1. Introduction

The full potential of the reversed-field pinch (RFP) confinement concept can probably be realized through the external control of the profile to stabilize the high-beta plasma since RFP confinement has the possibility of providing an economical reactor with a low external magnetic field due to the strong paramagnetism and the attainment of the ignition condition without further heating. In our previous studies, the benefit of radio-frequency wave current drive to widen the force-free magnetic field region was theoretically demonstrated through the significant reduction of the nonlinearly turbulent level associated with the dynamo activity in RFPs, which suggested a substantial improvement of energy confinement time [1-4]. Recent experimental results show that current profile control enhances the energy confinement time to a tokamak-like scaling. Magnetic fluctuations normally present in the RFP are reduced via parallel current drive in the outer region of the plasma. The confinement time increases tenfold (to ~10 ms), which is comparable L- and H-mode scaling values for a tokamak with the same plasma current, density, heating power, size and shape [5]. The experiment performed by the application of an oscillating poloidal electric field to the plasma edge demonstrate that, in principle, a stationary current profile and magnetic fluctuation control technique is feasible in the RFP [6]. The combination of increased shear stabilization through the reduction of magnetic islands and stabilization of tearing modes can realize improved confinement in high-$\Theta$ discharges [7]. The next problem to be resolved is maintaining the dynamo-free, stable RFP configuration and realizing its steady state with less wave power.

For steady-state reactor operation, the circulating power must be minimized if the engineering power gain $Q_E$ is to be maximized. This goal is achieved by reducing the noninductive seed current due to the bootstrap current effect. For that purpose, we are concerned with low-aspect-ratio equilibrium. A low-aspect-ratio approach enhances the neoclassical viscosity or the bootstrap current in both the banana regime and the Pfirsch-Schluter regime [8], and also has less dynamo action in the RFP with a quasi-single-helicity (QSH) state [9].

In this work, the steady-state neoclassical magnetohydrodynamic (MHD) equilibrium with a low aspect ratio, which is solved self-consistently considering the self-induced plasma current, is investigated by broadening the plasma pressure/temperature profile. The equilibrium obtained has a large magnetic shear due to its hollow current profile, which gives a high-stability beta, and simultaneously, a dominant

According to a simplified costing algorithm for a steady-state fusion reactor power plant, the relative attractiveness of advanced physics modes mainly depends on the stability and noninductive current drive of the equilibrium. The high-stability beta and the good alignment of the equilibrium current profile with a self-induced plasma current profile are compatible with the low-aspect-ratio neoclassical Reversed-Field-Pinch equilibrium solved self-consistently considering the self-induced plasma current. The high-stability beta is due to the hollow current profile making the magnetic shear increase and the force-free field dominant. The good alignment of the current profile significantly reduces the power required for noninductive current drive to generate the steady-state magnetic field configuration. As a result, the fusion power plant based on the neoclassical RFP equilibrium with low aspect ratio enables electricity to be generated at relatively low cost.

Keyword: low-aspect ratio, neoclassical RFP, relaxed-state equilibrium, stability beta limit, steady-state reactor
plasma self-induced current, which allows for an easier approach to steady-state confinement compared with the “TITAN” RFP reactor with a high aspect ratio [10]. We also show that the low-aspect-ratio RFP configuration is close to the relaxed-equilibrium state with minimum energy, which is predicted by the extremum of the Lyapunov functional [11], and is also robust against microinstabilities. These attractive features allow the economical design of compact steady-state fusion power plants with low cost of electricity (COE).

2. Plasma Self-Induced Current

The flux surface average total toroidal current \( I_p \) in the MHD equilibrium is given by

\[
I_p = -\left(1/2\pi\right) \int (d\psi/d\nu)((B^2_p)/(B^2)) \, dV + \left(1/2\pi\right) \int G((j \cdot B)/(B^2)) (1/R^2) \, dV, \tag{1}
\]

where \( V \) is the volume of the region enclosed by a flux surface, \( G = RB_pB \) is the toroidal magnetic field, \( G \) and pressure \( p \) functions are poloidal flux \( \psi \), \( X \) is the flux surface average of \( X \), \( X = \int (X/B_p) dV/dL(B_p) \), \( B_p \) is the poloidal magnetic field, and \( B = (B^2_p + B^2_e)^{1/2} \). The first term represents the contribution of perpendicular current \( I_{\text{PRP}} \), namely, the toroidal component of the Pfirsch-Schluter (PS) current \( I_{\text{BS}P} \) and the diamagnetic current \( I_{\text{BS}D} \), which plays a substantial role in RFP plasma where \( B_p \geq B_e \). The second term is from the parallel current, namely, the bootstrap current of the bulk plasma \( I_{\text{BS}B} \) and the ohmic current \( I_{\text{BS}O} \). Note that \( I_{\text{BS}O} \) depends on the plasma pressure and temperature profiles. We also consider the alpha-particle-induced bootstrap current \( I_{\text{BS}A} \) due to fusion-produced alpha particles. We define here the self-induced plasma current as \( I_{\text{SI}} = I_{\text{BS}B} + I_{\text{BS}O} \). The bootstrap current \( I_{\text{BS}B} \) increases as the aspect ratio \( A \) decreases because \( B_0 \) increases due to the increasing anisotropy in the electron toroidal field or viscosity with decreasing \( A \). The viscosity is a sensitive function of the magnetic structure and is enhanced by the divergent tendency of the geometric factor \( (n \cdot \nabla B)^2 \) as A approaches unity, where \( n = B/B \) and \( B \) is the equilibrium magnetic field [12]. Accordingly, \( I_{\text{BS}A} \) is high in low-aspect-ratio RFPs. The toroidal current density is hereafter required by \( j_b \) in place of \( I_p \).

The bulk parallel bootstrap current density is expressed as

\[
\begin{align*}
\langle j \rangle_{bB} & = (B^2/(B^2)) \left[ n_e T_e/(T_e + T_i) \right] \langle j \rangle_{bB0} \\
\langle j \rangle_{bO} & = -L_1 \left[ (p_e/(p_e + T_e + T_i)) \langle j \rangle_{bB0} - \langle j \rangle_{bO} \right].
\end{align*}
\tag{2}
\]

where \( n_e, T_e, \) and \( p_e = n_e T_e \) are the number density, temperature, and pressure, respectively, \( Z \) is the effective ionic charge, \( L_1(k = 1, 2, 3) \) is the transport coefficient and the prime denotes the differentiation with respect to the local minor radius \( r \). We here use the transport coefficient \( L_0 \) given by the Hirshman model [13], which has been developed for an axisymmetric configuration with arbitrary flux surface, aspect ratio and \( q \) value. It is derived for a plasma composed of electrons and a single ion species in the collisionless \((v_e \to 0)\) limit.

We write the alpha particle-induced bootstrap current density in the form

\[
j_{bA} = (B^2/(B^2)) [m_\alpha v_{\alpha}^2 \tau_d (dn/d\tau)/2B_0] [1 - (Z/\alpha)F] j_{b0}, \tag{3}
\]

with \( j_{b0} = -L_0 \langle \partial \log (\nu, \partial / \partial r) \rangle + L_0 \langle \partial \log (v^2_e) / \partial r \rangle \) where the subscript \( \alpha \) denotes the values for the \( \alpha \) particle, \( m_\alpha, \) the mass, \( v_\alpha, \) the initial velocity \((3.5 \text{ MeV})\), \( dn / dr \) the fusion production rate, and \( \tau_d, \) the slowing-down time. The critical velocity is defined as \( v_c = (3\pi^{1/2}/4) \Sigma(m_e n_e c^2/m_\alpha c^2) v_\alpha \) with \( \Sigma \) denoting a summation over only ion species and \( \nu \) the electron thermal velocity: \( 1 - (Z/\alpha)F \) represents the shielding of alpha current by the electron return current with \( F \) being the shielding factor [14]; and the thermal forces \( \langle \partial \log (\nu, \partial / \partial r) \rangle \) and \( \langle \partial \log (v^2_e) / \partial r \rangle \) can be calculated using the analytical fit for the deuterium-tritium (D-T) reaction rate [15]. The transport coefficients \( L_0 \) and \( L_\alpha \) are calculated on the basis of the conventional neo-classical transport theory [16,17].

