Contributed Paper

Evaluation of Para-Perp Type Mach Probe by Using a Fast Flowing Plasma

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Mach probe with two collecting tips facing to the direction parallel and perpendicular to the plasma flow, named para-perp type one, is evaluated by using a directional Langmuir probe (DLP) in a fast flowing plasma produced by an MPD (Magneto-Plasma-Dynamic) arcjet. We present simple formulas to determine an ion acoustic Mach number M_i from the ratio of collected ion saturation currents. The formulas and experimentally-obtained data are compared with Hutchinson's simulation results which are calculated using a particle-in-cell(PIC) code in an unmagnetized plasma condition. [I.H. Hutchinson, Plasma Phys. Control. Fusion, **44** 1953 (2002).] Good agreements are obtained among these data in both subsonic and supersonic plasma flows. Correction factor κ to determine M_i is presented under various conditions of T_i/T_e and the specific heat ratio for ions γ_i.

Keywords:

Mach probe, para-perp type Mach probe, directional Langmuir probe, supersonic plasma flow, unmagnetized plasma

1. Introduction

Recently a plasma flow has been recognized to play an important role in magneto-hydro-dynamic phenomena observed in space and fusion plasmas. Production of a high-beta, supersonic plasma flow is quite useful for these basic researches and also various industrial applications. Intensive researches to develop a fast flowing plasma with high particle and heat fluxes have been also performed for the purpose of various wall material researches and space applications.

In order to evaluate plasma flow properties, it is necessary to measure a flow velocity and Mach number of the flow. Cylindrical and spherical electrostatic probes were firstly used to measure a drifting plasma and effects of plasma flow on the probe characteristics were discussed in many researches. Mott-Smith and Langmuir [1] discussed collection characteristics of charged particles with a drifting Maxwellian distribution function. Kanal [2] derived mathematical formulas of ion current collected by a cylindrical probe set in a drifting plasma by taking account of ion orbital motion around the collecting surface. Stangeby and Allen [3] investigated density profile around a spherical obstacle set in a drifting plasma and discussed ion currents as a function of collecting angle with respect to the plasma flow direction. In experiments, Sonin [4] measured ion currents collected in a flowing plasma by cylindrical Langmuir probes with their axes aligned to and normal to the plasma flow direction. In order to clarify the probe characteristics in a drifting plasma, Makita and Kuriki [5] performed unique experiments by using a fast rotating arm. They fixed a spherical or cylindrical probe on a rotating arm in a static plasma and obtained current-voltage probe characteristics in a drifting plasma with a well-defined velocity.

From these researches arose an idea that a plasma drift velocity was able to be estimated by comparing two ion-current-collecting probes facing to different directions. Johnson and Murphree [6] discussed ion currents collected by two cylindrical probes with their axes perpendicular and parallel to the plasma flow and this combined type of probes was used to determine a plasma flow velocity in plasma jets [7,8]. Hudis and Lidsky [9] derived a simplified formula to obtain the drifting velocity from ratio of ion saturation currents collected by a directional probe facing to upstream and downstream directions. Harbour and Proudfoot [10] measured a plasma flow near a divertor plate in the DITE tokamak by a pair of electrostatic probes separated by a barrier. The probe with these types of structure has been called as "Mach" probe, since an ion Mach number can be derived from the current ratio.

An ion acoustic Mach number M_i is defined by ratio of a plasma flow velocity U_p to an ion acoustic velocity C_s,

$$M_i = \frac{U_p}{C_s} = \frac{U_p}{\sqrt{(\gamma_e T_e + \gamma_i T_i)/m_i}} \ , \qquad (1)$$

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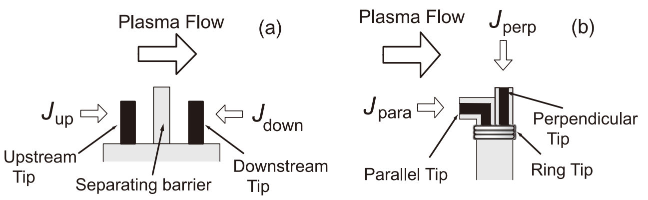

Fig. 1 Schematic of two types of the Mach probes.

where T_e and T_i are electron and ion temperatures, respectively, and m_i is ion mass. γ_e and γ_i are the specific heat ratios for electrons and ions, respectively.

