Rapid Communications

A New Method of Electron Density Measurement by Fabry-Perot Interferometry

HOJO Hitoshi[1)] and MASE Atsushi[2)]

[1)]*Plasma Research Center, University of Tsukuba, Tsukuba 305-8577, Japan*

[2)]*Art, Science and Technology Center for Cooperative Research, Kyushu University, Kasuga 816-8580, Japan*

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A new method for determining the electron density of a thin plasma by means of Fabry-Perot interferometry is proposed. The interferometer consists of two plasma layers and dielectric material surrounded by two plasma layers. The transmittance of electromagnetic waves across the interferometer is calculated, and Fabry-Perot resonances are demonstrated. It is shown that the plasma density can be determined based on the measurement of the resonance frequency when the width of a plasma layer is known.

Keywords:

Fabry-Perot resonance, interferometry, electromagnetic wave, density measurement, micro-plasma

A Fabry-Perot interferometer is often used for spectroscopic measurements of visible light [1-3] and x-ray [4]. We here propose a new method for determining the electron density of a thin plasma by means of Fabry-Perot interferometry. A Fabry-Perot interferometer using thin plasmas as the resonator is shown in Figure 1. We assume that uniform plasma is confined by a very thin material which is transparent for electromagnetic waves, and for the sake of simplicity, we here neglect the plasma-confining material. A dielectric material with the dielectric constant ε_a is inserted between two thin-plasma layers. If the wave frequency ω is lager than the electron plasma frequency ω_{pe}, the wave is in a propagating mode, and otherwise the wave becomes a evanescent mode. In this article, we can show that the Fabry-Perot resonance occurs and has very sharp peak in ω for wave-evanescent over-dense plasmas ($\omega < \omega_{pe}$), and thus the Fabry-Perot interferometer can attain its high resolution [5,6]. We can therefore determine the electron density of a thin plasma from the measurement of the resonance frequency because the resonance frequency is dependent on ω_{pe}.

Our starting point is a one-dimensional Maxwell wave equation given by

$$\left[\frac{d^2}{dz^2} + k^2 \varepsilon(\omega, z)\right] E(z) = 0 , \tag{1}$$

with

$$\varepsilon(\omega, z) = \begin{cases} 1, & z < 0 \\ 1 - \left(\dfrac{\omega_{pe}}{\omega}\right)^2, & 0 \le z \le L \\ \varepsilon_a , & L < z < 2L \\ 1 - \left(\dfrac{\omega_{pe}}{\omega}\right)^2, & 2L \le z \le 3L \\ 1 , & z > 3L \end{cases} \tag{2}$$

author's e-mail: hojo@prc.tsukuba.ac.jp

where $k = \omega/c$, c is the speed of light, $\omega_{pe} = (e^2 n_p/\varepsilon_0 m)^{1/2}$ is the electron plasma frequency with a constant density n_p, m the electron mass, e the electric charge, and ε_0 the permittivity of free space. The solution of eq.(1) with eq.(2) is given by, for over-dense plasmas ($\omega < \omega_{pe}$),

$$E = \begin{cases} E_0 e^{ikz} + b e^{-ikz}, & z < 0 \\ c_1 e^{\lambda z} + d_1 e^{-\lambda z} , & 0 \le z \le L \\ f e^{ik_a z} + g e^{-ik_a z} , & L < z < 2L \\ c_2 e^{\lambda z} + d_2 e^{-\lambda z} , & 2L \le z \le 3L \\ a e^{ikz} , & z > 3L \end{cases} \tag{3}$$

with

$$\lambda = k\sqrt{\delta} = k\sqrt{(\omega_{pe}/\omega)^2 - 1} \ \text{ and } \ k_a = k\sqrt{\varepsilon_a} \tag{4}$$

where the eight coefficients a, b, c_1, c_2, d_1, d_2, f and g are determined from the continuity conditions of E and its derivative at z = 0, L, $2L$, and $3L$. After rather lengthy calculations, we can obtain the transmittance T of electromagnetic waves traversing this Fabry-Perot interferometer. We note that the transmittance T is a function of three parameters, that is, ω/ω_{pe}, ε_a, and $\omega_{pe}L/c$.

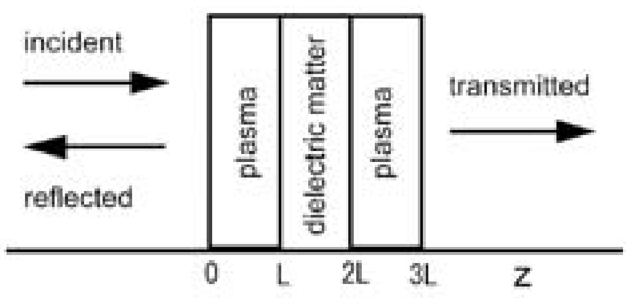

Fig. 1 Schematic of Fabry-Perot interferometer using plasma.