The Ohmic current can be written as

\[
j_{oh} = (e^2 n_e \tau_{\alpha} / m_\alpha) \langle j \rangle_{bO} (BE_0)/B/(B^2), \tag{4}
\]

where \( \tau_{\alpha} = 3\pi^{1/2}/4v_{\text{cr}}, \) with \( v_{\text{cr}} \) being the el-el collision frequency. \( E_0 \) is given in refs. 16 and 18, and \( E_{\text{ef}} \) is the parallel electric field.

3. Target Equilibrium and Stability

The dependence of bulk bootstrap current on aspect ratio \( (A) \), cross sectional ellipticity \( (\kappa) \) and plasma pressure/temperature profiles is investigated for the classical (PRSM) MHD equilibrium model for simplicity. Here, we specify the current flux function \( I(\psi) = 2\pi G(\psi)/\mu_0 \) under the condition of PRSM: \( j_b = \lambda B_0/\mu_0 \) or \( j_\nu d\psi/\mu_0 = \lambda \) [19] with \( \lambda = \text{const.} \), which is reasonably close to the relaxed-equilibrium state [2,3,20]. The results indicate that the ratio of bootstrap current fraction to equilibrium plasma current \( (F_{\alpha} = I_{\text{BS}A}/I_{\text{BS}O}) \) increases with decreasing \( A \), with increasing \( \kappa \) and with flattening of the pressure profile. In addition, the alignment factor \( A_0, \) defined as the normalized deviation of parallel bulk bootstrap current density \( \langle j \rangle_{bB0} \) from the equilibrium plasma current density \( \langle j \rangle_{B0} \), \( A_0 = \int d\psi (\langle j \rangle_{bB0} - \langle j \rangle_{B0})^{1/2} \langle j \rangle_{B0}^{1/2} \rangle, \) is larger for a flatter profile (or a higher \( \beta \)) and for a larger \( \kappa \) elongation, while it is nearly independent of \( A \). The flat pressure profile results in the high-stability \( \beta \) because of the large magnetic shear [21]. The larger \( A_0 \) means that more power is required for the noninductive RFCD to generate the steady-state configuration at the given pressure/temperature profiles. Note, however, that the high-stability \( \beta \) and the small \( A_0 \) are incompatible in the classical (PRSM) RFP equilibrium, and the ratio \( F_{\alpha} \) by itself is not an adequate figure of merit when the equilibrium current profile is poorly aligned with the
bootstrap current profile.

### 3.1 Target (neoclassical) equilibrium

To remove this incompatibility, the neoclassical RFP equilibrium, which is solved consistently considering self-induced plasma current, is employed here since the equilibrium current profile aligns well with the bootstrap current profile, resulting in a feasible generation of the steady-state configuration with a smaller amount of power for RFCD.

The parallel currents, \( I_{\phi}^{\text{BSb}}, I_{\phi}^{\text{BSa}} \) and \( I_{\phi}^{\text{OH}} \) as well as the perpendicular current \( I_{\phi}^{\text{PRP}} \) in RFP are self-consistently solved with the given initial functions \( p(\psi) \) and \( G(\psi) \) in order to simultaneously satisfy the Grad-Shafranov equation and the general MHD equation, using the formulae for the constituents of \( I_{\phi}^{\text{SI}} \) as described above, and keeping the total toroidal current \( I_{\phi}^{\text{EQ}} \) constant where \( G(\psi) \) is solved iteratively until the summation of \( G(dG/d\psi) \) at all the flux surfaces converges with an accuracy on the order of \( 10^{-3} \) (Fig. 1).

Then, we obtain the neoclassical RFP equilibrium. First, we consider \( I_{\phi}^{\text{BSb}}, I_{\phi}^{\text{OH}} \) and \( I_{\phi}^{\text{PRP}} \). The result shows that \( I_{\phi}^{\text{BSb}} \) has a hollow current profile with relatively broad plasma pressure and temperature profiles, namely, \( p(\psi) = p_0(1 - \psi_0^2) \) and \( T(\psi) = T_0(1 - \psi_0^2) \), \( \psi_0 \) is the normalized poloidal flux function; \( \psi_0 = (\psi - \psi_0) / (\psi_{\text{lim}} - \psi_0) \), \( \psi_{\text{lim}} = 0, \alpha_p = 1.0, \beta_p = 3.0, \beta_t = 3.0 \), and the plasma parameters are listed in Table 1, where \( W_n(=P_n/A_w) \) is the neutron wall load for the total area of \( A_w = 305 \text{ m}^2 \). The ratio of peak electron and ion temperatures is set at a value typical for power balance calculations for commercial plants, \( T_e / T_i = 1.07 \) [22].

Next, the steady-state neoclassical RFP equilibrium

Table 1: Design points for economic analysis of a low-A RFP.

<table>
<thead>
<tr>
<th>( A )</th>
<th>( R_0 )</th>
<th>( \kappa )</th>
<th>( \beta )</th>
<th>( q_0 )</th>
<th>( B_0 )</th>
<th>( I_{\phi}^{\text{SO}} )</th>
<th>( \langle n_e \rangle_{20} )</th>
<th>( \langle T_e \rangle )</th>
<th>( \langle T_i \rangle )</th>
<th>( P_{\text{CD}} )</th>
<th>( P_{\text{E}} )</th>
<th>( W_n )</th>
<th>( M_{\text{FPC}} )</th>
<th>( M_{\text{PD}} )</th>
<th>( Q_8 )</th>
<th>( \text{COE} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>%</td>
<td>T</td>
<td>MA</td>
<td>m⁻²</td>
<td>keV</td>
<td>MW</td>
<td>MW</td>
<td>MW/m²</td>
<td>kton</td>
<td>kW/kWe</td>
<td>kW/kWe·h</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2.8</td>
<td>1.4</td>
<td>63</td>
<td>-1.0</td>
<td>3.0</td>
<td>30.7</td>
<td>1.88</td>
<td>29.6</td>
<td>27.7</td>
<td>16.0</td>
<td>778</td>
<td>4.44</td>
<td>2.51</td>
<td>311</td>
<td>13.4</td>
<td>-50.0</td>
</tr>
</tbody>
</table>

Table 2: Compositions of toroidal current in steady-state neoclassical RFP equilibrium with \( A = 2.0 \) and \( \kappa/\delta = 1.4/0.4 \).

<table>
<thead>
<tr>
<th>( I_{\phi}^{\text{SO}}[\text{MA}] )</th>
<th>( I_{\phi}^{\text{BSb}}[\text{MA}] )</th>
<th>( I_{\phi}^{\text{BSa}}[\text{MA}] )</th>
<th>( I_{\phi}^{\text{PRP}}[\text{MA}] )</th>
<th>( I_{\phi}^{\text{RF}}[\text{MA}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.72</td>
<td>17.80</td>
<td>4.82</td>
<td>6.39</td>
<td>1.71</td>
</tr>
<tr>
<td>100%</td>
<td>57.94%</td>
<td>15.68%</td>
<td>20.8%</td>
<td>5.58%</td>
</tr>
</tbody>
</table>

