A New Explanation for Toroidal Spin-Up of a Field-Reversed Configuration

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A new explanation regarding the toroidal spin-up of a field-reversed configuration (FRC) is provided. A physical picture showing that the poloidal flux can convert directly to kinetic angular momentum is described. Through the use of an ion orbit calculation in resistively decaying FRC plasma, toroidal rotation at both the separatrix and the field-null is found to occur.

Keywords: field-reversed configuration, toroidal spin-up, resistive flux decay, canonical angular momentum, inductive electric field

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An $n = 2$ rotational instability of a field-reversed configuration (FRC) plasma has been observed experimentally and reported in several papers [1–3]. This instability originates from the centrifugal force which acts on a rotating FRC plasma. The origin of toroidal spin-up has not yet been identified, though particle loss [4] and end-shorting [5] are now considered the two most probable mechanisms governing this phenomenon. Recently, Belova et al. claimed that particle loss associated with a resistive flux decay may contribute to FRC plasma rotation [6]. Both particle loss and end-shorting occur locally, however, in the vicinity or outside of the separatrix. Therefore, an explanation of the toroidal spin-up of the entire FRC plasma is questionable. We now propose here a possible mechanism to rotate the whole FRC plasma inside the separatrix.

Suppose the FRC plasma is axisymmetric. This assumption is valid until the rotational instability is triggered. In this case, the canonical angular momentum

$$P_\theta = mv_\theta r + q\psi(r, z) \tag{1}$$

of every particle is conserved, where m, q are the mass and charge, respectively, v_θ is the toroidal velocity component, and $\psi(r, z)$ is the poloidal flux function. If the poloidal flux decays due to resistivity and toroidal axisymmetry is still valid, then

$$m\Delta(v_\theta r) = -q\Delta\psi. \tag{2}$$

Equation (2) shows that when the poloidal flux decays, every ion gains angular momentum in the ion diamagnetic direction. Generally, the separatrix radius decreases during the decay phase. If the guiding center r is also decreased, the toroidal velocity v_θ is further increased.

We can also explain FRC plasma rotation from the viewpoint of particle trajectories. In FRC plasma, a small-gyroradius drift orbit, a figure-8 orbit, and a betatron orbit are three possible types of trajectories. In contrast to the betatron particles, small-gyroradius drift particles and figure-8 particles have smaller angular momentum. If the poloidal flux decays, the figure-8 particles can change abruptly to betatron particles due to the increase of the Larmor radius. The transition of trajectory type results in the increment of the toroidal angular momentum.

The inductive electric field is also a possible cause of the rotation. The betatron particles move around the field-null. The toroidal electric field always accelerates the betatron particles in the ion diamagnetic direction. Here, the radial $\boldsymbol{E} \times \boldsymbol{B}$ drift motion contributes to the betatron oscillation, and therefore the guiding center is fixed at the equilibrium position at which the centrifugal force and the Lorentz force are balanced.

Since the discussion above is based on a single-particle picture, we need to confirm a collective effect due to the flux decay. A number of super-particles are traced numerically in the decaying FRC plasma. The poloidal flux decay is reproduced by

$$\frac{\partial\psi}{\partial t} = -r\eta J_\theta. \tag{3}$$

Here, the electric resistivity η equals $f_A\eta_{cl}$, where f_A is the anomaly factor and η_{cl} is the classical resistivity. The flux lifetime is controlled by the parameter f_A. By integrating

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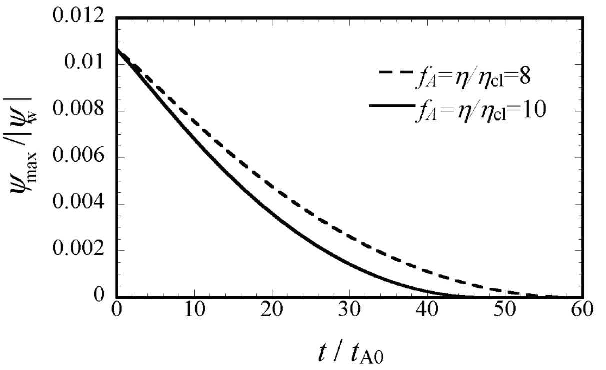

Fig. 1 The time evolution of the maximum trapped poloidal flux ψ_{max}. The quantities ψ_w and t_{A0} are the flux function at the wall and the midplane and the Alfvén time $r_w\sqrt{\mu_0 m_i n_0}/B_{ex}$, respectively. The ratio η/η_{cl} is the anomaly factor f_A, where η_{cl} is the classical resistivity.

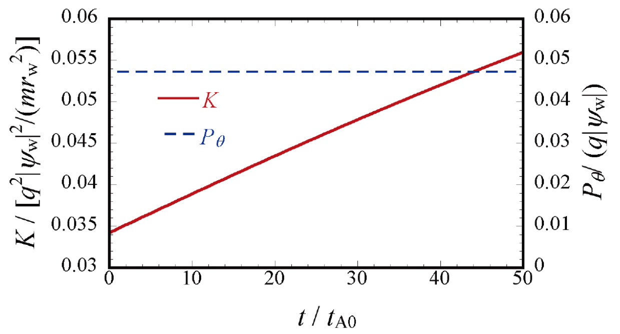

Fig. 2 The time evolution of the kinetic energy K (the red solid line) and canonical angular momentum P_θ (the blue dashed line) for a sample betatron particle.

