Townsend Avalanche Breakdown Assisted by Radio Frequency Wave in Tokamaks

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A simple model of radio frequency wave (RF) assisted breakdown based on Townsend avalanche theory is proposed for the purpose of evaluating the effect of RF on Townsend avalanche breakdown. According to this model, the required minimum electric field for RF-assisted breakdown can be decreased down to half of that for breakdown by the induction electric field alone. The electric field of RF reaches a minimum when the frequency of the RF is equal to the electron cyclotron frequency.

Keywords: Townsend avalanche, breakdown, induction electric field, radio frequency wave, RF

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In ITER, the induction electric field constrained by terminal voltage limitations on multi-turn superconducting poloidal field (PF) coils is about 0.3 V/m [1]. Since this means avalanche margins are small in relation to Townsend avalanche theory, present ITER plans are to provide a 90-140 GHz electron cyclotron (EC) system to assist during the breakdown phase [1] in addition to a 170 GHz EC system for plasma heating. Reduction of breakdown loop voltage using ECH pre-ionization has been investigated in DIII-D, where a decrease of the minimum electric field from 0.25 V/m to 0.15 V/m was achieved with 700 kW of 60 GHz EC assist power [2]. In the higher frequency region such as 90-140 GHz of the ITER case, the required RF power for breakdown without one-turn voltage has been investigated on the superconducting tokamak TRIAM-1M, which can generate a toroidal magnetic field up to 8 T, and has been observed to be 50 kW at 1.3×10^{-3} Pa with a 170 GHz 200 kW EC system [3]. However, the conditions for breakdown in the presence of both radio frequency waves (RF) and one-turn voltage have not been clear until now.

The purpose of this paper is to model the breakdown process with reference to Townsend avalanche theory so that it includes both the effects of RF and an induction electric field, and to evaluate the effect of RF on the minimum induction electric field defined by Townsend avalanche.

According to Townsend avalanche theory, the ionization coefficient α (ionizations per unit length) is

$$\alpha = (1/\lambda)\exp(-x/\lambda), \tag{1}$$

where λ is the electron's mean free path. An electron gains energy eE_0x while moving a distance x along a static electric field E_0. A breakdown occurs if this energy exceeds the ionization potential eV_i. By substituting x into Eq. (1), the well-known equation of Townsend avalanche can be obtained as

$$\frac{\alpha}{p} = A\exp\left(-\frac{B}{E_0/p}\right), \tag{2}$$

where p is the neutral gas pressure, and A $(= 1/(\lambda p))$ and B $(= AV_\mathrm{i})$ are coefficients that depend on gas species: for hydrogen, $A = 3.8\,\mathrm{m^{-1}Pa^{-1}}$ and $B = 94\,\mathrm{Vm^{-1}Pa^{-1}}$. An aspect of breakdown assisted by RF is that the electron also gains energy from RF. The averaged velocity of an electron under E_0 is given by $\upsilon = \mu E_0$, where μ is the mobility defined as $\mu = e/(m\nu)$, and ν is the frequency of the electron's collisions with neutral gas, assuming here $\nu\,(1/\mathrm{sec}) = 1.78\times10^7 p$ (Pa) [4]. Hence, the time to move distance x is given by

$$\Delta t = x/\upsilon = x/(\mu E_0) = (m\nu/(eE_0))\,x. \tag{3}$$

The electron gains energy eE_0x from the static electric field and energy $\Delta t\delta_\mathrm{E}$ from RF while moving distance x, where δ_E is the energy that the electron gains per unit time from RF. A breakdown is assumed to occur if the sum of these two energies exceeds the ionization potential.

$$eE_0x + \Delta t\delta_\mathrm{E} = eV_\mathrm{i}. \tag{4}$$

Substitution of Δt from Eq. (3) into Eq. (4) gives the distance x for breakdown.

$$x = \frac{V_\mathrm{i}/E_0}{1+\delta_\mathrm{E}(m\nu/(e^2E_0^2))}. \tag{5}$$

Hence, from Eq. (1), Townsend avalanche breakdown as-

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sisted by RF is given by

$$\frac{\alpha}{p} = A \exp\left\{-\frac{B}{E_0/p}\,\frac{1}{1+\delta_E(m\nu/(e^2E_0^2))}\right\}. \tag{6}$$

By comparing Eq. (6) to Eq. (2), one can notice that the effective electric field can be defined as

$$E_{\mathrm{eff}} = E_0\,\{1+\delta_{\mathrm{E}}(m\nu/(e^2E_0^2))\}. \tag{7}$$

Setting $\alpha = 1/L$ in Eq. (6), where L is the ionization mean free path leading to the breakdown, gives the minimum electric field

$$E_{\mathrm{eff}} = \frac{B\,p}{\ln(A\,p\,L)}. \tag{8}$$

Hence, the minimum electric field E_0 for breakdown assisted by RF can be determined by Eq. (7), where E_{eff} is defined by Eq. (8). Since Eq. (7) is a quadratic equation in E_0, the solution is

$$E_0 = \frac{1}{2}\left(E_{\mathrm{eff}} + \sqrt{E_{\mathrm{eff}}^2 - 4m\nu\delta_{\mathrm{E}}/e^2}\right). \tag{9}$$