Fig. 1 Flow diagram of the equilibrium code with specified current sources.
without \( I_{P}^{OH} \) is calculated by replacing \( I_{P}^{OH} \) with \( I_{P}^{BS} \) and the noninductive RF-driven current \( (I_{P}^{RF}) \). The good alignment of the current profile with the profile of \( I_{P}^{PRP} \) including \( I_{P}^{PRP} \) and \( I_{P}^{BS} \) significantly lowers the requirements for the noninductive seed current [Table 2]. Figure 2(a) shows the magnetic flux surface of the neoclassical equilibrium studied here, to the plasma boundary, and Fig. 2(b) shows the \( q \)-profile. As shown in Fig. 2(c), the toroidal current density \( j_{\psi}^{EQ} \) has a hollow profile due mainly to \( j_{BS}^{Bb} \), where \( j \) is substituted with \( j \) for current density. \( I_{P}^{BS} \) and \( I_{P}^{BS} \) are \( 57.94% \) and \( 15.68% \) of \( I_{P}^{EQ} (= 30.72 \text{ MA}) \), respectively, for the D-T-fueled plasma with \( A = 2, R_{0} = 2.8, \kappa = 1.4, \) triangularity \( \delta = 0.4, Z = 2.0, \) and central ion and electron temperatures \( T_{e0} = 35 \text{ keV} \) and \( T_{i0} = 37 \text{ keV} \), respectively, considering the plasma power balance [22]. \( I_{P}^{RF} \) is required to compensate \( I_{P}^{OH} \) in conjunction with \( I_{P}^{BS} \), making \( q_{0} \) smaller than unity at the magnetic axis where the bootstrap current goes to zero, and therefore, RF current drive is necessary there. Using a low-frequency fast wave (LFFW; \( \omega \sim 2\Omega_{c} \); \( \Omega_{q} \) is deuterium cyclotron frequency) that provides the seed current in the core region with high density and temperature, the required RF current is \( I_{P}^{RF} = 1.71 \text{ MA} \) (5.58% of \( I_{P}^{EQ} \)) within the region of the normalized flux function \( \psi_{00} = 0 \) at the magnetic axis) [22,23]. Note that \( j_{BS}^{RF} \) is driven in the central region [Fig. 2(c)]. The rest of the current \( I_{P}^{PRP} \) is 20.8% of \( I_{P}^{EQ} \).

### 3.2 MHD stability

The stability beta limit in the low-aspect-ratio RFP configuration is determined by Mercier’s localized instability because the safety factor increases and then the local magnetic shear decreases, with decreasing aspect ratio at a given plasma pressure profile and cross-sectional shape (ellipticity \( \kappa \) and triangularity \( \delta \)). For comparison with the stability of the neoclassical RFP equilibrium, the stability analysis is carried out for the classical RFP equilibrium although it has a smaller magnetic shear. The plasma pressure profiles are specified as follows: \( (a_{p}, b_{p}) = (2.0, 1.0), (1.4, 1.0) \) and polynomial form \( p(\psi) = p_{0}(1 - 0.469\psi_{00} - 3.65\psi_{0}^{n+1} + 3.15\psi_{0}^{n+2}) \) (\( n = 4, \psi_{00} = 0 \) at the magnetic axis); the corresponding \( (\beta) \) are 22.5%, 33.4% and 65.6%, respectively. The stability beta limit increases with flattening the plasma pressure profile and increasing ellipticity \( \kappa \). An optimum \( \beta \) value, such that the Mercier criterion is satisfied over the whole plasma region at the highest beta, is found to be \( \beta_{0} = 50.6% \) \( (\langle \beta \rangle = 65.6%) \) at \( A = 2.0 \) and \( \kappa / \delta = 2.0 / 0.4 \) in the case of a relatively flat pressure profile. For the neoclassical RFP equilibrium with a flat plasma pressure profile \( (a_{p} = 1.0, b_{p} = 3.0) \) similar to the polynomial form above, Mercier mode stable configuration is attained with a large bootstrap current ratio at \( \beta_{0} = 63% \) \( (\langle \beta \rangle = 66.0%) \) at \( A = 2.0 \) and \( \kappa / \delta = 1.4 / 0.4 \) although these parameters are not yet optimized. Next, the stability of the ideal kink mode instability is examined by adopting Robinson’s sufficient stability condition in a cylindrical geometry approximation with circular cross-sectional shape at a free boundary [24], \( K = \pi \rho_{0} (B_{0}) - (B_{0}) / (2r) \) \( (P(r) / P(a) + 1)^{1/2} < 0 \) for \( 0 < r < b \), where \( P(r) \) is the pitch length. This stability condition gives two stability conditions for localized modes, i.e. \( |P(0)| / P(b) > 1 \) for small \( r \) (internal mode) and the axial current reversal \( j_{z} \leq 0 \) for large \( r \) (surface mode). If there is a conducting shell at \( r = b \), the necessary stability condition is given by the inequality \( P(b) < 3 \) \( P(0) \), which means that \( B_{b} \) cannot be much smaller than \( B_{a} \) at the shell. The conducting shell is not considered in the present study, and we define the plasma boundary so that it satisfies the given geometrical constraints. Therefore, the assumption of the conducting shell does not affect the neoclassical RFP equilibrium shown in

![Fig. 2 Steady-state neoclassical RFP equilibrium obtained by replacing Ohmic current \( I_{P}^{OH} \) with alpha-particle-induced bootstrap current \( I_{P}^{BS} \) and RF-driven current \( I_{P}^{RF} \) for \( q_{0} \leq 1 \). Magnetic flux surface (a), safety factor (b) and toroidal current density in midplane (c).](image-url)
Fig. 3 Ideal kink mode stability of the low-aspect-ratio (A = 2), classical equilibrium with k/δ = 1.4/0.4 for three plasma pressure profiles, using the stability criterion in cylindrical geometry approximation.

In conclusion, the relaxed-equilibrium equation leads to a hollow profile of the toroidal current density that is higher by a factor of 3β0 at the plasma boundary than at the axis in the case of a broad plasma pressure profile, and thus the neoclassical RFP equilibrium satisfies the relaxed-equilibrium equation. Considering the force-free current due to the increasing neoclassical effects with the decrease of the aspect ratio in the relaxed-equilibrium equation, the current profile becomes more hollow in the relaxed-equilibrium state, as shown in Fig. 2 (c), indicating that the neoclassical RFP equilibrium is close to the relaxed-equilibrium state with a minimum energy.

The high current density at the boundary may not be convenient for fusion devices, but its existence and the presence of $I_0^{B_{SP}}$ current flowing near the boundary would help in external control to sustain the $I_0^{PPR}$ current if necessary, because the plasma current reversal is not only essentially

3.3 Relaxed-state features of neoclassical equilibrium

The features of the minimum energy state for the low-aspect-ratio neoclassical equilibrium with a relatively broad plasma pressure profile might be explained by a minimum Lyapunov functional $L$, which is proposed as a condition for a minimum of energy and pressure and offers the possibility of finding finite solutions with a significant maximum pressure at the center of the plasma [11,25]. Their approach again uses the energy relaxation principle at a given constraint. This constraint is obtained in the frame of the conventional set of MHD equations:

$$M = \int h f(p^{1/\gamma}/h) dV = \text{const.} \quad (5)$$

Here, $h$ is the local magnetic helicity $A \cdot B$ where $A$ is the vector potential of the magnetic field $B$, $f(x)$ is an arbitrary function of its argument and $\gamma$ is the adiabatic index. Under this more restrictive condition, the plasma can relax to a state in which $p$ has a significant maximum at the center of the plasma. In other words, a turbulent relaxation realizes the minimum Lyapunov functional $L = E + M$, where $E = \int \left(p/2 + p/(\gamma - 1) + B^2/2\mu_0\right) dV$, $p$ is the mass density, and $v$ is the plasma velocity. When $p \to 0$, the function $f(0) = \text{const.}$ and $M$ are proportional to the total magnetic helicity $K(= \int h dV)$. However, when $p$ is not zero and $f \neq \text{const.}$, the variational principle of the extremum $\delta L = 0$ leads to the relaxed-equilibrium equation:

$$\nabla \times B = -2HB - \nabla H \times A, \quad H = H(h). \quad (6)$$

Here, $H(h)$ is a function of $h$ related to the $f$-function. It gives the steady-state pressure as a function of the helicity:

$$p(h) = \int h(\partial H/\partial h) dh/\mu_0 + \text{const.} \quad (7)$$

The system of eqs (6) and (7) singles out a very limited subset of the set of plasma equilibria. In the case of $H = \text{const.}$, the current is parallel to $B$ according to eq. (6), and there is a force-free state. The corresponding equation has been solved analytically by Taylor [26]. We find other solutions of eq. (6) numerically in a cylindrical geometry with the coordinate system $(r, \theta, z)$.