A typical Mach probe consists of two conducting tips separated by an insulator between the tips, as illustrated in Fig. 1(a), so that each tip can collect ions from the direction opposite to the other tip. A Mach number can be obtained from ratio of ion saturation current densities collected by each tip, $R = J_{up}/J_{down}$, where J_{up} and J_{down} are current densities collected by upstream and downstream probes, respectively. Theoretical models have been developed and experimental researches have been done to determine M_i by this type of Mach probe, named as up-down type Mach probe [11-17]. The models were discussed under "magnetized" or "unmagnetized" condition and were classified as either kinetic or fluid models. The magnetized or unmagnetized condition depends on whether the probe dimension (typically a probe radius r_p) is larger or smaller than ion Larmor radius ρ_i. Fluid models derive a solution by using continuity and momentum equations while kinetic models solve the Boltzmann equation.

According to the progress of diagnostic tools, spectroscopic measurement and LIF (Laser Induced Fluorescence) diagnostics are recently used to determine ion temperature and local velocity. Among several diagnostics to measure a plasma flow velocity, a Mach probe is one of the most simple and useful tools to obtain an ion Mach number with a good spatial resolution.

To obtain an accurate Mach number by the Mach probe, precise calibration by other diagnostics is necessary. Moreover, it is not assured that the formula deduced under the condition of a subsonic ($M_i < 1$) flow can be extended to a supersonic ($M_i > 1$) flow because these model formulas include the presheath condition that is not applicable in the supersonic flow.

For the purpose of measurement in a wide range of M_i including a supersonic regime, another type of Mach probe, the para-perp type Mach probe illustrated in Fig.1(b), has been utilized in the research of collisionless shock waves in a hypersonic ($M_i \gg 1$) plasma flow [18].

The para-perp type Mach probe consists of two plane-probe tips facing parallel and perpendicular to the plasma flow. M_i can be derived from the ratio of J_{para}/J_{perp}, where J_{para} and J_{perp} are ion saturation current densities collected by the two tips, respectively. A simple formula to obtain M_i in both subsonic and supersonic flows from the ratio of J_{para}/J_{perp} is presented in the following section. It is based on the simple considerations that J_{perp} is not affected by the flow and that a presheath region in front of the J_{para} probe tip should disappear in a hypersonic flow with $M_i \gg 1$. The para-perp type Mach probe can be used in an unmagnetized plasma flow and has the advantage of being able to simultaneously obtain the plasma density.

In this paper are reported characteristics of the para-perp type Mach probe and calibration results by using a directional Langmuir probe (DLP) to simulate the Mach probe in a fast flowing plasma. Plasma flow velocities and ion temperature are measured by spectroscopy and compared with the DLP data under various conditions of the plasma flow generated by an Magneto-Plasma-Dynamic arcjet (MPDA). The formula and experimentally-obtained results are compared with the PIC (particle-in-cell) simulation by Hutchinson [19]. The correction factor to determine M_i under various conditions of T_i/T_e is also discussed.

2. Model Formula of the Para-Perp Type Mach Probe

In order to evaluate characteristics of the para-perp type Mach probe, we should present a simple formula to derive M_i from the ratio of ion saturation currents collected by the two probe tips. We assume that in an unmagnetized plasma flow an electrostatic sheath condition does not change in the direction perpendicular to the flow. The validity of this assumption is confirmed by the Hutchinson's simulation results in Ref. 19, where the ion flux collected at the surface perpendicular to the plasma flow direction is almost constant in a wide range of the flow velocity.

Then, the ion saturation current density J_{perp} collected by a perpendicular tip of the Mach probe is simply expressed as,

$$J_{perp} = \kappa_0 q n_i \sqrt{\frac{T_e}{m_i}} \ , \tag{2}$$

where q is the charge of an electron, n_i is ion density, and the coefficient κ_0 depends on ratio T_i/T_e. When $T_i/T_e \ll 1$, κ_0 equals to exp (–1/2) = 0.607, which is derived from the Bohm sheath criterion [20].

When T_i/T_e is larger than order of unity, J_{perp} can be determined from a random ion flux [1] without being influenced by the Bohm sheath criterion as follows,

$$J_{perp} = \frac{q n_i \langle v_i \rangle}{4} = \frac{q n_i}{4}\sqrt{\frac{8\,T_i}{\pi m_i}} = 0.4\, q n_i \sqrt{\frac{T_i}{m_i}} \ . \tag{3}$$

Then, it can be expressed in the same formula as eq. (2) by using κ_0,

$$J_{perp} = 0.4\, q n_i \sqrt{\frac{T_i}{T_e}}\sqrt{\frac{T_e}{m_i}} = \kappa_0 q n_i \sqrt{\frac{T_e}{m_i}} \ . \tag{4}$$

Here, in the region of $T_i/T_e \gg 1$, κ_0 is expressed as follows;

$$\kappa_0 = 0.4\sqrt{\frac{T_i}{T_e}} \ . \tag{5}$$

The expression of κ_0 as a function of T_i/T_e will be discussed in Sec. **4**.

