 to 1.0, and the CCS discrestized number M is 6.

In case of double-null plasmas which is large elongated and triangular, typical plasma shapes corresponded to $\ell_i = 0.6{\sim}1.0$ ($\beta_\mathrm{p} = 0.5$) are calculated. As shown in Fig. 7, the shape difference is less than 6.5 mm, while M is 6.

## 4. Summary and Future Work

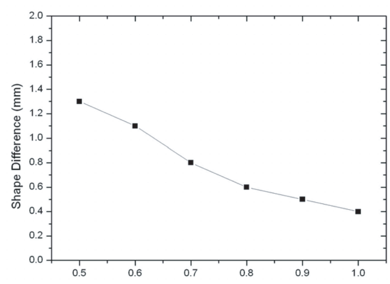
The Cauchy Condition Surface method is based on an analytical exact solution for magnetic field in the multiply-connected vacuum region and numerically reconstructs plasma shape with good precision.

The CCS method can reproduce plasma shape of



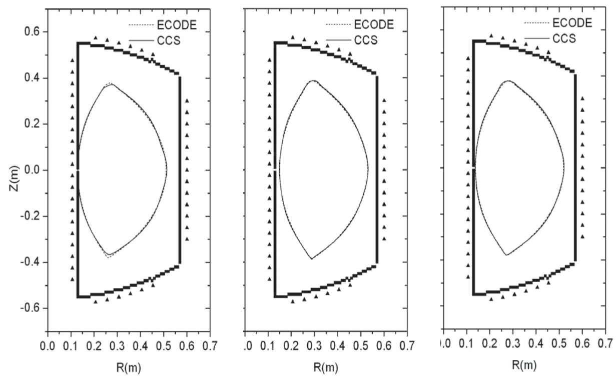

($\ell_i$ =0.8) ($\ell_i$ =0.9) ($\ell_i$ =1.0)

(a) Reconstructed plasma shape of different $\ell_i$ ($\beta_\mathrm{p}$ = 0.3)



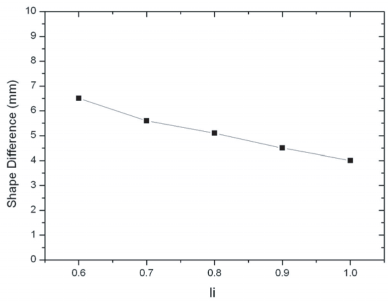

(b) Shape difference of different $\ell_i$

Fig. 6 $\ell_i$ Dependence of limiter plasma.




($\ell_i$ =0.8) ($\ell_i$ =0.9) ($\ell_i$ =1.0)

(a) Reconstructed plasma shape of different $\ell_i$ ($\beta_p = 0.5$)




(b) Shape difference of different $\ell_i$

Fig. 7 $\ell_i$ Dependence of double-null plasma.

spherical tokamak with different number of flux loops measurement (45, 35 and 23), while the shape difference is less than 1 cm.

Even in case of much elongated and triangular plasmas in spherical tokamak, good precision can be achieved by increase in degrees of parametric freedom representing the Cauchy condition.

The CCS method can reproduce spherical tokamak plasma shape with good precision in different plasma current profiles ($\ell_i = 0.5\sim1.0$). In case of small elongated plasma, normally the shape difference is less than 3 mm, and in case of large elongated and triangular double-null plasma, the shape difference is less than 8 mm, while the mesh precision is 1 cm.

As mentioned above the shape difference of large triangular and elongated plasma is a little larger than that of limiter plasma in spherical tokamak. In order to solve this problem the Cauchy-Condition Surface should be optimized.

Furthermore in the real plasma discharge experiment, eddy current effect is important, especially at the ramp-up stage of discharge. The effect of eddy current should be taken into account of Cauchy Condition Surface method in future.

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