In conclusion, the relaxed-equilibrium equation leads to a hollow profile of the toroidal current density that is higher by a factor of $3\beta_0$ at the plasma boundary than at the axis in the case of a broad plasma pressure profile, and thus the neoclassical RFP equilibrium satisfies the relaxed-equilibrium equation. Considering the force-free current due to the increasing neoclassical effects with the decrease of the aspect ratio in the relaxed-equilibrium equation, the current profile becomes more hollow in the relaxed-equilibrium state, as shown in Fig. 2 (c), indicating that the neoclassical RFP equilibrium is close to the relaxed-equilibrium state with a minimum energy.

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affected by the increasing resistivity near the boundary but also the presence of $I_{th}^{th}$ current plays a role of preventing the resistivity from increasing near the boundary as well as the presence of magnetic diverters [27-29]. This role is enhanced by the RF-driven current near the boundary, as described below. We have proposed to drive the force-free current near the boundary to strengthen the magnetic shear and to achieve the weak dependence of stability on the conducting wall [30]. For this purpose, a lower hybrid wave (LHW; $\omega \geq 2\omega_{th}$, $\omega_{th}$ is a lower hybrid frequency), which is useful for the current drive in low density region with divertor operation, can be injected with a power of ~11 MW in the classical equilibrium model with a cylindrical geometry: Suydam parameter $S_0 = 0.8$ ($\langle b_0 \rangle = 16.1\%$), $F/\Theta = -0.2/1.7$, $B_0 = 1.5$ T, $R_0/a = 3.02$ m/1.4 m, $n_{e0} = 1.58 \times 10^{20}/m^3$, $T_{e0} = 28.5$ keV, $T_{i0} = T_{e0}/1.07$ and $Z_{eq} = 2.0$, $\lambda = (2\pi dG/dy) = \lambda_0(1 - (\psi - \psi_0)/(\psi_{lim} - \psi_0))^{m/2}$ where $\psi_{lim} = 0$, $m_0 = 5.0$, $n_0 = 2.0$ and $n_0 = 5.0$, $n_0 = 2.0$. The normalized current drive efficiency is given by $j^2/B^2 = 8u^2/(5+Z) + 12(6+Z)/(5+Z)+3+Z)2D/2u$, where $Z = Z_{eq}$, $D = 3.76$, and $u = \omega/k_T\bar{v}_T$ (relative wave phase velocity $\omega/k_T$). The magnetic shear is enhanced particularly near the boundary (Fig. 4(a)) through the enhancement of the $\lambda$-profile near the boundary (Fig. 4(b)) by RF power injection. The profile of the force-free current $|j-B|^2/B^2$ is shown to be approximately the same as the $\lambda$-profile in the classical RFP equilibrium [20]. The requisite RF power is decreased by flattening the $\lambda$-profile as well as the force-free current profile. Hence, the steady-state neoclassical RFP configuration with a hollow current profile can be maintained with a dominant plasma self-induced current.

### 3.4 Microinstability

Even in MHD stable, dynamo-free RFPs, the energy confinement time might be determined by micro-instabilities. The behavior of the magnetic lines of force in the low-aspect-ratio RFP equilibrium has some favorable stability properties for both micro- and macro-instabilities. The magnetic lines of force do not have the poloidal curvature in a wide region on the inboard side of the low-aspect-ratio torus, which results in a high pitch of the magnetic lines of force. Furthermore, the equilibrium with a modest poloidal beta ($\beta_p < 1$), which is realized in low-aspect-ratio RFPs and tokamaks, is essentially force-free, that is, it has a paramagnetism with the current density nearly parallel to the magnetic field in the plasma core region. Since the magnetic lines of force have a high pitch, a large poloidal current component is produced to enhance the toroidal magnetic field strength or the safety factor at the plasma axis and hence to lead to near-omnigenity [31]. In the omnigenous systems the bounce-averaged current remains within a flux surface. Since the bounce-averaged drift conserves the bounce (or longitudinal) action, this implies that the bounce action is constant on a surface. Thus, we must find systems in which the bounce action, or bounce adiabatic invariant,

\[
J = \int m dB \cdot d\sigma = m(\alpha B_0 + B_0)\int mb \cdot dB, \\
u = [2(E - \mu B - c\Phi)/m]^{1/2},
\]

is constant on a magnetic surface. Here, $\nu$ is determined by energy conservation, $\iota$ is the rotation transform, the loop integral is along a magnetic field line between reflection points, and $E = \mu B$ neglecting electric potential $\Phi$. The immediate consequence of the condition that $J$ is constant on a magnetic surface is that the local minima of the magnetic field along field lines have the same value of $B$, and also that the magnetic maxima and the action of particles at the trapped-passing boundary have the same value on a surface and, hence, transition orbits are absent in omnigenous systems. The maximum longitudinal adiabatic invariant ($J_{\max}$) at the axis because of the longest loop length, or $VpVJ > 0$ is attained at a low aspect ratio with strong paramagnetism, which improves the stability for the ion mode of trapped particle micro-instabilities, the toroidal ion temperature gradient (ITG) mode (with fluid approximation neglecting the density gradient and the parallel wave-number) to be unstable in the outside region of the torus with $VpVJ > 0$, as well as the MHD interchange mode and ballooning mode and also to be considered as a cause of the anomalous ion heat transport in the high temperature core region of a toroidal plasma, and probably the confinement degradation [32] of high-energy particles due to the nonclosure contours of magnetic field strength resulting from a small Shafranov shift in the tokamak with negative magnetic shear. For the trapped-electron-$\iota$ mode, which is
associated with the electron drift wave, the stability condition for the electrostatic toroidal drift mode is \( \eta_i(= \text{dln} T_e/\ln n_i) < 2/3 \) in the collisionless case, which means to be stable for the flatter \( T_e \)-profile for a given pressure profile. For the ITG mode, which is associated with the sound wave, the effects of nonadiabatic response of electron by trapped electrons, more detailed equilibrium configuration, impurity ions, and magnetic fluctuation may make the ITG mode more unstable. The micro-tearing mode with high poloidal mode number, which is associated with the Alfvén wave, is unstable due to two effects resulting in the growth of magnetic perturbation to a saturated island: the kinetic effects of plasma inertia and resistivity and the nonlinear effect of island structure, which depends on the parameter \( \eta_i \) (electron temperature gradient; \text{dln} \( T_e/\ln n_i \)). If the island width associated with a perturbation in the radial component of magnetic field exceeds the width of this so-called “resistive layer”, then the linear theory is inadequate and the effect of the island structure on the plasma density and temperature profiles must be taken into account. These nonlinear effects can lead to extra drives for the island growth and theory suggest that small magnetic island may be present even in the low and high collision-frequency regimes in which the linear theory predicts stability. The toroidicity-induced Alfvén eigenmodes (TAEs), although the modes are MHD modes but sometimes being referred to as the gap mode in the frequency curves \( \text{a}(r) \), can be expected to be reduced for the RFPs with a small safety factor and a strong poloidal magnetic field at the plasma boundary.

