Deformation of Weakly Unstable Density Distribution of Non-Neutral Plasma Stimulated by Resonant Clumps

Daiju FUJITA[1,a)], Yasuhito KIWAMOTO[1,2)], Yukihiro SOGA[2)] and Nobuya HASHIZUME[2)]

[1)] *Faculty of Integrated Human Studies, Kyoto University, Kyoto 606–8501, Japan*

[2)] *Graduate School of Human and Environmental Studies, Kyoto University, Kyoto 606–8501, Japan*

(Received 27 March 2006 / Accepted 20 July 2006)

Accelerated deformations are observed in the density distribution of a pure electron plasma that is weakly diocotron-unstable when ℓ clumps are introduced along a circle where the rotation frequency advected by the target plasma is close to the phase-rotation frequency of an unstable wave with the mode number of ℓ. When the advection frequency is not close to the phase frequency, the clumps are modified in their orbit by the most unstable wave of the target plasma and bunched azimuthally into the distribution modulated by the wave.

Keywords: diocotron instability, non-neutral plasma, resonant clumps, external perturbation

DOI: 10.1585/pfr.1.044

We report an experimental study of a weakly unstable system's response to external perturbations imposed as an initial condition. The target system is the annular distribution of electrons which is susceptible to diocotron instability [1, 2]. A pure electron plasma with a typical density of $n \sim 10^{12}\,\mathrm{m}^{-3}$, a total number of $N \sim 1.6\times10^{8}$, and a length of 220 mm is held in a Malmberg trap with various distribution shapes [3]. The growth rate of the diocotron mode is reduced by decreasing the degree of the distribution's concave form [4]. The panels of Fig. 1 (a) show spontaneous deformation with the azimuthal mode of $\ell = 2$. The observed growth rate is evaluated as $\gamma_{2\mathrm{obs}} = 2.2 \times 10^4\,\mathrm{s}^{-1}$. This density distribution is taken as the target system in the following examination.

We introduce a set of string-shaped distributions of high-density electrons (clumps) as perturbations to the target system. Each clump is characterized by the peak density $n_\mathrm{c} \sim 3 \times 10^{13}\,\mathrm{m}^{-3}$ and the total number of electrons $N_\mathrm{c} \sim 1 \times 10^6$. The first examination is focused on the response of the $\ell = 2$ unstable target to a two-clump perturbation. The experimental results are shown in Figs. 1 (b) and (c). The clumps are observed to accelerate the $\ell = 2$ deformation. The acceleration is greater when the clumps are placed outside the ring-shaped distribution (exterior clumps) than when the clumps exist inside it (interior clumps). The interior clumps are advected and bunched into a single blob indicating a relatively weak influence on the target.

These observations are summarized in Table 1 in terms of the growth rate as obtained by time-resolved mode analysis of the image data. The bottom row gives the predictions of the linear theory that are determined as the eigen values of the potential perturbation $\delta\phi_\ell(r)\exp(i\ell\theta - i\omega_\ell t)$ which is required to satisfy the Poisson equation

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\delta\phi_\ell(r) - \frac{\ell^2}{r^2}\delta\phi_\ell(r) + \frac{\ell\,\omega_\mathrm{p}^2}{\omega_\mathrm{c} r\, n_0}\frac{\partial n_0}{\partial r}\frac{\delta\phi_\ell(r)}{\omega_\ell - \ell\omega_\mathrm{r}(r)} = 0, \quad (1)$$

with a boundary condition of $\delta\phi_\ell(r) = 0$ both on the axis

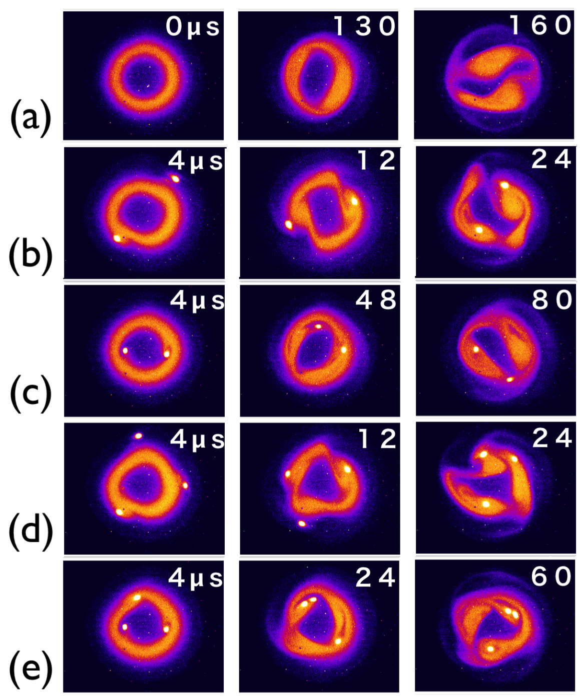

Fig. 1 Photographs showing the deformation of the hollow density distribution; (a) spontaneous deformation, (b, c, d, e) deformations stimulated by clumps.

author's e-mail: dfujita@bio.phys.nagoya-u.ac.jp

[a)] Present address: Division of Material Science, Graduate School of Science, Nagoya University, Nagoya 464-8602 Japan