We introduce a new coefficient κ to represent J_{perp}, which is expressed in eq. (2), by using the ion acoustic velocity $C_s = ((\gamma_e T_e + \gamma_i T_i)/m_i)^{1/2}$ in order to relate it to an ion Mach number for arbitrary ratio of T_i/T_e,

$$J_{perp} = \kappa_0 q n_i \sqrt{\frac{T_e}{m_i}} = \kappa q n_i \sqrt{\frac{\gamma_e T_e + \gamma_i T_i}{m_i}} \quad . \tag{6}$$

The coefficient κ is related to κ_0 as,

$$\kappa = \frac{\kappa_0}{\sqrt{\gamma_e + \gamma_i (T_i/T_e)}} \quad . \tag{7}$$

Here κ also depends on the ratio T_i/T_e and asymptotically equals to κ_0 in case of $T_i/T_e \ll 1$ and $\gamma_e = 1$.

In a hypersonic flow ($M_i \gg 1$), the presheath region in front of the probe tip parallel to the flow disappears and J_{para} is asymptotically expressed in the simple formula for single-ionized ions ($Z = 1$),

$$J_{para} = q n_i U_p \tag{8}$$

Then the Mach number in a hypersonic flow is calculated as follows;

$$M_i = \frac{U_p}{C_s} = \frac{U_p}{\sqrt{(\gamma_e T_e + \gamma_i T_i)/m_i}} = \kappa \frac{J_{para}}{J_{perp}} \quad . \tag{9}$$

In a subsonic flow ($M_i \ll 1$), Stangeby and Allen [3] obtained ion density around a cylindrical obstacle located in a plasma flow. The ion density at the stagnation point, which corresponds to the sheath boundary of the parallel probe tip, was expressed as,

$$n = n_0 \exp\left(-\frac{1}{2}\right) \exp\left(\frac{1}{2} M_i^2\right) \tag{10}$$

As the ion density is $n_0 \exp(-1/2)$ at the sheath boundary of the perpendicular probe for an ion flow, the current ratio J_{para}/J_{perp} should be given by $\exp(M_i^2/2)$. Hence, Kuriki and Inutake [18] expressed the ratio J_{para}/J_{perp} in the subsonic flow ($M_i < 1$) of $T_i/T_e \ll 1$ as,

$$\frac{J_{para}}{J_{perp}} = \exp\left(\frac{1}{2} M_i^2\right) \quad . \tag{11}$$

At the point of $M_i = 1$, this function is smoothly connected to that of eq. (9), since $\kappa = \exp(-1/2) = 0.607$ in case of $T_i/T_e \ll 1$ and $\gamma_e = 1$.

Here, we are going to derive a formula representing the relation between the ratio J_{para}/J_{perp} and M_i for wide range of T_i/T_e. It should satisfy the following conditions;(a) J_{para} is asymptotically expressed as eq. (8) at $M_i \gg 1$, (b) J_{para}/J_{perp} equals to unity at $M_i = 0$, and (c) the formula is expressed as eq. (11) under the conditions that $T_i/T_e \ll 1$ and $M_i < 1$.

In a hypersonic flow ($M_i \gg 1$) we can adopt eq. (9) to satisfy the condition (a). In the subsonic flow ($M_i < 1$) we introduce a new asymptotic expression of the current ratio as follows,

$$\frac{J_{para}}{J_{perp}} = \exp\left(a M_i^b\right) \tag{12}$$

where the coefficients a and b are arbitrary factors to be determined in order to satisfy the above conditions (b) and (c). We can derive the coefficients so as to connect smoothly the curve of eq. (12) to eq. (9) at $M_i = 1$. Then, we obtain

$$a = -\ln\kappa, \quad b = -1/\ln\kappa \quad . \tag{13}$$

Therefore, the ratio J_{para}/J_{perp} in our model is expressed as follows [21,22],

$$\frac{J_{para}}{J_{perp}} = \frac{M_i}{\kappa} \qquad (M_i > 1) \tag{14}$$

$$\frac{J_{para}}{J_{perp}} = \exp\left(a M_i^{1/a}\right), \quad a = -\ln\kappa \qquad (M_i < 1) \tag{15}$$

The equation (15) satisfies the conditions of (b) and (c), since $\ln\kappa = -1/2$ in case of $T_i/T_e \ll 1$ and $\gamma_e = 1$ as mentioned above. The coefficient κ is a key factor to derive M_i from J_{para}/J_{perp}, and we call it a correction factor, hereinafter.

