Rapid Communications

Reflectionless Transmission of Electromagnetic Wave in One-Dimensional Multi-Layer Plasmas

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The reflectionless transmission of electromagnetic waves in one-dimensional multi-layer plasmas is studied. The wave transmittance is obtained analytically for single-layer underdense plasma as well as for two-layer critical plasma where the wave frequency ω is equal to the electron plasma frequency ω_{pe}, and it is shown that reflectionless transmission can be possible for both cases. Reflectionless transmission in two-layer critical plasma as well as in single-layer underdense plasma should be considered Fabry-Perot resonance well-known in optics.

Keywords:

electromagnetic wave, transmittance, reflectionless transmission, multi-layer plasma, Fabry-Perot resonance

The reflection and transmission of electromagnetic waves in plasma layers is a basic problem in plasma physics, and its solution such as reflectionless transmission is of particular significance in regard to plasma's technological applications.

Here, we study electromagnetic-wave transmission in one-dimensional multi-layer plasma. Electromagnetic waves which are launched into a plasma layer are generally reflected from the plasma; however, it is well-known that waves can be perfectly transmitted without receiving reflections if a certain condition is satisfied. This is called the Fabry-Perot resonance [1]. This phenomenon can be realized for an underdense-plasma layer, that is, $\omega > \omega_{pe}$. For a critical density or overdense plasma layer satisfying $\omega \le \omega_{pe}$, reflectionless transmission does not occur since the waves are not propagating. However, we can show that reflectionless transmission can be possible for multi-layer plasmas even if $\omega = \omega_{pe}$. This should be considered Fabry-Perot resonance.

Our starting point is a one-dimensional Maxwell wave equation given by

$$\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial z^2} + \omega_{pe}^2\right)E(z,t) = 0\,, \tag{1}$$

where c is the speed of light, $\omega_{pe} = (e^2 n_p/\varepsilon_0 m)^{1/2}$, and n_p is a plasma density. For the stationary wave propagation, assuming $E(t) \propto \exp(-i\omega t)$, we obtain

$$\left(\frac{d^2}{dz^2} + \frac{\omega^2 - \omega_{pe}^2}{c^2}\right)E(z) = 0\,. \tag{2}$$

Here, we study electromagnetic-wave transmission based on eq.(2) for the multi-layer plasma shown in Fig.1. We first consider the case of a single-layer plasma ($n = 1$). We assume the plasma density $n_p(z)$ given by

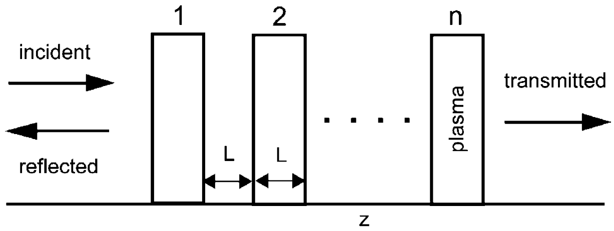

Fig. 1 Schematic of multi-layer plasma

$$n_p(z) = \begin{cases} 0\,, & z < 0 \\ n_0\,, & 0 \le z \le L\,. \\ 0\,, & z > L \end{cases} \tag{3}$$

For $\omega > \omega_{pe}$, the solution of eq.(2) with eq.(3) is given by

$$E = \begin{cases} E_0 e^{ikz} + be^{-ikz}\,, & z < 0 \\ ce^{ik_p z} + de^{-ik_p z}\,, & 0 \le z \le L\,, \\ ae^{ikz}\,, & z > L \end{cases} \tag{4}$$

where $k = \omega/c$, $k_p = (\omega^2 - \omega_{pe}^2)^{1/2}/c$, and E_0 is the incident-wave amplitude. The four coefficients a, b, c, and d are determined from the continuity conditions of E and dE/dz at $z = 0$ and $z = L$. Substituting eq.(4) into the continuity conditions of E and dE/dz, we can obtain the wave transmittance T ($= |a/E_0|^2$) given by

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$$T = \frac{16\alpha^2}{(1+\alpha)^4 + (1-\alpha)^4 - 2(1-\alpha^2)^2 \cos(2kL\alpha)}, \qquad (5)$$

where $\alpha = [1 - (\omega_{pe}/\omega)^2]^{1/2}$. We show the transmittance T as a function of $(\omega_{pe}/\omega)^2$ for kL = 1, 6, and 10 in Fig.2. We see that reflectionless transmission (T = 1) can occur for kL = 6 and 10, which is well-known in optics as the Fabry-Perot resonance [1]. The number of the resonant frequency that corresponds to reflectionless transmission increases with the increase in the width of the plasma layer. For a critical-density plasma satisfying $\omega = \omega_{pe}$, taking a limit of $\alpha \to 0$, we obtain

$$T = \frac{1}{1 + \left(\frac{kL}{2}\right)^2}. \qquad (6)$$

In this case, the transmittance T becomes a monotone decreasing function of kL. That is, the transmittance decreases with the increase of the plasma-layer width.

