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Numerical Study of the Location of the Microwave Imaging Reflectometer Object Plane

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This paper is devoted to numerical study of the location of the imaging optics object plane. The simulation shows that in the case of the plane plasma cutoff, the object plane is located at the virtual cutoff position defined by E. Mazzucato [Nuclear Fusion **45**, 203 (2001)], while in the case of cylindrical plasma, the object plane is shifted toward the cutoff position.

Keywords:

microwave imaging reflectometry, numerical simulation

Microwave reflectometry is a diagnostic tool that uses radar to measure density profiles and their fluctuations. Although this diagnostic tool has been in use a long time, interpretation of the reflectometer signal in terms of plasma parameters is still an issue due to destructive interference of the signal wavefronts scattered by random fluctuations [1]. One of the possible solutions to this problem is to project the signal from the cutoff region to the receiving plane by means of imaging optics [2].

Plasma changes the optical path of the signal and thus affects the location of the optics object plane. In this paper we present the results of a numerical study of the location of the object plane in the presence of plasma.

The model employed in this study comprises analytical and numerical (FDTD) solutions of 2D Maxwell's equations in a vacuum and of the equation for the induced current density $\boldsymbol{j}$, which includes plasma parameters in the model (Fig. 1). (Further details on the calculation are given in [3]).

Poloidal fluctuations $\delta n/n$ were generated separately for cylindrical and plane geometries:

$$\left(\frac{\delta n}{n}\right)_{\mathrm{cyl}} = \gamma \cos[k_{\mathrm{p}} r_{\mathrm{cut}} (\phi - w_{\mathrm{p}} t)]$$

$$\left(\frac{\delta n}{n}\right)_{\mathrm{pln}} = \gamma \cos[k_{\mathrm{p}} (z - w_{\mathrm{p}} r_{\mathrm{cut}} t)] \ . \qquad (1)$$

Here, $\gamma = 0.06$ is the amplitude of the poloidal fluctuations, ϕ is the poloidal angle, k_{p} is the simulated poloidal wavenumber, r_{cut} is the cutoff position (for modelled parameters $r_{\mathrm{cut}} = 10.3$ cm) and w_{p} is the angular speed of rotation selected such that speed of rotation at the plasma edge is equal to 6 km/s. The microwave imaging system projects an image of the plasma cutoff to the receiving plane. The quality of the image is evaluated by the value of the cross-correlation coefficient between the shape of the cutoff density fluctuations and the shape of the image within the received signal spot-size. The closer the cross-correlation coefficient is to unity, the higher quality of the image.

Within the limits of free space paraxial signal propagation, a single lens Fourier-transforms the incident field distribution [4], such that

$$E_{\mathrm{f}}(z) \sim \tilde{E}_{\mathrm{obj}}\left(k_0 \frac{z}{f}\right) \exp\left[i \frac{k_0 z^2}{2f}\left(1 - \frac{d_{\mathrm{obj}}}{f}\right)\right], \qquad (2)$$

where E_{f} is the distribution of the field at the rear focal plane, $\tilde{E}_{\mathrm{obj}}$ is the Fourier image of the object field and d_{obj} is the distance from the object to the lens. The second lens of the modelled two-lenses imaging system causes a Fourier transformation again and thus restores the distribution of the object field at the receiving plane.

As can be seen from Eq. (2), the field distribution at the

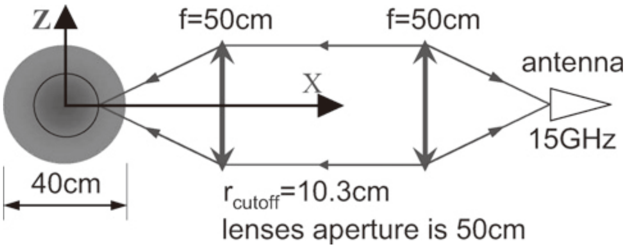

Fig. 1 Scheme of the simulated MIR system.