Another expression of the ratio can be derived in terms of a kinetic model based on the work of Mott-Smith and Langmuir [1]. They calculated ion currents assuming that ions flow into a probe tip without any effect of the sheath potential. The ion saturation current density collected by the parallel probe tip is given by,

$$J_{para} = q \int_0^\infty v f(v)\, \mathrm{d}v \quad , \tag{16}$$

where $f(v)$ is a drifting Maxwellian distribution function expressed as,

$$f(v) = n_i \sqrt{\frac{m_i}{2\pi T_i}} \exp\left(-\frac{m\left(v - U_p\right)^2}{2T_i}\right) . \tag{17}$$

Then the current density is calculated as,

$$J_{para} = q n_i \sqrt{\frac{T_i}{2\pi m_i}} \times \left\{\exp\left(-W^2\right) + \sqrt{\pi}\, W\, [erf(W) + 1]\right\}, \tag{18}$$

where $erf(W)$ is an error function and

$$W = \frac{U_p}{\sqrt{2T_i/m_i}} = M_i \sqrt{\frac{\gamma_e + \gamma_i\,(T_i/T_e)}{2\,(T_i/T_e)}} \quad . \tag{19}$$

Whereas, the random current density collected by the perpendicular tip is given by

$$J_{perp} = \frac{q n_i \langle v_i \rangle}{4} = q n_i \sqrt{\frac{T_i}{2\pi m_i}} \quad . \tag{20}$$

Then the ratio J_{para}/J_{perp} is expressed as

$$\frac{J_{para}}{J_{perp}} = \exp\left(-W^2\right) + \sqrt{\pi}\, W\left[erf(W) + 1\right] . \tag{21}$$

Relationships between the ratio J_{para}/J_{perp} and the Mach number M_i are plotted in Fig. 2 for both our formula; eqs. (14) and (15), and the kinetic formula; eqs. (19) and (21).

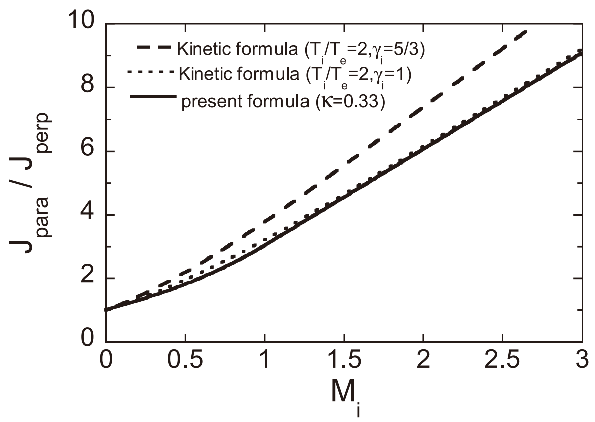

Fig. 2 Ratio of J_{para}/J_{perp} is calculated as a fuction of ion Mach number M_i. Solid line represents our present formula as eq. (14) and (15) with $\kappa = 0.33$. Dashed and dotted lines represent kinetic formula as eq. (21) with $\gamma_i = 5/3$ and $\gamma_i = 1$, respectively. $T_i/T_e = 2$ and $\gamma_e = 1$ are assumed in the kinetic formula.

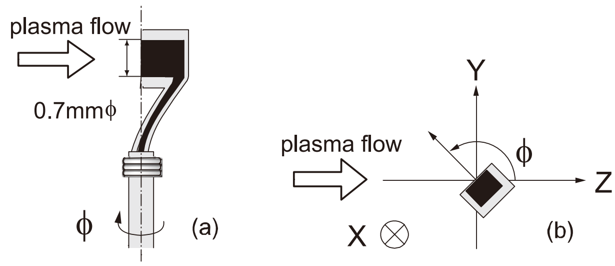

Fig. 3 Schematic of a directional Langmuir probe.

As shown in the figures, our simple formula is well fitted to the kinetic formula by selecting the appropriate correction factor κ.

It is noted that both our formula and the kinetic formula are based on simple assumptions and there is no theoretical proof in these equations when M_i is nearly unity. Experimental verification of the present formula on M_i expressed by eqs. (14) and (15) is necessary for subsonic and supersonic flows and the correction factor κ should be determined by using other diagnostics.