The other solution is negligible. The condition for E_0 to be a real number in Eq. (9) is

$$E_{\mathrm{eff}}^2 \geq 4m\nu\delta_{\mathrm{E}}/e^2. \tag{10}$$

When there is no assist by RF ($\delta_{\mathrm{E}} = 0$), the required electric field E_0 is equal to E_{eff}, which is defined by Townsend avalanche in Eq. (8). And, E_0 decreases with an increase in δ_{E}. Especially, when the equality in Eq. (10) is satisfied, the required electric field E_0 for the RF-assisted breakdown becomes a minimum value, and this value is half of E_{eff} by Eq. (9).

To evaluate δ_{E}, the equation of motion for an electron is simply assumed to be

$$m\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = e\boldsymbol{v}\times\boldsymbol{B}_0 - m\nu\boldsymbol{v} + e\boldsymbol{E}_{\mathrm{RF}}, \tag{11}$$

where $\boldsymbol{B}_0$ is the static magnetic field and $\boldsymbol{E}_{\mathrm{RF}}$ is electric field of RF. Choosing the z-coordinate along the static magnetic field and the y-coordinate along the perpendicular electric field of RF, the components of Eq. (11) are

$$\begin{aligned} m\frac{\mathrm{d}v_x}{\mathrm{d}t} &= ev_yB_0 - m\nu v_x\,, \\ m\frac{\mathrm{d}v_y}{\mathrm{d}t} &= -ev_xB_0 - m\nu v_y + eE_{\mathrm{RF}}\,. \end{aligned} \tag{12}$$

When $v_x = v_{x0}\exp(j\omega t), v_y = v_{y0}\exp(j\omega t)$ and $E_{\mathrm{RF}} = E_1\exp(j\omega t)$ are applied, v_{x0} and v_{y0} are given by

$$\begin{aligned} v_{x0} &= \frac{\omega_{\mathrm{c}}}{\nu + j\,\omega}v_{y0}\,, \\ v_{y0} &= \frac{eE_1}{m}\frac{\nu + j\,\omega}{(\nu + j\,\omega)^2 + \omega_{\mathrm{c}}^2}\,, \end{aligned} \tag{13}$$

where $\omega_{\mathrm{c}} = eB_0/m$ is the electron cyclotron frequency. With Eq. (13), $\mathrm{Re}[v_{y0}]$ is given by

$$\begin{aligned} \mathrm{Re}[v_{y0}] &= \frac{\nu}{2}\frac{eE_1}{m}\left\{\frac{1}{(\omega+\omega_{\mathrm{c}})^2+\nu^2} + \frac{1}{(\omega-\omega_{\mathrm{c}})^2+\nu^2}\right\} \\ &\approx \frac{\nu}{2}\frac{eE_1}{m}\frac{1}{(\omega-\omega_{\mathrm{c}})^2+\nu^2}\,, \end{aligned} \tag{14}$$

where $\nu \ll \omega$. Since the energy δ_{E}, which an electron gains per unit time from RF, is given by

$$\delta_{\mathrm{E}} = e\langle \mathrm{Re}[v_y]\cdot\mathrm{Re}[E_{\mathrm{RF}}]\rangle = e\mathrm{Re}[v_{y0}]\cdot E_1, \tag{15}$$

substitution of $\mathrm{Re}[v_{y0}]$ from Eq. (14) into Eq. (15) gives

$$\delta_{\mathrm{E}} = \frac{\nu}{2}\frac{e^2E_1^2}{m}\frac{1}{(\omega-\omega_{\mathrm{c}})^2+\nu^2}. \tag{16}$$

This gives a new condition for minimum E_0 in Eq. (9) by substitution of δ_{E} into Eq. (10):

$$E_{\mathrm{eff}} = E_1\sqrt{\frac{2}{(\omega-\omega_{\mathrm{c}})^2/\nu^2+1}}, \tag{17}$$

assuming the case that the equality of Eq. (10) is satisfied. When the RF frequency ω is equal to the electron cyclotron frequency ω_{c}, the RF electric field E_1 becomes a minimum value.

A simple model of RF-assisted breakdown based on Townsend avalanche theory is proposed. The required minimum electric field for RF-assisted breakdown can be decreased down to half of the electric field required for unassisted breakdown when the condition of Eq. (17) is satisfied. In this condition, the electric field of RF becomes a minimum value when the RF frequency is equal to the electron cyclotron frequency.

For the estimation of δ_{E} in this model, an electron is simply assumed to gain energy by RF and lose it by collision with neutral gas. However, for a more detailed model, other effects should be considered such as the energy dependence of electron collision cross-section. This will constitute our future work.

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