Recently a hierarchical model for plasma transport has been proposed, which includes nonlinear interaction between different scale lengths. As a result, nonlocal transport based on the transport-MHD model [33,34] and multi-scale turbulence [35] is produced; the zonal \( \mathbf{E} \times \mathbf{B} \) shear flow and the non-adiabatic electron response have been found to give a new physical mechanism for the non-local transport. The zonal \( \mathbf{E} \times \mathbf{B} \) shear flow, which is observed in the simulation to be generated by Reynolds stress in mode plasma turbulence itself, is one of significant effects on the magnitude of anoma-

4. Characteristics of Steady-State RFP Reactor Concept

We desire to achieve the compact power plants with high “mass power density” (MPD) as well as high “engineering power gain” (QE), which will strongly affect the economic attractiveness of fusion power plants. The MPD is defined as the ratio of net electric power \( P_E \) to the grid divided by the total mass of the fusion power core \( (M_{\text{FPC}}; \text{plasma chamber, blanket shield, reflector, plasma heaters, magnets and primary support structure}) \). The QE is the ratio of plant gross electric power production \( P_{\text{ET}} \) to circulation power \( P_C \) for sustaining plant operation. Although highly simplified and intended primarily to provide a costing gauge for magnetic fusion energy power plants, the cost-gauge model agrees with the predictions of complete systems studies that wrestle with the nonlinearities and connectivities of physics, engineering, and financial design space, indicating the importance of MPD and QE in projecting electric-engineering costs for the given (calibrated) set of top-level unit direct costs (UDC) and cost estimating relations (CERs) used herein [36]. The parametric of MPD on \( Q_E \) in goal values of COE and net electric power \( P_E \) for the otherwise fixed economic parameters is examined. Specification of net electric power \( P_E \) allows the gross electric and thermal powers, \( P_{\text{ET}} \) and \( P_{\text{TH}} \), respectively, to be determined for use in the appropriate CERs. In this way, MPD-QE trade-offs for a range of specified (goal) COE results as shown in Fig. 5. This economic gauge is then used to compare and assess results from detailed conceptual magnetic fusion energy (MFE) reactor design studies. The simplified engineering-economic approach permits a graphic comparison of the economic competitiveness of the MHD equilibria. We aim to minimize the cost of electricity (COE), and this COE depends predominantly on only two variables, \( Q_E \) and MPD for a given \( P_E \). We caution that COE should be interpreted as representing mainly the relative economic potential offered among various equilibria.

4.1 Power plant engineering

In order to maximize MPD, the fusion power density averaged over the plasma volume should be high, which means that the plasma beta should be maximized. For steady-state operation, if \( Q_E \) is to be maximized, the circulation power for the current drive must be minimized; this goal is achieved with the low toroidal current equilibria and by optimizing the bootstrap current contribution, thereby minimizing the non-inductive seed current for the bootstrap current.

High MPD is made possibly by achieving a high fusion power density with a compact, high pressure plasma. Since
the plasma’s fusion power density is roughly proportional to $\beta^2 B_0^2$, we need a large magnetic field as well as a high beta. The higher beta-plasma as in RFPs can be confined with the smaller magnetic field for a given $\beta^2 B_0^2$. The potential feasibility of a spherical torus (ST) with superconducting toroidal field (TF) coil system has been shown. Major specifications of the coil are the center post radius of 0.7 m, the average current density of 40 A/mm² and the winding turns of 10 – 20. Under the consideration of 65 cm thick neutron shield structure (reflector, absorber and slowing down) and 5 cm vacuum (scrape-off), the resultant plasma major/minor radii is 2.8 m/1.4 m ($A = 2.0$). Then it is sacrificed to use the tritium breeder blanket in torus inboard region. The on-axis vacuum magnetic $B_{ov}$ field is lower than the peak magnetic field $B_M$ at the TF coil surface, $B_{ov} = B_M [1 – A^{-1} – (\Delta'/R_0)]$, where $\Delta'$ (= 70 cm) is the inboard distance between the plasma surface and the TF coil. The total outboard distance from the plasma surface, $\Delta$, is composed of vacuum (surface-off) and various blanket and shield components. We set $\Delta' = 1.35$ m, which should be possible [37,38] for the power plants using liquid lithium breeder/coolant and a vanadium alloy structure in the blanket [39,40]. In the present RFP equilibria, fortunately, the on-axis magnetic field in plasma, $B_{ov}$, is enhanced owing to the strong paramagnetism represented by $1 + (I_{ps}/I_{tc})$ and the externally supplied on-axis vacuum magnetic field, $B_{ov}$, is lowered by a factor of (paramagnetism)$^{-1}$ than $B_M$, then the peak magnetic filed at the center post radius of TF coil $B_{ov}$ becomes ~ $4 B_{M}$, value of which is much smaller by an order of magnitude than $B_M$, meaning much smaller radius of superconducting TF coil (~ 0.07 m). In this circumstances, it is possible to use the tritium breeder blanket in the torus inboard region even in the low-aspect-ratio torus with a superconducting TF coil system. The design points for typical RFP equilibria in economic analysis are listed in Table 1.

The MPD can be determined approximately because of the simplified level of analysis. However specific plant designs obtained through more detailed system studies [40,41] shows a strong correlation between MPD and $W_n$, the surface-averaged neutron wall load. For the lithium vanadium blanket technology, the ratio $MPD/W_n = 300\, (\text{kWe/tonne})/(\text{MW/m²})$ with little variation from a constant over 1.0 $\leq W_n$ (MW/m²) $\leq 5.0$; the MPD is inferred by applying this ratio to the $W_n$ calculated for power balance in this range of $W_n$ and otherwise evaluated directly using the size scaling of $M_{RFC}$. This blanket operates at a relatively high temperature, allowing a thermal-to-electric power conversion efficiency of $\eta_n = 0.46$. Detailed neutronics calculations of specific designs prompt to specify the blanket neutron energy multiplication factor as $M_n = 1.207$.

Part of the gross electric power $P_{ERT}$ is recirculated for radio-frequency (RF) current drive (CD) and to maintain plant operations. The recirculation power is evaluated according to the RFCD routine [42]. A low-frequency fast magnetosonic wave (LFFW) provides the seed current near the magnetic axis. The frequency of LFFW is chosen to place the deuterium cyclotron harmonic $\omega = 2\Omega_d$ resonance just outside the low field edge of the plasma; for $A = 2.0$, this places the tritium cyclotron harmonic $\omega = 3\Omega_d$ well inboard of the axis, with the result that the LFFW deposits most of its power on electrons in a single pass through the plasma. The aggressive conversion efficiency of the electric-to-RF power, $\eta_{CD}$, has a strong bearing on the COE. For the calculated RFCD equilibria [42], an aggressive development program will deliver LFFW efficiencies of 78%.

### 4.2 Plasma power balance

The power, $P_{CD}$, injected for RFCD depends on volume averaged electron temperature, $\langle T_e \rangle$, for a given density profile. In the present study, the ratio of peak electron and ion temperatures is set at a value typical of power balance calculations for commercial plants, $T_{ei}/T_{ee} = 1.07$ [22]. The impurity ion content for the RFCD calculation is composed of a nominal moderate Z component (fully stripped vanadium, $n_i/n_A = 0.12\%$) plus an alpha particle fraction which increases with temperature, $n_{\alpha}/n_A = 0.00164 [^7\text{Li} (\text{keV})]^{1.74}$, this being a fit to a large number of power balance runs for which effective alpha particle confinement time is ten times the ion energy confinement time, $t_{\alpha}^e = 10t_{\alpha}^F$. Over the range of $6 \leq T_e$ (keV) $\leq 25$, this results in $Z_{eff} = 1.6 – 1.7$ and a ratio of electron to total ion density of $Z_{n} = n_i/n_e = 1.1 – 1.2$. In the calculation of the $P_{CD}$, the fact that part of the pressure is due to nonthermalized alpha particles is taken into account; this fast alpha pressure is set to be a constant fraction of the thermal pressure at the magnetic axis, $p_{\alpha}/p_{ei} = 0.10$, which is typical of the results of detailed slowing down and particle balance calculations for $Z_{eff} \geq 1.6$, over a wide range of central temperature. The fast alpha pressure fraction decreases away from the axis and becomes negligible for plasma temperatures less than about 5 keV. As expected, the required $\eta_{\alpha} t_{\alpha}^F$ for ignition increases with $Z_{eff}$ by a factor of ~ 1.5, increasing the required temperature by a small factor, but the thermal power density is independent of impurity concentration, in a range of $Z_{eff}$ = 1.3 to 2.9. There remain in subtle consequences of increasing $Z_{eff}$, which deteriorates RF current driving efficiency resulting in higher circulation power and lower $Q_{ps}$, thus, higher COE. Using a low-frequency fast wave (LFFW; $\omega \sim 2\Omega_d$, $\omega_d$ is deuterium cyclotron frequency) to provide the seed current in the core region with a high density/temperature, the required RF power ($P_{CD}$) is given by