3. Experimental Setup

Experiments are carried out in the HITOP device [23-25]. The HITOP consists of a large cylindrical vacuum chamber (diameter D = 0.8 m, length L = 3.3m) with eleven main and six auxiliary magnetic coils, which generate a uniform magnetic field up to 0.1T. A high power, quasi-steady MPDA is installed at one end-port of the HITOP as a source of a fast flowing plasma. An MPDA has coaxial electrodes with a central cathode rod and an annular anode. The discharge current I_d up to 10kA is supplied with a quasi-steady duration of 1ms. A high density plasma (more than 10^{20} m^{-3}) is produced with helium as a working gas and accelerated axially by electromagnetic force generated by a radial discharge current J_r and an azimuthal self-induced magnetic field B_θ [26,27]. The M_i in a uniform magnetic field configuration is nearly unity at the muzzle region and can be varied up to 3 in a diverging field configuration.

Plasma flow characteristics are measured by several diagnostics installed in the HITOP device. Spatial profiles of electron temperature T_e and density n_e are measured by a movable triple probe and a fast-voltage-scanning Langmuir probe. T_e and n_e are derived from a current-voltage characteristic line detected by an electrostatic Langmuir probe. Its current-correction tip is a plane surface of 0.9 mmϕ in diameter and faces perpendicularly to the plasma flow. Ion temperature T_i and plasma flow velocity U_p are measured from Doppler broadening and shift of HeII line spectra (λ = 468.575 nm) by a spectrometer. Line spectrum emission from the plasma is collected by a quartz lens and is transferred to a 1m Czerny-Turner spectrometer with a grating of 2,400 grooves/mm through a single fiber cable. The emission is detected by a CCD camera coupled with an image intensifier tube (ICCD), set at the exit plane of the spectrometer. The line spectra are obtained in every 0.1 ms time interval during a shot with the spectral resolution of 0.02 nm. T_i is also measured by an electrostatic energy analyzer. The measurement of T_i and U_p are described in detail in ref [23].

A directional Langmuir probe (DLP) shown in Fig. 3(a) is used to measure dependence of the ion saturation current on the angle between the plasma flow and the normal to the plane probe surface by rotating the probe around its axis as shown in Fig. 3(b). The probe tip is made of tungsten with 0.7 mmϕ in diameter and set on the axis so as not to change the measurement point by the rotation. It is biased at –40 V against a conducting wire with a large collection area that is wound around the ceramic support tube. The probe is a kind of an asymmetric double probe. The ion saturation current can be measured in spite of a significant time variation of plasma potential usually observed in an initial phase of the discharge. We measure the current by a current sensor to isolate a data-acquisition circuit from the probe biasing circuit.

4. Experimental Results and Discussion

Experiments are performed under unmagnetized plasma conditions. Typical parameters of the plasma flow are n_i = 2 ~4.5 × 10^{20} m^{-3}, T_e = 5 ~8 eV, T_i = 9 ~15 eV, U_p = 1 ~3 × 10^4 m/s with B_z = 5 ~8.7 × 10^{-2} T in the present experiments. This plasma conditions can be controlled by the MPDA operation. The ratio of the DLP-tip radius r_p (= 0.35 mm) to Debye length λ_d is nearly 10^3, and the ratio of the ion Larmor radius ρ_i to r_p is nearly 10^2.

Figure 4 shows a typical dependence of the ion saturation current J_{is} on the angle ϕ. Definition of the surface angle ϕ is illustrated in Fig. 3(b). We measured the dependence of J_{is} on cosϕ under two different flow conditions of subsonic (M_i = 0.8) and supersonic (M_i = 1.3) flows and plotted the results in Fig. 5. These Mach numbers are calculated from eq. (1) by using the experimentally obtained U_p, T_e and T_i assuming γ_e = 1 and γ_i = 5/3. In the MPDA plasma an ion temperature is relatively higher than an electron temperature in a uniform magnetic field configuration.

The experimental data are compared with Hutchinson's PIC simulation results under the condition of $T_i/T_e = 2$. Ion collection flux densities are calculated by PIC simulation under unmagnetized plasma condition and are represented as a function of the normalized plasma flow velocity $v_f = U_p/(T_e/m_i)^{1/2}$ in Ref. 19. The simulation results with nearly the same M_i as the experiments are also plotted as solid ($M_i = 1.2$) and dotted ($M_i = 0.77$) lines in Fig. 5. Here, M_i is calculated from v_f using $T_i = 2T_e$, $\gamma_e = 1$ and $\gamma_i = 5/3$. The simulation results are in good agreement with the experimental results in both subsonic and supersonic plasma flows.