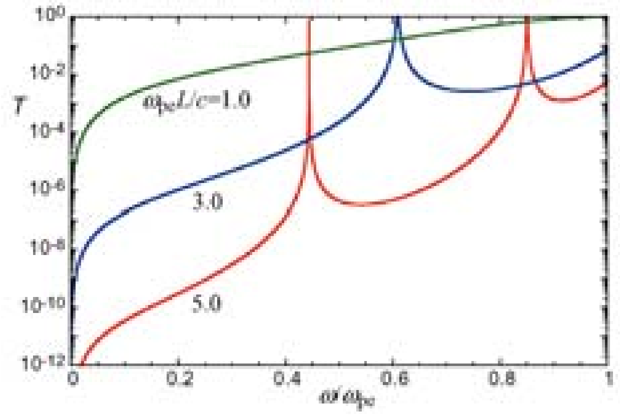

Fig. 2 Transmittance T as a function of ω/ω_{pe} for ε_a = 1 and $\omega_{pe}L/c$ = 1, 3, and 5.

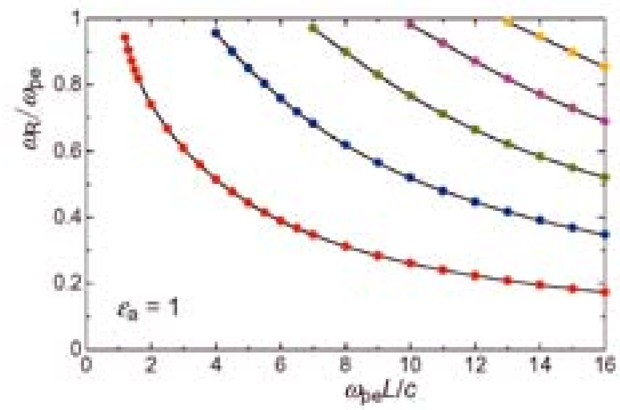

Fig. 3 Resonance frequencies ω_R/ω_{pe} as a function of $\omega_{pe}L/c$ for ε_a = 1.

We first show the wave transmittance T as a function of ω/ω_{pe} for ε_a = 1 (i.e., a vacuum) and three different values of $\omega_{pe}L/c$ (= 1, 3 and 5) in Fig. 2. When $\omega_{pe}L/c$ = 1, the transmittance T monotonously decreases with the decrease of ω/ω_{pe}, and no Fabry-Perot resonances appear. However, Fabry-Perot resonances can arise for $\omega_{pe}L/c$ = 3 and 5. We have one resonance for $\omega_{pe}L/c$ = 3 and two resonances for $\omega_{pe}L/c$ = 5. We see that the number of the resonances increases with the increase of $\omega_{pe}L/c$, and the resonance peak becomes sharper for the larger value of $\omega_{pe}L/c$. In Figure 3, we show the Fabry-Perot resonance frequencies as a function of $\omega_{pe}L/c$ for ε_a = 1. The resonances up to the fifth resonance are shown in the figure. Each resonance frequency decreases with the increase of $\omega_{pe}L/c$.

We next mention a method for determining the electron density of thin plasmas used in the Fabry-Perot interferometer. We concentrate on the first resonance frequency, which is detected primarily by upward frequency sweeping. The first resonance frequency ω_R shown in Fig. 3 can be well fitted by exponential functions as, for $1.2 \le x \le 16$,

$$\omega_R(x) = \omega_{pe}\left[a_0 + \sum_{i=1}^{3} a_i \exp(-x/d_i)\right], \qquad (5)$$

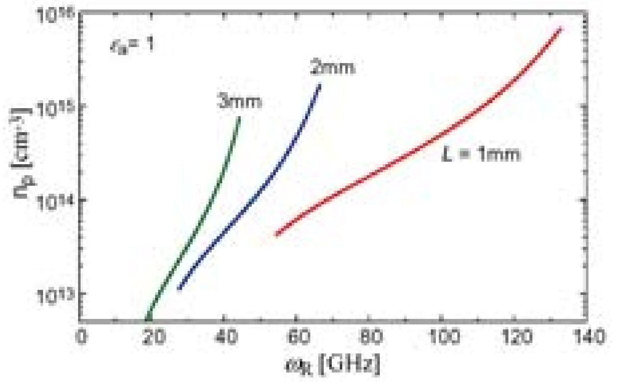

Fig. 4 Plasma density n_p as a function of the first resonance frequency ω_R for ε_a = 1 and L = 1, 2 and 3 mm.

$a_0 = 0.09659,$
$a_1 = 0.56455,$ $d_1 = 2.35393,$
$a_2 = 2.86458,$ $d_2 = 0.32293,$
$a_3 = 0.50245,$ $d_3 = 8.56968,$

where $x = \omega_{pe}L/c$. If the plasma thickness L is known in eq.(5), we can determine the plasma density n_p through ω_{pe} by measuring the first resonance frequency ω_R, because eq.(5) is a function of ω_{pe} and ω_R. In Figure 4, we show the relationship between the plasma density n_p and the first resonance frequency ω_R for ε_a = 1 and different values of L. Thus, we see that we can determine the electron density from the measurement of the first resonance frequency by Fabry-Perot interferometry. It is also found that the resonance frequency shifts to the lower frequency side for the fixed L and n_p when the dielectric constant ε_a increases. Finally, we consider that the present method can be applied to the electron density measurement of micro-plasmas such as PDP plasmas and semiconductor plasmas. However, a more realistic model should be necessary for the electron density measurement of industrial PDP plasmas.

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