$$P_{RF}^e [\text{A}] / P_{CD} [\text{W}] = (0.122 \left( T_e \right) [\text{keV}] / R_0 [\text{m}] \left(n_i\right)_{20} [\text{m}^{-3}]) \ln \Lambda \times \frac{f \cdot p^*}{t^*},$$

where $f \cdot p^*$ is the normalized current driving efficiency and a function of parallel wave phase velocity normalized to the electron thermal velocity, which depends only on the current driving system and the aspect ratio of magnetic configuration, and $\ln \Lambda$ the Coulomb logarithm (~15). The RF power spectrum is selected such that RFCD creates a current profile $R(t) = \langle j \cdot B \rangle_{\text{RFF}} / (B^2) - 4\langle j \cdot B \rangle_{\text{RFF}} / (B^2) – D(t)$, where $D(t)$ is the bootstrap current density. The value of $f \cdot p^*$ has been found to be 0.0175 at $Z_{eff} = 2.0$ and $A = 2.0$ from the global RFCD
efficiency in the dominant bootstrap current-airduced reversed-shear (RS) tokamak with a similar R(τ)-profile using the same LFFW current driving system [22]. Then the required RF power for $I_\text{q}^{\text{FS}} = 1.71$ [MA] is evaluated to be $P_{\text{CD}} = 16.0$ [MW] for the D-T-fueled plasma.

During steady-state operation a fusion power plant produces a gross (total) electric power of $P_{\text{ET}} = \eta_{\text{TH}}(M_a + 0.25) P_a + P_{\text{CD}} + \eta_{\text{TH}} P_{\text{pump}} P_{\text{pump}}$ [41]. The first term in brackets are, the neutron power (including the blanket multiplication factor $M_a$), the “alpha particle transport power” liberated as radiation and particle losses to plasma-facing surfaces (about one-quarter of the virgin neutron power), and power, $P_{\text{CD}}$, injected for RFCD, respectively. The final term in this expression is very small ($P_{\text{pump}} = 0.01 P_{\text{ET}}$ and $\eta_{\text{pump}} = 0.9$) and represents partial recovery of the coolant power. Part of the gross electric power is recirculated for RFCD and to maintain plant operations: $P_C = (P_{\text{CD}}/\eta_{\text{CD}}) + P_{\text{pump}} + P_{\text{aux}}$ with $P_{\text{aux}} = 0.04 P_{\text{ET}}$ for plant auxiliaries. The net electric power supplied to the utility grid is thus $P_{\text{ET}} - P_C = P_E$. The following economic analysis uses the ratio $Q_E = P_E/P_C$, the engineering power gain, as a figure of merit [36]. In the present design criteria, the limiting value (for $P_{\text{CD}} = 0$) is $Q_E \rightarrow 20$. The neutron power density depends on the profile constants of plasma pressure ($a_p = a_T + a_b, b_1 = 1.0$), temperature ($a_T$), density ($a_b$) and the volume-averaged electron temperature $T (\approx T_e (\approx T))$. We expect that for a fixed $a_b$ and $T$, the density power density increases with increasing $a_T$ (decreasing $a_b$) for $(1 + a_T) (T) < 13$ keV and decreases with increasing $a_T$ for $(1 + a_T) (T) > 13$ keV, showing the power density to be almost independent of the ratio of $a_T$ to $a_b$ as long as $(1 + a_T) (T) \approx 13$ keV. Due to this very mild dependence of power density and ignition condition ($\eta_{\text{TH}}$), on $a_T$ and $a_b$, at fixed $a_b$, we feel comfortable in selecting a reference value for $a_b$. The significant effect of increasing $a_b$ is the reduction of the $(T)$ at which the maximum thermal power is produced.

### 4.3 Economic Analysis

Our goal is to realize a steady-state compact power plant with a high MPD at a high $Q_E$, namely, to realize a low COE. The calculation of the COE of each design point is achieved by using the costing algorithms described in detail in ref. [36]. This economic model reveals that $Q_E$ and MPD are the primary variables determining COE, but there are secondary factors that exert smaller influences. Since we intend to compare the performance of different steady-state equilibria — a question dealing with plasma physics issues of MHD stability and current drive — it is appropriate to use relatively simple engineering and economic modeling, which reduces the complexity of power plant design to algorithms involving a rather small number of parameters. As an example, the economic model depends on a certain parameter, the unit cost of the fusion power core, $U_{\text{FPC}}$. The core of a fusion power plant would, of course, be built with thousands of components, each made of various materials, with often sophisticated fabrication techniques; a detailed cost accounting of such an elaborate facility is obviously speculative. Instead, the parameter $U_{\text{FPC}}$, taken to be US $100/kg in this model, represents a highly aggregated accounting of the cost of the overall fusion power core. The RFCF system, a pivotal component of steady state RFPs, is priced separately from the fusion power core and is assigned the unit cost, $U_{\text{FPC}} = $2.00/ W, where the dominator is the power, $P_{\text{CD}}$, injected into the plasma [41]. Computation of the COE requires that the sum of all annual charges, $AC = FCR (1 + f_{\text{DC}})$ TDC + OM + FUL + DD, be divided by the net electric energy sold during a given year, $\sim p_{ET} = p (1 - 1/Q_E) P_{ET}$. In this expression, $FCR$ is the fixed charge rate [in constant dollar (1/yr)] on the total direct cost (TDC) (in million dollars) (including individual contingency factor CONT), $f_{\text{DC}}$ is an indirect cost (IDC) factor, OM is an annual fuel charge (expected to be nearly zero for D-T-fueled fusion power plants), and DD (1/yr) is an annual escrow payment made to assure that a fraction $f_{\text{DD}}$ of TDC is available for decontamination and decommission (D&D) operations at the end of the plant life $T_L$. If CRF $(X_a, T_L)$ is the capital recovery factor [43] (1/yr) for a real cost of money $X_a$ (i.e., corrected for inflation), $DD = FCR_{\text{DD}}$ TDC, where $FCR_{\text{DD}} = f_{\text{DD}}$CRF $(X_a, T_L)$ is an effective FCR for the D&D escrow payment.

Defining $f_{\text{OM}}(1/yr) = OM/TDC, f_{\text{FUL}}(1/yr) = FUL/TDC, and the unit direct costs (UDC) [in dollars per watt (electric)] as $UC = TDC/P_E$, the following expressions for COE results:

\[
\text{COE} = (10^9/8760) \times (\text{UDC}/p) \
\times (FCR (1 + f_{DC}) + f_{OM} + f_{FUL} + FCR_{\text{DD}}).
\]

Assuming the cost model input parameters of FCR (1/yr) = 0.0860, $f_{\text{DC}} = 0.96$, $f_{\text{OM}} (1/yr) = 0.0400$, $f_{\text{FUL}} (1/yr) = 0.0$ and $FCR_{\text{DD}} = 0.0017; f_{\text{DD}} = 0.20, X_a (1/yr) = 0.05, T_L (yr) = 40, CRF (X_a, T_L) (1/yr) = 0.0583$, the sum of $AC/TDC (1 + [ ]$ in eq.(8)) = 0.21026.