In order to evaluate the present formula (eqs. (14) and (15)) and to determine the correction factor κ of the para-perp type Mach probe, the specroscopically-obtained Mach numbers are compared with ratios J_{para}/J_{perp} of the ion saturation currents detected by the Mach probe. Figure 6(a) shows the ratios of J_{para} to J_{perp} obtained from the DLP data at $\phi = \pi$ and $\pi/2$, respectively, as a function of M_i obtained by spectroscopy. The error bars represent several experimental data points scattered under the same experimental conditions. We obtain U_p, T_e, T_i and J_{para}/J_{perp} in every shot and calculate M_i from eq. (1) by assuming $\gamma_e = 1$ and $\gamma_i = 5/3$. Calculated curves according to eqs. (14) and (15) with $\kappa = 0.3$, 0.33 and

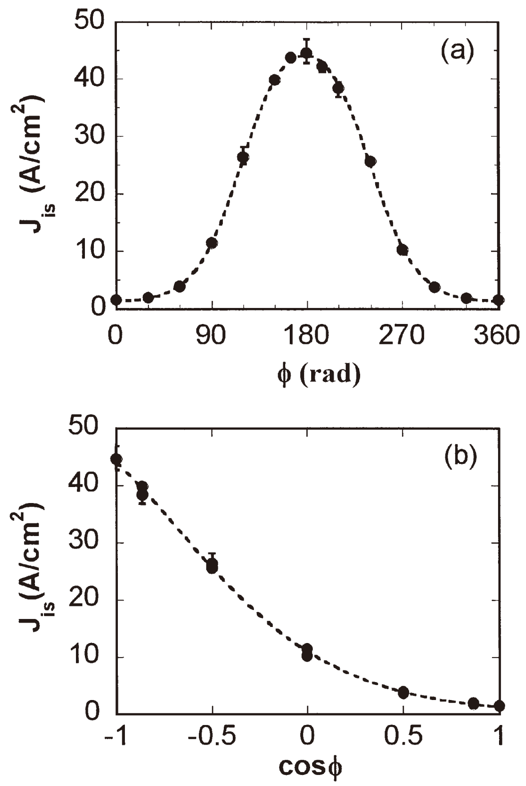

Fig. 4 (a) Typical dependence of ion saturation current J_{is} on the angle ϕ between the plasma flow and the normal to the tip surface. (b) The same J_{is} is plotted as a function of cosϕ . $n_i = 1.4 \times 10^{20}$ m^{-3}, $B_z = 0.05$ T.

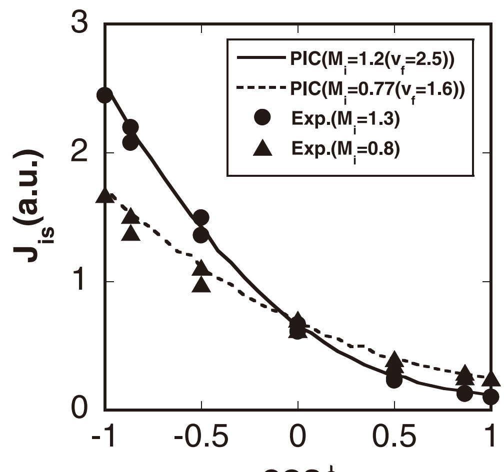

Fig. 5 Dependences under two different flow conditions of subsonic (▲: $M_i = 0.8$, $U_p = 19$ km/s, $T_i = 11$ eV and $T_e = 6.2$ eV) and supersonic (●: $M_i = 1.3$: $U_p = 28$ km/s, $T_i = 8.6$ eV and $T_e = 5.3$ eV) flows. $v_f = U_p/(T_e/m_i)^{1/2}$. The experimental data are normalized at cos$\phi = -1$.