We next consider the wave transmission in the case of two-layer plasma (n = 2). For the sake of simplicity, we here assume a critical-density plasma with $\omega = \omega_{pe}$. In this case, the solution is given by

$$E = \begin{cases} E_0 e^{ikz} + be^{-ikz}, & z < 0 \\ c_1 z + d_1, & 0 \le z \le L \\ fe^{ikz} + ge^{-ikz}, & L < z < 2L \\ c_2 z + d_2, & 2L \le z \le 3L \\ ae^{ikz}, & z > 3L \end{cases}, \qquad (7)$$

where the coefficients a, b, c_1, c_2, d_1, d_2, f, and g are determined from the continuity conditions of E and its derivative at z = 0, L, 2L, and 3L. By means of a moderately lengthy calculation, we obtain the wave transmittance in the case of two-layer plasma as

$$T = \frac{\mu^4}{1 + (1+\mu^2)^2 - 4\mu \sin\left(\frac{4}{\mu}\right) - 2(1-\mu^2)\cos\left(\frac{4}{\mu}\right)}, \qquad (8)$$

with $\mu = 2/kL$. In this case, the transmittance becomes a function of kL only. In Fig.3, we show the transmittance T as a function of kL. We see that in this case as well, the reflectionless transmission of electromagnetic waves can be possible as shown in Fig.3. This, as in the case of single-layer underdense plasma ($\omega > \omega_{pe}$), should be considered the Fabry-Perot resonance.

We note that the reflectionless transmission of electromagnetic waves due to the Fabry-Perot resonance as shown in Fig.3 can arise for over-dense plasmas with $\omega < \omega_{pe}$ [2], and is also possible for plasmas of diffusive profiles from the analogy with the reflectionless potential scattering discussed in quantum mechanics [3]. A similar method for measuring the electron density of sheet plasmas is proposed in Ref.4, though this method is not concerned with the Fabry-Perot resonance. Such reflectionless transmission due to the Fabry-Perot resonance in multi-layer plasmas can be applied to frequency filters [5,6] and interferometers [7,8] in the micro and millimeter-wave range, and also might be applied to measurements of the electron density of plasma display panel (PDP) plasmas.

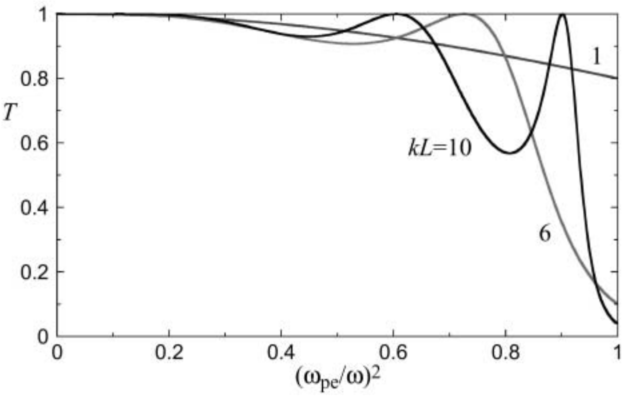

Fig. 2 Transmittance T as a function of $(\omega_{pe}/\omega)^2$ for single-layer plasma, where kL = 1, 6 and 10 ($k = \omega/c$).

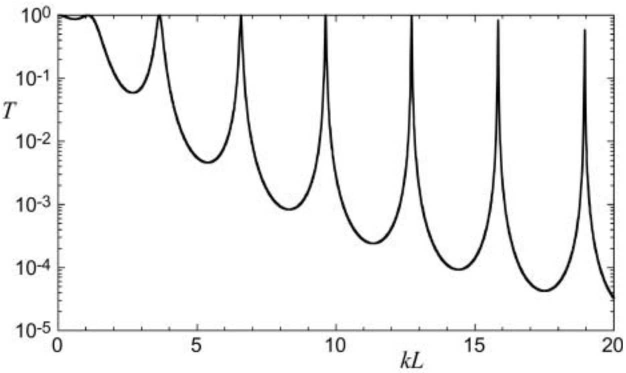

Fig. 3 Transmittance T as a function of kL for two-layer plasma with $\omega = \omega_{pe}$ ($k = \omega/c$).

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[1] M. Born and E. Wolf, *Principles of Optics* (Pergamon Press, New York, 1970).

[2] H. Hojo and A. Mase, Extended Abstracts in Optics Japan 2003 (Hamamatsu, 2003), p.230.

[3] L.D. Landau and E.M. Lifshitz, *Quantum mechanics, non-relativistic theory* (Pergamon Press, 1958).

[4] A. Ejiri and K. Kawahata, Jpn. J. App. Phys. **39**, 2839 (2000).

[5] M. Scalora *et al.*, J. Appl. Phys. **83**, 2377 (1998).

[6] C. Pacher *et al.*, Appl. Phys. Lett. **79**, 1486 (2001).

[7] R.L. Waters and M.E. Aklufi, Appl. Phys. Lett. **81**, 3320 (2002).

[8] Yu.V. Shvyd'ko *et al.*, Phys. Rev. Lett. **90**, 013904 (2003).