Maintaining the generic nature of this model is accomplished by including in the parameter $f_{\text{om}}$ the annual charge associated with first-wall and blanket replacement costs (similar to a fuel charge, but usually accounted separately in the detailed, concept-specific models). If the unit cost for the $j$-th subsystem is $U_C$ and the associated contingency factor is $\text{CONT}_j$, the TDC is given by

\[
TDC = \sum_{j=1}^{n} U_C \left[ M_j, P_j \right] (1 + \text{CONT}_j).
\]

where $[M_j, P_j]$ is either a mass- or power-related capacity appropriate for the subsystem in question. Inserting the specific value of $UC$ and defining $UC_j = UC (1 + \text{CONT}_j)$ give the following expression for UDC:

\[
\text{UDC} = \frac{U_{\text{FPC}}^{\text{UC}}}{\text{MPD}} \frac{1}{\eta_{\text{TH}} (1 - 1/Q_E)} \times \left[ \frac{U_{\text{HTG}}^{\text{UC}}}{\eta_{\text{HTG}} Q_e M} + U_{\text{PHD}}^{\text{UC}} + \eta_{\text{TH}} (U_{\text{HOP}}^{\text{UC}} + U_{\text{XTC}}^{\text{UC}}) \right].
\]
where FPC denotes the fusion power core, HTG the plasma heating, PHT the primary heat transport, BOP the balance of plant and SITE the structure and the site facilities. In this expression, \( M = 1/Q_e + 0.8 M_e + 0.2 \), and the plasma \( Q \)-value, \( Q_e \), defined as the ratio of D-T fusion power to plasma heating power, is related to the engineering gain or \( Q \)-value, \( Q_e \), by the following relationship:

\[
1/Q_e = f_{\text{AUX}} + 1/(\eta_{\text{HTG}}Q_e M).
\]  (11)

The following system of equations allows the dependence of MPD on \( Q_e \) to be examined parametrically in goal values of COE and net electric power \( P_e \) for the otherwise fixed economic parameters of

\[
\begin{align*}
U_{\text{FPC}}^* &= UC_{\text{FPC}} \times 1.30, \quad UC_{\text{FPC}} = 100 \text{ [$/kg$]},
U_{\text{PHT}}^* &= UC_{\text{PHT}} \times 1.2, \quad UC_{\text{PHT}} = 0.80/P_{\text{TH}}^{0.45} \text{ [$/$/We]},
U_{\text{BOP}}^* &= UC_{\text{BOP}} \times 1.15, \quad UC_{\text{BOP}} = 0.7705/(\eta_{\text{HTG}}P_{\text{TH}})^{0.16} + 4.2665/(\eta_{\text{HTG}}P_{\text{TH}})^{0.51} + 1.0005/(\eta_{\text{HTG}}P_{\text{TH}})^{0.41} \text{ [$/$/We]},
U_{\text{SIE}}^* &= UC_{\text{SIE}} \times 1.10, \quad UC_{\text{SIE}} = 0.30 \text{ [$/$/We]},
\end{align*}
\]

where \( P_{\text{TH}} \) [MW (thermal)] is fusion thermal power (\( P_{\text{IT}}/\eta_{\text{HTG}} \)). These parameters allow the ratio COE/UDC to be computed from Eq. (8) for subsequent use in parametric evaluations of MPD versus \( Q_e \) for a range of target or goal COE values. Specification of net electric power \( P_e \) allows the gross electric and total thermal powers \( P_{\text{IT}} \) and \( P_{\text{TH}} \), respectively, to be determined for use in the appropriate CERs; for a given \( Q_e \), Eq. (11) allows \( Q_e \) (plasma \( Q \)-value or gain \( M_e/P_{\text{CD}} \)) to be evaluated for use along with a specified COE in Eq. (10) to determined the corresponding MPD value. In this way, the MPD-\( Q_e \) trade-offs for a range of specified (goal) COE result. This economic gauge is then used to compare and assess results from detailed conceptual magnetic fusion energy (MFE) reactor studies.

The essential elements of the costing-gauge model are embodied in Eqs. (8) and (10) along with the parameters described above. This set of expressions is evaluated parametrically as shown in Fig. 5 where \( P_e = 778.5 \) [MWe], \( P_e = 0.75 \), auxiliary power fraction \( f_{\text{AUX}} = P_{\text{AUX}}/P_{\text{IT}} = 0.04 \), \( \eta_{\text{HTG}} = 0.46 \) and \( \eta_{\text{HTG}} = 0.65 \). For the classical equilibrium with \( P_e = 778.5 \) [MWe], \( P_{\text{IT}} = 841.0 \) [MWe], \( P_e = 62.5 \) [MWe], and \( M_{\text{FPC}} = 2.51 \times 10^3 \) [ton], MPD is 311 [kWe/ton] and \( Q_e \) = 13.4; then the COE of \(-50 \text{ mill/kWe} \) is attained with the plasma transport power incident on the first wall surface, however the wall load is predicted to be slightly higher, because both \( \eta_{\text{CD}} \) and \( \eta_{\text{CD}} \) are determined with the possibility of compact size.

### 4.4 Neutron wall load and divertor heat flux

A possible engineering design goal might be to achieve a surface averaged neutron wall load (\( W_n \)) of less than 10 MW/m². The \( W_n \) value achieves \(~4.44 \text{ MW/m}^2\) for the classical equilibrium. This \( W_n \) value is an averaged one over the first wall surface, however the wall load is predicted to be higher than 10 MW/m² in outer equatorial plane because of a large peaking factor in the case of low aspect ratio.

An additional engineering challenge in the economic analysis is how to remove the plasma transport power incident on the diverter surfaces. We do compute, for the design point, a simple ratio of the diverted power divided by an approximate diverting surface area: \( W_d = P_{\text{diff}}D/(2\pi R_0) \), where \( D = (1.25) \text{ m} \). The value of the constant \( D \) was chosen such that \( W_d \) agrees with the peak heat flux reported for ARIES I, the double null divertor design [43]. Since \( P_{\text{diff}} = 0.25P_e + P_{\text{CD}} - P_{\text{CD}} \), where the first term is “alpha particle transport power” liberated as radiation and particle losses to plasma-facing surfaces (about one-quarter of the virgin neutron power) and \( P_{\text{CD}} \) is the power uniformly radiated to the first wall, we see that \( P_{\text{diff}} \) and \( W_d \) will be large when \( P_{\text{CD}} \) is large (low \( Q_e \)). Conversely, high \( T_e \) operation increases the radiation from bremsstrahlung and synchrotron (\( T_e \geq 20 \text{ keV} \)), which lowers both \( P_{\text{diff}} \) and \( W_d \). The obtained \( P_{\text{diff}} \) and \( W_d \) for the present design point with a relatively low \( T_e \) and such a small major radius are relatively high. The \( W_d \) value in the classical RFP equilibrium is \( 33.16 \text{ MW/m}^2 \) at \( T_e = 12.0 \text{ keV} \) and \( (n_e)_{20} = 4.63 \text{ m}^{-3} \) (D-T-fueled system without considering the alpha particle bootstrap current), and reduces to \(~14.8 \text{ MW/m}^2\) at the design points for economic analysis of \( T_e = 29.6 \text{ keV} \) and \( (n_e)_{20} = 1.88 \text{ m}^{-3} \) (D-T-fueled system considering the alpha particle bootstrap current). The engineering design goal might be to achieve \( W_d \) less than 10 MW/m². For the reduction of both \( W_d \) and \( W_d \), it is required to operate at a higher temperature and lower density for a given beta value even though a lower temperature and a higher density would provide a larger MPD and a lower COE. As the result, the MPD becomes smaller but COE slightly higher, because of higher \( Q_e \), as indicated by the arrow in Fig. 5. For an example, \( W_d = 4.26 \text{ MW/m}^2 \), \( W_d = 1.15 \text{ MW/m}^2 \) at \( T_e = 40 \text{ keV} \), \( (n_e)_{20} = 1.39 \text{ m}^{-3} \). For the further reduction of \( W_d \), density profiles will be controlled by fuelling techniques and particle transport physics which can not be predicted with certainty at present. The peaked densities invariably result in lower COE than flat profiles. There is a subtle effect contributing to this result: for a given plasma pressure and volume averaged \( T_e \), peaked density profile has \( \langle n_e \rangle \) about 20% smaller than a flat density, leading to somewhat smaller \( P_{\text{CD}} \) and higher \( Q_e \) at the same MPD. Then the \( W_d \) value is reduced to \(~9.24 \text{ MW/m}^2\) at \( T_e = 50 \text{ keV} \), \( (n_e)_{20} = 1.11 \text{ m}^{-3} \). More significantly, the bootstrap aided current driving efficiency \( \eta_{\text{CD}} = \langle n_e \rangle_{20}^{\text{BO}}R_0/P_{\text{CD}} \) generally increases for more peaked density, resulting in substantially higher \( Q_e \) at a given \( T_e \). Accordingly the low-aspect-ratio RFP D-T reactor leads to an attractive option with the possibility of compact size.
In the discussion, an optimization study of the possibility of achieving a lower COE is desired, varying A and the cross-sectional shape such that they may further enhance MPD and $Q_E$. Some different features exist between the RFP and the tokamak neoclassical equilibrium. The strong paramagnetism would simplify the structure of the superconducting toroidal coil in the RFP, resulting in a compact device with a lower aspect ratio. The present engineering techniques of the blanket and superconducting coil set the aspect ratio to $A \geq 2$ to attain the steady-state reactor, then the stability window of the ballooning mode between the first and the second stability regime does not exist in the tokamak. A neoclassical tokamak with the stability of $\beta_0 = 55\%$ at $A = 1.4$, $\kappa = 3.0$ has been reported to have a completely self-sustained current [44]. The neoclassical tearing modes, which are driven by the perturbed Pfirsch-Schluter current in negative magnetic field, are also reduced in RFPs, where the safety factor is small and $R_b$ is large at the plasma boundary.