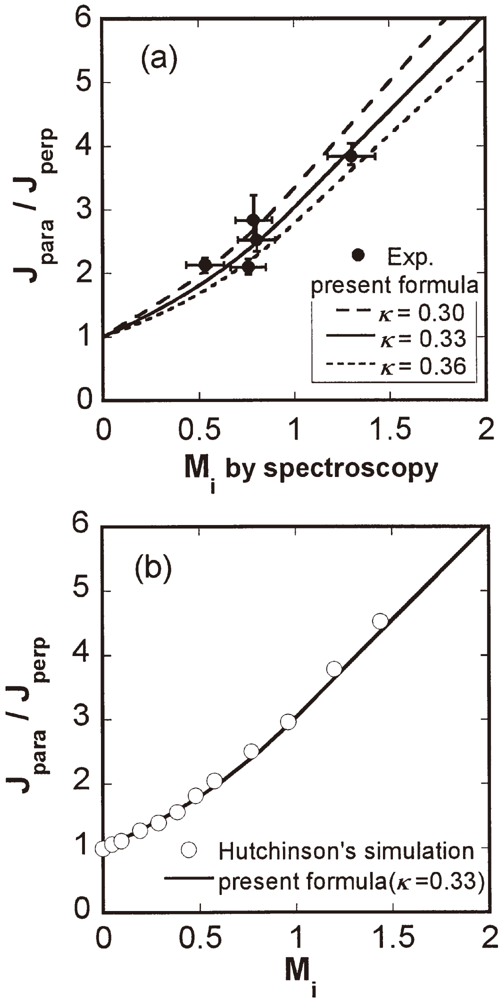

Fig. 6 (a) Ratios of J_{para} to J_{perp} obtained from the DLP data at $\phi = \pi$ and $\pi/2$, respectively, are plotted as closed circles as a function of M_i. Curves calculated by the present formula (eqs. (14) and (15)) with $\kappa = 0.3$, 0.33 and 0.36 are also plotted. (b) Hutchinson's simulation results ($T_i/T_e = 2$, $\gamma_i = 5/3$) are plotted as open circles and the curve calculated by the present formula with $\kappa = 0.33$ is represented as solid line.

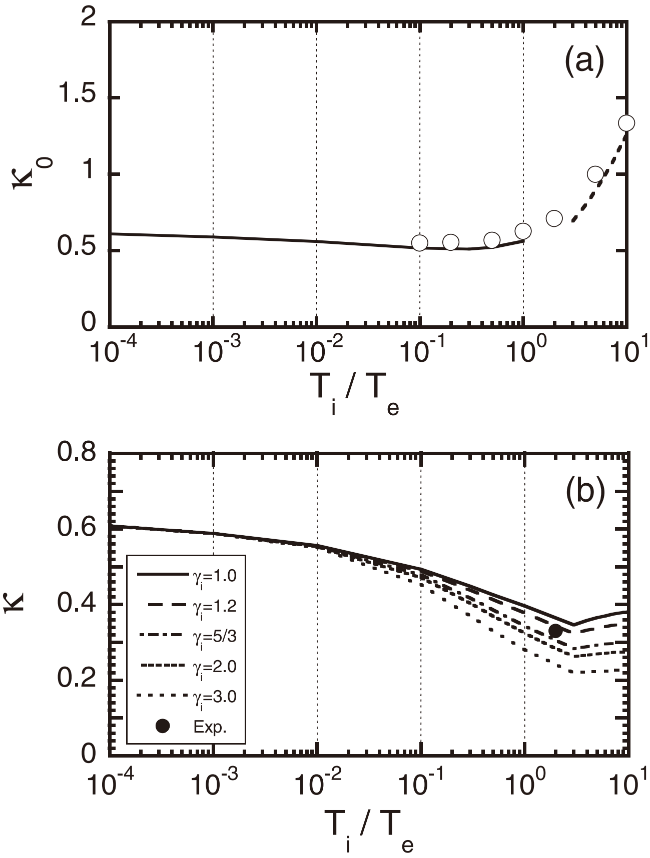

Fig. 7 (a) Coefficient κ_0 as a function of T_i/T_e. Open circles are Hutchinson's simulation results. (b) Coefficient κ as a function of T_i/T_e for various γ_i. $\kappa = \kappa_0/(\gamma_e + \gamma_i(T_i/T_e))^{1/2}$. Closed circle is the experimental data of $\kappa = 0.33$ under the conditions of $T_i/T_e = 2$ and $\gamma_i = 5/3$.

0.36 are also plotted in the figure. The experimental values are well fitted to the curve with $\kappa = 0.33$. It is remarkable that this curve with $\kappa = 0.33$ is also fitted to the simulation results with $T_i/T_e = 2$, $\gamma_e = 1$ and $\gamma_i = 5/3$ as shown in Fig. 6(b).

The correction factor κ in eq. (9) depends on T_i/T_e, γ_i and γ_e. In the present research we assume $\gamma_e = 1$, since electrons behave as isothermal media. It is useful to derive κ for various values of T_i/T_e and γ_i. When the ratio T_i/T_e increases, the coefficient κ_0 in eq. (2) is given as a function of T_i/T_e as presented by Swift and Schwar [28] in the range of $10^{-4} < T_i/T_e < 1$. It is also represented as eq. (5); $\kappa_0 = 0.4\ (T_i/T_e)^{1/2}$ in the region of $T_i/T_e \gg 1$. In Fig. 7(a) κ_0 in Ref. [28] is shown in the range of $10^{-4} < T_i/T_e < 1$ and κ_0 calculated by eq. (5) is shown in the range of $3 < T_i/T_e < 10$ with solid and dotted lines, respectively. Hutchinson also presented the PIC simulation results of ion current densities collected in a static plasma. We evaluated κ_0 for several cases of T_i/T_e by using his results. The calculated results are plotted as a function of T_i/T_e in the same figure. The simulation results coincide remarkably well with the calculated results of κ_0.