Eq. (3) by means of the Runge-Kutta method, a flux function at a calculation point is found. The electromagnetic fields are then written by the obtained ψ as

$$\boldsymbol{B} = \nabla\times\boldsymbol{A} = \nabla\times\left(\frac{\psi}{r}\boldsymbol{e}_\theta\right), \quad \boldsymbol{J} = \frac{1}{\mu_0}\nabla\times\boldsymbol{B}, \quad \boldsymbol{E} = \eta\boldsymbol{J}. \tag{4}$$

Initially, ions are loaded in the r-z plane uniformly. The ion as a super-particle is weighted by the Maxwellian distribution. The equation of motion is solved for ions in $\boldsymbol{B}$ and $\boldsymbol{E}$ fields given by Eq. (4), and the ion density and toroidal flow velocity are obtained by a PIC method at each calculation time step.

The time evolution of the maximum trapped flux is presented in Fig. 1, where ψ_w is the flux function at the chamber wall and the midplane, t_{A0} is the Alfvén time $t_{A0} \equiv r_w/v_{A0}$, r_w is the wall radius, and $v_{A0} \equiv B_{ex}/\sqrt{\mu_0 m_i n_0}$ (B_{ex}: the external magnetic field at $t = 0$, m_i: the plasma ion mass, n_0: the density at the field-null and $t = 0$). Note that $\psi > 0$ inside the separatrix region in our paper. As the anomaly factor f_A increases from 8 to 10, the flux lifetime clearly decreases. Consequently, the magnetic field reversal disappears at $t = 45\,t_{A0}$, when f_A is 10; the lifetime is comparable with the experiment data measured on the NUCTE (Nihon University Compact Torus Experiment)-III device. In the present paper, $B_{ex} = 0.4$ T, $r_w = 0.17$ m, and $n_0 = 2.0\times10^{21}$ m^{-3}, these being typical experiment parameters of the NUCTE-III device.

The time evolution of the kinetic energy K and canonical angular momentum P_θ for a sample betatron particle is shown in Fig. 2. Although P_θ is conserved, K is increased in time. The increase in K shows a continuous acceleration in the ion diamagnetic direction, and can result in a rotating motion at the field-null. In addition, the conservation of P_θ guarantees the validity of our calculation. In order to show the fact that a flux decay can rotate the FRC plasma, we

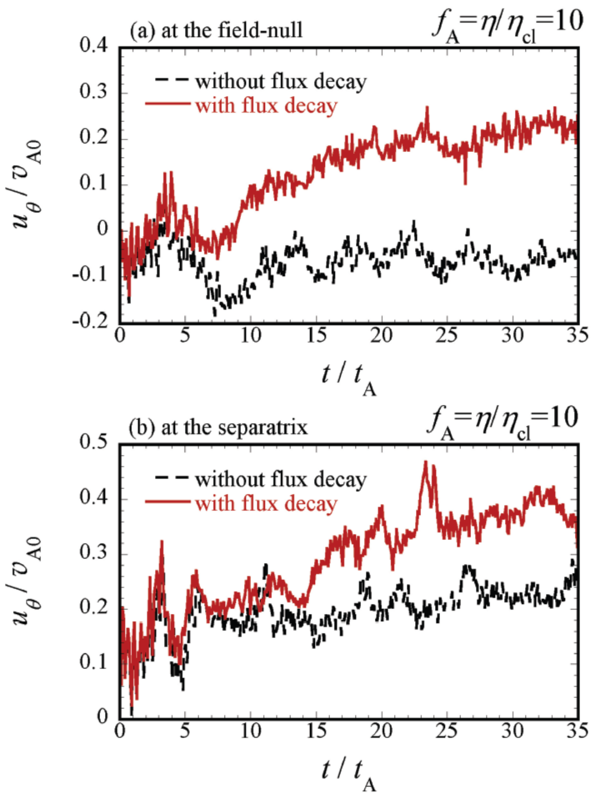

Fig. 3 Comparison of the toroidal flow velocity at (a) the field-null and (b) the separatrix between the absence of (the black dashed line) and the presence of (the red solid line) the resistive flux decay. Here the anomaly factor f_A (= η/η_{cl}) is 10.

compare the toroidal flow velocity between the presence of and absence of the flux decay; the comparison is shown in Fig. 3. It is found from Fig. 3 (a) that the toroidal flow velocity at the field-null increases to $0.2v_{A0}$ when the flux decays. On the other hand, when the flux is maintained rotation in the ion diamagnetic direction is never observed. At the separatrix, the increase in the toroidal flow is observed for both cases as shown in Fig. 3 (b). For the case without the flux decay, particle loss is responsible for the increase, because ions initially near the separatrix can be lost. One can find that the flux decay enhances the rotation. A direct change from the poloidal flux to the angular momentum is responsible for this enhancement.

To conclude our report, a rotation at the field-null of an FRC plasma can be explained by a resistive flux decay alone; a viscous torque is not always necessary to explain a rigid-like rotation.

The contribution of loss particles to the rotation and a comparison of the present numerical results with experimental results will be reported in a subsequent paper. Though the toroidal flow affects the electromagnetic fields and the ion motion, this effect is neglected in the present paper and is left as a subject for future study.

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