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Surface Wave Analysis with Plasma Resonance

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Surface waves are studied in axially non-uniform cylindrical cold plasma with a linear density profile. The real frequency, damping rate, and eigenfunction for the transverse magnetic mode in pure surface waves are obtained for collisional plasmas, where the plasma resonance is taken into account. It is shown that the eigenfunction peaks at the position of the plasma resonance layer.

Keywords:

surface wave, transverse magnetic mode, plasma resonance, non-uniform processing plasma

Surface waves have attracted much interest in the context of heating and diagnostics for processing plasmas. The surface waves discussed by Ghanashev *et al.* [1-3] are based on a simple *uniform* plasma model; however, strictly speaking, the plasma is non-uniform.

In this paper, we study surface waves in cold cylindrical plasmas having a *non-uniform* axial density profile. In this case, we encounter the problem of plasma resonance [4,5]; that is, the wave equation becomes singular at the layer where the wave frequency ω is equal to the electron plasma frequency ω_{pe} when there are no collisions. Here, we solve the dispersion relation and eigenfunction for the transverse magnetic (TM) modes of surface waves, taking into account this plasma resonance.

Our starting point is Maxwell's equations for electromagnetic wave fields $\boldsymbol{E}$ and $\boldsymbol{B}$ given by

$$\frac{\partial}{\partial t}\boldsymbol{B} = -\nabla \times \boldsymbol{E}, \tag{1}$$

$$\frac{\partial}{\partial t}(\varepsilon_r \boldsymbol{E}) = c^2 \nabla \times \boldsymbol{B}, \tag{2}$$

where $\varepsilon_r = \varepsilon/\varepsilon_0$, and ε_0 and μ_0 are the permittivity and permeability of free space, respectively, and c is the speed of light. If we assume an $\exp(-i\omega t)$ dependence for $\boldsymbol{E}$ and $\boldsymbol{B}$, we obtain from eqs.(1) and (2),

$$\nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E} - k_0^2 \varepsilon_r \boldsymbol{E} = 0 . \tag{3}$$

where $k_0 = \omega/c$. We assume here that ε_r is radially uniform and is a function of z only. In this case, eq.(3) is divided into the following two equations:

$$\nabla_\perp^2 \frac{\partial}{\partial z} E_z - \left(\frac{\partial^2}{\partial z^2} + k_0^2 \varepsilon_r\right) \nabla_\perp \cdot \boldsymbol{E}_\perp = 0 , \tag{4}$$

$$\frac{\partial}{\partial z} \nabla_\perp \cdot \boldsymbol{E}_\perp - \left(\nabla_\perp^2 + k_0^2 \varepsilon_r\right) E_z = 0 . \tag{5}$$

In this case we assume that a radially uniform plasma is contained in a metal chamber having radius a. We also assume an axial model shown in Fig. 1. The metal plate corresponding to the slot antenna is located at $z = -h$, quartz of $\varepsilon_1 = 4$ exists for $-h < z < 0$, the plasma density increases linearly for $0 < z < d$, and the density is uniform with N_0 for $z > d$. When we assume a separable wave form as, for E_z and $\nabla_\perp \cdot \boldsymbol{E}_\perp$,

$$E_z(r, \theta, z) = \psi(r, \theta) F(z) , \tag{6}$$

$$\nabla_\perp \cdot \boldsymbol{E}_\perp(r, \theta, z) = \psi(r, \theta) G(z) , \tag{7}$$

and furthermore, if we assume that ψ satisfies

$$\left(\nabla_\perp^2 + \lambda^2\right) \psi(r, \theta) = 0 , \tag{8}$$

we obtain coupled equations for F and G as

$$\lambda^2 \frac{\mathrm{d}}{\mathrm{d}z} F + \left[\frac{\mathrm{d}^2}{\mathrm{d}z^2} + k_0^2 \varepsilon_r(z)\right] G = 0 , \tag{9}$$

$$\frac{\mathrm{d}}{\mathrm{d}z} G + \left[\lambda^2 - k_0^2 \varepsilon_r(z)\right] F = 0 . \tag{10}$$