	Clumps	ℓ= 2 mode	ℓ= 3 mode
Observation	none	2.2 ×10^4	1.3 ×10^4
	2-exterior	7.1×10^4	-
	2-interior	2.5 ×10^4	-
	3-exterior	-	10.2 ×10^4
	3-interior	2.7 ×10^4	-
Linear theory	none	2.2×10^4	1.4×10^4

Table 1 Growth rate of the perturbation (1/s).

and at the conducting wall placed at r = 32 mm. Here, ω_p is the plasma frequency associated with the local density $n_0(r)$, ω_c is the gyro-frequency under the uniform magnetic field at B = 0.048T, and $\omega_r(r)$ is the rotation frequency of the plasma including both the self-field and the bounce-averaged radial field of the confining potential. Both $n_0(r)$ and $\omega_r(r)$ are plotted in Fig. 2 (a) with a dotted brown curve and a green solid curve, respectively. The amplitude of the complex eigen function $\delta\phi_\ell(r)$ is plotted in Fig. 2 (b) for the modes $\ell = 2, 3$.

The growth rate of the $\ell = 3$ mode is evaluated experimentally by means of a mode-analysis in the very initial stage of the evolution to confirm the linear theory, though this mode is readily masked by the ever-growing $\ell = 2$ mode as seen in Fig. 1 (a). However, when three clumps are introduced, the dominant mode in the deformation turns to be $\ell = 3$ as shown in Fig. 1 (d). In the condition when the clumps exist outside the ring-shaped distribution, the observed growth rate exceeds the theoretical prediction by a factor of 7. In the case of three interior clumps, as shown in Fig. 1 (e), slight $\ell = 3$ deformations observed in the early stage are quickly masked by $\ell = 2$ deformations, and the clumps are bunched into two blobs.

Now we briefly discuss the observed results on the basis of linear theory. The density perturbation $\delta n_\ell(r)$ with mode ℓ is connected to the potential perturbation $\delta\phi_\ell(r)$ by

$$\delta n_\ell(r) = -\frac{1}{rB}\frac{\delta\phi_\ell(r)}{\omega_\ell/\ell - \omega_r(r)}\frac{\partial n_0}{\partial r}. \tag{2}$$

Here, ω_ℓ is the complex eigen value of Eq. (1). The phase-rotation frequencies $\omega_r(r)/\ell$ are plotted in Fig. 2(a) by the dot-dashed lines in red for $\ell = 2$ and in blue for $\ell = 3$. Substantial enhancement of $\delta n_\ell(r)$ is expected at the resonant radius that satisfies $\omega_r(r) = \omega_\ell/\ell$. Though no contributions of the clumps are included in Eq. (2), it can be shown that the externally imposed array of m clumps creates a periodic charge distribution with azimuthal mode m to appear at the right hand side of Eq. (1) with a denominator of $\omega_\ell - \ell\omega_r(r)$. Therefore, $\delta\phi_\ell(r)$ can be enhanced resonantly provided $\ell = m$ and the clumps are placed near the resonant radius. The radial locations of the interior and exterior clumps are indicated by vertical lines in Figs. 2 (a) and (b). Comparisons between the contributions of the interior and exterior clumps also suggest the importance of the radial matching. This should be appreciated by observing in Fig. 2 (b) that $|\delta\phi_\ell(r)|$ is larger at the location of the interior clumps while a stronger contribution of the exterior clumps is apparent in Figs. 1 (b) and (d). The stronger influence of the exterior clumps may be attributed to the observation of Fig. 2 (a) that the angular frequency $\omega_r(r)$ of the advection of the exterior clumps is closer to the rotation frequency ω_ℓ/ℓ of the mode ℓ wave than that of the interior clumps.

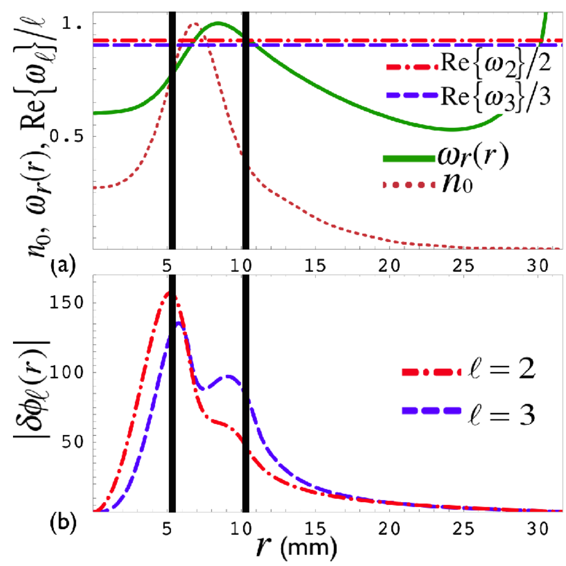

Fig. 2 Radial profiles of (a) the initial density distribution $n_0(r)$, the rotation frequency $\omega_r(r)$, the angular frequencies $\omega_r(r)/\ell$ of the wave modes and (b) the absolute values of the complex eigen functions. Vertical lines represent the radial locations of the clumps with thickness corresponding to the clumps' 1/e width.

In summary we have observed the stimulated growth of weakly unstable modes of density perturbation resonantly triggered and enhanced by externally imposed azimuthal arrays of clumps.

This work was supported by a Grant-in-Aid for Scientific Research from the JSPS and partly by the collaborative program of NIFS.

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