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author's e-mail: max@astec.kyushu-u.ac.jp rear focal plane is proportional to the Fourier transformation of the object multiplied by a quadratic phase factor. And only if the object is located at the front focal plane, i.e., $d_{\mathrm{obj}} = f$, will the phase factor become unity and the lens causes a precise Fourier transformation. In spite of the relatively rough compliance of the simulated system conditions to the validity region of Eq. (2) (namely, the limited aperture of the lenses and large deviation of the scattered wavefronts from the optical axis), our simulation confirmed the feasibility of Eq. (2) for use in microwave imaging reflectometry (MIR) (Fig. 2(a)). The region of the high cross-correlation coefficient is associated with the object plane and is located approximately symmetrically relatively focal plane position. In the case of the stratified plasma with the plane cutoff it is possible to show that

$$d_{\mathrm{obj}} = \int_{x_{\mathrm{lns}}}^{x_{\mathrm{cut}}} \frac{dx}{\sqrt{\varepsilon(x)}}, \tag{3}$$

where $\varepsilon(x)$ is the unperturbed permittivity of the medium, x_{lns} is the x-coordinate of the nearest to the plasma lens and the optics is assumed to transmit the signal from cutoff position. Equation. (3) is similar to the expression obtained by E. Mazzucato ([2], Eq.13) for virtual cutoff position. For modelled parameters, the virtual cutoff is distant from real one by 3.4 cm and the distance is coincident with the results of the simulation (Fig. 2(b)).

In the case of cylindrical plasma our simulation (Fig. 2(c)) proves that the object plane is located close to the plasma cutoff position. Theoretical displacement of the object plane from the cutoff position ([5], Eq.4) equals to 3.7 cm. Such discrepancy reveals the complicated nature of image formation.

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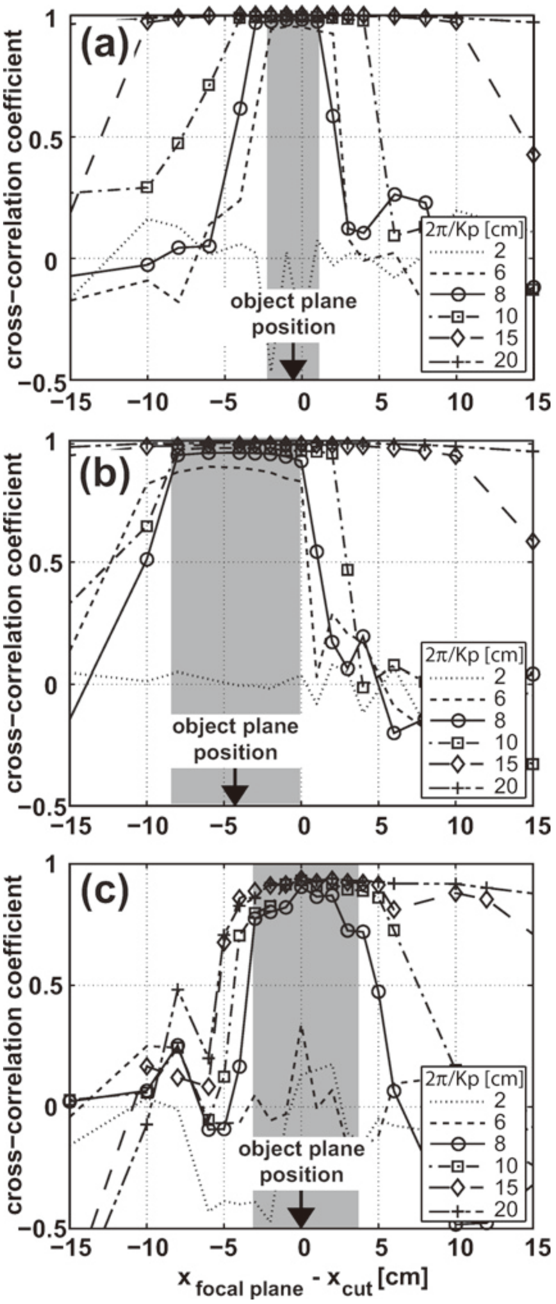

Fig. 2 Cross-correlation coefficient between the shape of received signal phase fluctuations and the shape of density fluctuations as function of the optics front focal plane position shift relatively cutoff location. (a) Free space propagation (phase screen model) and imaging of (b) plane and (c) cylindrical plasma. Region of high cross-correlation coefficient is also shown.