The solution of eq.(8) is given by

$$\psi(r, \theta) = J_m(\lambda r)[c_1 \cos(m\theta) + c_2 \sin(m\theta)] , \tag{11}$$

where J_m is the Bessel function of the first kind, c_1 and c_2 are

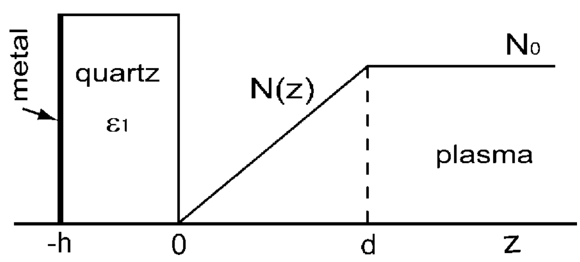

Fig. 1 Model with a non-uniform density profile.

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the integration constants, and m is an integer. From the boundary condition that $E_\theta = E_z = 0$ at $r = a$, that is, $J_m(\lambda a) = 0$, we obtain

$$\lambda = j_{mn}/a \ , \tag{12}$$

where j_{mn} is the n-th root of $J_m(x) = 0$.

We next consider the solutions of eqs.(9) and (10). From the boundary conditions in which $E_\theta = E_r = 0$ at $z = -h$, and $E_z = 0$ at $z = \infty$, we obtain, for $-h < z < 0$,

$$F = \alpha_0 \cosh[\,p_1(z+h)]\ , \tag{13}$$

$$G = -\mathrm{d}F/\mathrm{d}z = -\alpha_0 p_1 \sinh[\,p_1(z+h)]\ , \tag{14}$$

($k_0^2 < \lambda^2/\varepsilon_1$) and for $z > d$,

$$F = \alpha_3 \exp(-p_3 z)\ , \tag{15}$$

$$G = -\mathrm{d}F/\mathrm{d}z = p_3\alpha_3 \exp(-p_3 z)\ , \tag{16}$$

where $p_j = (\lambda^2 - k_0^2\varepsilon_j)^{1/2}$, $\varepsilon_1 = 4$, $\varepsilon_3 = 1 - (\omega_{p0}/\omega)^2$, and $\omega_{p0} = (e^2N_0/\varepsilon_0 m)^{1/2}$. For $0 < z < d$, where the plasma is non-uniform, we can obtain an equation for G as

$$\frac{\mathrm{d}^2G}{\mathrm{d}z^2} + \frac{\lambda^2}{\lambda^2 - k_0^2\varepsilon_2}\frac{1}{\varepsilon_2}\frac{\mathrm{d}\varepsilon_2}{\mathrm{d}z}\frac{\mathrm{d}G}{\mathrm{d}z} - \left(\lambda^2 - k_0^2\varepsilon_2\right)G = 0\ , \tag{17}$$

where $\varepsilon_2(z) = 1 - (\omega_{p0}/\omega)^2(z/d)$. For $\lambda^2 >> |\,k_0^2\varepsilon_2\,|$, which is justified for low-density plasmas, eq.(17) is reduced approximately to

$$\frac{\mathrm{d}^2G}{\mathrm{d}z^2} + \frac{1}{\varepsilon_2}\frac{\mathrm{d}\varepsilon_2}{\mathrm{d}z}\frac{\mathrm{d}G}{\mathrm{d}z} - \lambda^2 G = 0\ , \tag{18}$$

where the plasma resonance takes place at the zero $z_r = d(\omega/\omega_{p0})^2$ of $\varepsilon_2(z) = 0$. The solution of eq.(18) is then given by

$$G = \alpha_1 I_0[\lambda(z - z_r)] + \alpha_2 K_0[\lambda(z - z_r)]\ , \tag{19}$$

$$F = -\frac{1}{\lambda^2}\frac{\mathrm{d}}{\mathrm{d}z}G\ , \tag{20}$$

where I_0 and K_0 are the modified Bessel function of the first and second kinds, respectively. The dispersion relation and three coefficients among α_j (j = 0~3) are determined based on the continuity conditions of $\varepsilon(z)F(z)$ and $G(z)$ at two interfaces $z = 0$ and d; i.e.,

$$\left[\,\varepsilon F\,\right]_{z-0} = \left[\,\varepsilon F\,\right]_{z+0}\ , \quad \left[\,G\,\right]_{z-0} = \left[\,G\,\right]_{z+0}\ . \tag{21}$$