Then, the correction factor κ defined in eq. (7) can be calculated from the factor κ_0 shown in Fig. 7(a). In Fig. 7(b) is shown the calculated κ as a function of T_i/T_e for various cases of γ_i with $\gamma_e = 1$. The value of κ in the range of $1 < T_i/T_e < 3$ is interpolated in the figure. The experimental value $\kappa = 0.33$ for a case of $T_i/T_e = 2$ and $\gamma_i = 5/3$ is also plotted in the figure and shows a reasonable agreement with the curve for $\gamma_i = 5/3$. As the value of γ_i in plasmas is difficult to measure, we assume $\gamma_i = 5/3$ in this research. It may be smaller value than 5/3 due to many degrees of freedom, such as ionization excitation and recombination processes. It is further task to determine γ_i in plasmas in order to clarify the physics of a compressive plasma flow.

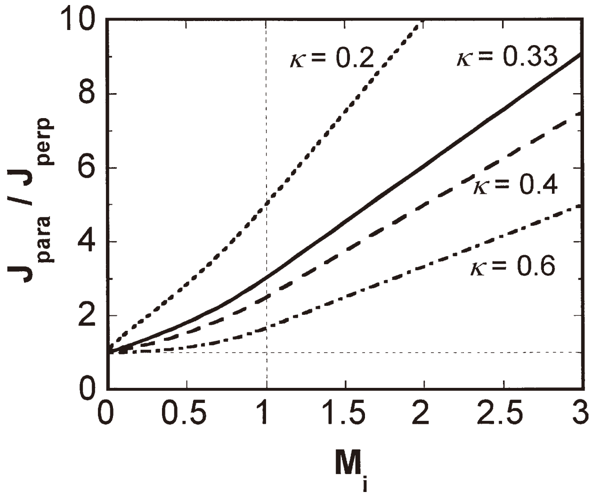

Fig. 8 Ratios of J_{para} to J_{perp} are plotted as a function of M_i. Curves are calculated by the present formula (eqs. (14) and (15)) with $\kappa = 0.2$, 0.33, 0.4 and 0.6.

As shown in Fig. 7(b), the coefficient κ varies from 0.2 to 0.6 depending on T_i/T_e and γ_i. The ratios J_{para}/J_{perp} calculated by the present formula (eqs. (14) and (15)) are shown as a function of M_i under the condition of $\kappa = 0.2$, 0.33, 0.4 and 0.6 in Fig. 8.

It is confirmed in this research that the para-perp type Mach probe is useful to measure M_i in both subsonic and supersonic plasma flows in an unmagnetized plasma with $\rho_i \gg r_p$. In a strongly magnetized plasma, however, the present formula should be modified since the ion current collected by the perpendicular tip will be influenced by a cross-field diffusion. The perpendicular tip should be adjusted carefully so as to face perpendicular to the plasma flow because the collected current is sensitive to the parallel flow.

5. Conclusions

A Mach probe with two collecting plane surfaces facing parallel and perpendicular to a plasma flow, named para-perp type, is evaluated by using a directional Langmuir probe (DLP) in a fast flowing plasma produced by an MPD (Magneto-Plasma-Dynamic) arcjet in the HITOP device. The obtained data are compared with Hutchinson's simulation results that are calculated using a PIC code in an unmagnetized plasma. Dependences of ion saturation current J_{is} on the angle ϕ between the plasma flow and the normal to the plane-probe surface are measured under the two different flow conditions of subsonic ($M_i = 0.8$) and supersonic ($M_i = 1.3$) flows. The obtained data are in good agreement with the PIC simulation results.

Simple formulas for the para-perp type Mach probe in both subsonic and supersonic flows are presented to deduce an ion acoustic Mach number M_i from the ratio of collected ion saturation currents. The present formulas and experimental data are compared with the Hutchinson's simulation results. These three are in good agreement among them in both subsonic and supersonic plasma flows. The correction factor κ is determined experimentally to be 0.33 under the condition of $T_i/T_e = 2$, $\gamma_i = 5/3$ and $\gamma_e = 1$. We evaluate κ in a wide range of T_i/T_e and γ_i for a convenient use in various experiments.

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