This study is the first step toward achieving the low-cost of electricity for the steady-state RFP fusion power plant to be compatible with the engineering design goal. From this study, key physics and engineering issues central to achieving the reactor have emerged. Issues that remain to be addressed include optimizing the pressure/temperature profiles, aspect ratio and cross-sectional shapes, and replacing ohmic current with RF noninductive current so that the stability beta limit and the plasma self-induced current fraction are maximized, applying the bootstrap current models considering the effects of finite banana width around the magnetic axis, collisionality, low-frequency fluctuations and the synergistic effects with RF-induced currents, assessing the need for current profile control, creating high-$\beta$ free-boundary equilibria with magnetic diverter using a realistic field-shaping coil set, and determining optimum pressure/temperature profiles consistent with a transport model. Finally, the ideal externally nonresonant kink mode stability in the low-aspect-ratio plasma surrounded by an external shell of finite conductivity should be investigated to examine the stability window on wall-ratio ($r_w/a$) stabilizing resistive-wall modes (RWM) in a steady-state machine and reactor, as well as perfectly conducting wall modes. In this case, the position of the conducting wall is adjusted by controlling the magnetic shear in the outer region of free boundary so that it is just behind the blanket if its width can be thinner so as to satisfy the condition of stability window on the wall ratio. For such stabilization, it is noteworthy that the increasing parallel viscosity with decreasing aspect ratio may contribute to the stabilization of RWM modes as well as the stabilization of the neoclassical resistive $g$-mode driven by perturbed Pfirsch-Schluter current in negative magnetic shear, as in RFPs.

4.5 D-3He reactor

D-3He-fueled reactor is desired to operate at a higher temperature (volume-averaged electron temperature of $\langle T_e \rangle = 50$ keV) and a lower density for a given beta value might be desired. The D-3He-fueled design requires a level of plasma performance that is significantly more advanced than is required to fuse D-T, but a significant reduction in neutron production and subsequent radioactivity generation in blanket structural materials (HT-9M) is anticipated. Furthermore, the reduced neutron environment makes possible a simpler shield (a tritium-breeding blanket per se is not required) that is designed to recover only heat and to protect the magnets while using materials (Fe-1422) and coolants (organic coolant) generally not applicable for use in the intense neutron fluxes associated with D-T-fueled systems. The fusion power core mass ($M_{FPC}$) is evaluated using the size scaling for the ARIES-III D-3He-fueled system. An important goal met by the D-3He-fueled system is a fusion power core that operates for the life of the plant. The net electric power, achieved in the D-3He-fueled system at the design point of $\langle T_e \rangle = 50$ keV and $\langle n_e \rangle_0 = 4.4$ m$^{-3}$ for the neoclassical RFP equilibrium considering the alpha particle bootstrap current, is $P_E \sim 179$ [MW] assuming the thermal-conversion efficiency of organic coolants $\eta_{TH} = 0.35$ and $\eta_{LCS} = 0.78$, then $MPD \sim 177$ [MW] using $M_{FPC} = 1.01 \times 10^3$ [tonne] and $Q_E \sim 5.84$, resulting in the COE $\sim 120$ [mill/kWe-h] with the lunar-3He-fuel supply contributing $\sim 20\%$ to the COE. The D-3He-fueled power plant design leads to smaller neutron wall loading but lower $Q_E$ and higher COE compared with those for the D-T-fueled power plant.

Neutron production due to the side reactions occurring in the D-3He-fueled cycle cause sufficient structural activation of the HT-9M alloy used and, along with the chemical energy stored in the low-pressure organic coolant (OC), maintains the safety rating to that of the D-T-fueled design. The OC could be exchanged for pressurized water to enhance the safety, but the decrease in thermal-conversion efficiency (from 46% to 35%) slightly overrides the increased safety cost credit to raise the COE by 1 [mill/kWe-h]. Furthermore, all the wall load is the surface thermal loading $W_{ST}$, which results from the “proton and alpha particle power”, $P_P$, liberated as radiation and particle losses to the first wall, the power, $P_{CD}$, injected for RFCD, and the power uniformly radiated to the first wall, $P_{rad}$. The $W_{ST}$ value defined as a simple ratio of the total thermal power to the first wall divided by an approximate first wall surface area $A_w \sim 2.21$ [MW/m$^2$] and the $W_d$ value is $\sim 27.7$ [MW/m$^2$], which makes the surface cooling more difficult and the diverter heat flux larger compared with D-T-fueled reactor. For the reduction of both $W_{ST}$ and $W_d$ at the nearly same MPD, it is required to operate at a higher temperature and a lower density for a given beta value and also at a peaked density profile and a flat temperature profile for a given plasma pressure profile, as well as for the D-T-fueled reactor mentioned above. Although these engineering problems are difficult to resolve, if possible, the low-aspect-ratio D-3He-fueled reactor would lead to an attractive option with the possibility of compact size and neutron reduction, but with smaller $P_E$ and lower $Q_E$ then higher COE compared with those for the D-T-fueled fueled power plant.
5. Conclusion

The neoclassical RFP equilibrium with the low-aspect-ratio of $A = 2$ is studied in a reactor-relevant parameter regime. It was shown that in a steady-state case, the equilibrium current with a flat pressure profile and $\kappa/\delta = 1.4/0.4$ can be sustained by an approximately complete (94%) self-induced plasma current with a hollow profile, thus a feasible steady-state operation in RFP is possible. The high-stability $\beta$ of $\beta = 63\%$ ($\langle \beta \rangle = 66\%$) to the ideal kink and Mercier localized modes is maintained when a more compact device compared with those in the classical RFP equilibrium with conventional values of $A (> 3)$ is used. The finite banana-width effect still remains to be clarified along with the aim of achieving a complete self-induced plasma current. The equilibrium is close to the relaxed-equilibrium state with minimum energy and is also robust against microinstabilities. These attractive features allow the economic design of a compact steady-state fusion power plant with low COE.

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References