We here introduce collisions between plasma and neutral particles to avoid the singularity of F and G at $z = z_r$, that is, we replace ω^2 by $\omega(\omega + i\nu)$ in ε_2 and ε_3. We thus obtain a complex dispersion equation of TM surface modes as

$$\frac{I_0(z_d) - t_2 I_1(z_d)}{K_0(z_d) + t_2 K_1(z_d)} = \frac{I_0(z_0) + t_1 I_1(z_0)}{K_0(z_0) - t_1 K_1(z_0)}\ , \tag{22}$$

where $z_d = \lambda(d - z_{rc})$, $z_0 = \lambda z_{rc}$, $z_{rc} = d\omega(\omega + i\nu)/\omega_{p0}^2$ and

$$t_1 = \frac{p_1}{\lambda}\tanh\left(\,p_1 h\,\right)\ , \quad t_2 = -\frac{p_3}{\lambda}\ . \tag{23}$$

In Fig.2, we show the real frequency and damping rate of the TM surface mode as a function of $(\omega_{p0}a/c)^2$, where h/a = 0.2, d/a = 0.3, and (m,n) = (8,1) and we assume $\nu a/c = 0.01N_0/N^*$, N^* being the density at $(\omega_{p0}a/c)^2 = 50$. In Fig.3, we also show the eigenfunction (G) of the surface mode for $(\omega_{pe}a/c)^2 = 50$, where the other parameters are the same as those in Fig.2. We can find that the eigenfunction of the surface mode becomes peaked at the position z_r of the plasma resonance satisfying $\omega = \omega_{pe}$ (for $\nu = 0$). We also find that the profile of $|F|$ is quite similar to that of $|G|$ shown in Fig.2 in the plasma region, but F is discontinuous at $z = 0$ based on eq.(21).

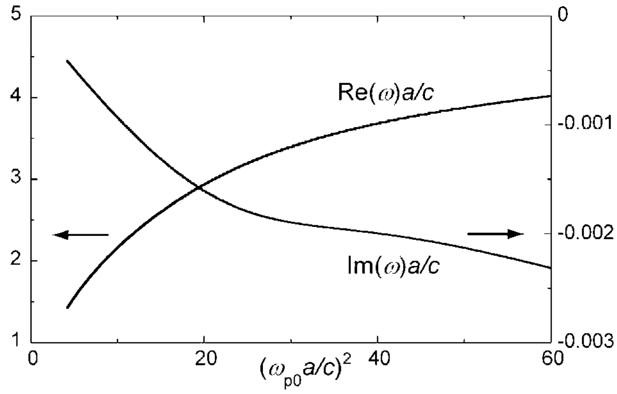

Fig. 2 Dispersion relation of TM surface mode.

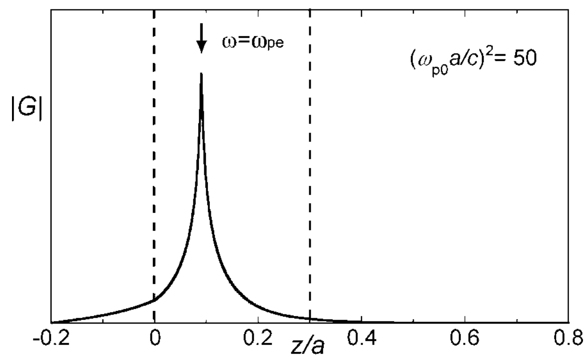

Fig. 3 Eigenfunction of G for TM surface mode.

Finally, we note that we can obtain a finite damping rate due to phase mixing effects caused by the plasma resonance even for collision-free ($\nu = 0$) plasmas. This problem is analogous to that of shear Alfven resonance [6]. The analysis for the case of $\nu = 0$ will be discussed elsewhere.

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