Reflectionless Transmission of Electromagnetic Wave in One-Dimensional Multi-Layer Plasmas

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The reflectionless transmission of electromagnetic waves in one-dimensional multi-layer plasmas is studied. The wave transmittance is obtained analytically for single-layer underdense plasma as well as for two-layer critical plasma where the wave frequency \( \omega \) is equal to the electron plasma frequency \( \omega_{pe} \), and it is shown that reflectionless transmission can be possible for both cases. Reflectionless transmission in two-layer critical plasma as well as in single-layer underdense plasma should be considered Fabry-Perot resonance well-known in optics.

Keywords: electromagnetic wave, transmittance, reflectionless transmission, multi-layer plasma, Fabry-Perot resonance

The reflection and transmission of electromagnetic waves in plasma layers is a basic problem in plasma physics, and its solution such as reflectionless transmission is of particular significance in regard to plasma’s technological applications.

Here, we study electromagnetic-wave transmission in one-dimensional multi-layer plasma. Electromagnetic waves which are launched into a plasma layer are generally reflected from the plasma; however, it is well-known that waves can be perfectly transmitted without receiving reflections if a certain condition is satisfied. This is called the Fabry-Perot resonance [1]. This phenomenon can be realized for an underdense-plasma layer, that is, \( \omega > \omega_{pe} \). For a critical density or overdense plasma layer satisfying \( \omega \leq \omega_{pe} \), reflectionless transmission does not occur since the waves are not propagating. However, we can show that reflectionless transmission can be possible for multi-layer plasmas even if \( \omega = \omega_{pe} \). This should be considered Fabry-Perot resonance.

Our starting point is a one-dimensional Maxwell wave equation given by

\[
\left( \frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial z^2} + \omega^2_{pe} \right) E(z, t) = 0, \tag{1}
\]

where \( c \) is the speed of light, \( \omega_{pe} = (e^2 n_p / \varepsilon_0 m)^{1/2} \), and \( n_p \) is a plasma density. For the stationary wave propagation, assuming \( E(t) \propto \exp(-i\omega t) \), we obtain

\[
\left( \frac{d^2}{dz^2} + \frac{i\omega - \omega_p^2}{c^2} \right) E(z) = 0. \tag{2}
\]

Here, we study electromagnetic-wave transmission based on eq.(2) for the multi-layer plasma shown in Fig.1. We first consider the case of a single-layer plasma (\( n = 1 \)). We assume the plasma density \( n_p(z) \) given by

\[
n_p(z) = \begin{cases} 0, & z < 0 \\ n_0, & 0 \leq z \leq L \\ 0, & z > L \end{cases} \tag{3}
\]

For \( \omega > \omega_{pe} \), the solution of eq.(2) with eq.(3) is given by

\[
E = \begin{cases} E_0 e^{ikz} + be^{-ikz}, & z < 0 \\ ce^{ikz} + de^{-ikz}, & 0 \leq z \leq L \\ ae^{ikz}, & z > L \end{cases} \tag{4}
\]

where \( k = \omega/c \), \( k_p = (\omega^2 - \omega_{pe}^2)^{1/2}/c \), and \( E_0 \) is the incident-wave amplitude. The four coefficients \( a, b, c, \) and \( d \) are determined from the continuity conditions of \( E \) and \( dE/dz \) at \( z = 0 \) and \( z = L \). Substituting eq.(4) into the continuity conditions of \( E \) and \( dE/dz \), we can obtain the wave transmittance \( T (= |aE_0|^2) \) given by
$T = \frac{16\alpha^2}{(1 + \alpha^4 + (1 - \alpha^4 - 2(1 - \alpha^2)\cos(2kL\alpha))} \quad (5)$

where $\alpha = [1 - (\omega_0/\omega)^2]^{1/2}$. We show the transmittance $T$ as a function of $(\omega_0/\omega)^2$ for $kL = 1, 6,$ and 10 in Fig.2. We see that reflectionless transmission ($T = 1$) can occur for $kL = 6$ and 10, which is well-known in optics as the Fabry-Perot resonance [1]. The number of the resonant frequency that corresponds to reflectionless transmission increases with the increase in the width of the plasma layer. For a critical-density plasma satisfying $\omega = \omega_{pe}$, taking a limit of $\alpha \to 0$, we obtain

$$T = \frac{1}{1 + \left(\frac{kL}{2}\right)^2}. \quad (6)$$

In this case, the transmittance $T$ becomes a monotone decreasing function of $kL$. That is, the transmittance decreases with the increase of the plasma-layer width.

We next consider the wave transmission in the case of two-layer plasma ($n = 2$). For the sake of simplicity, we here assume a critical-density plasma with $\omega = \omega_{pe}$. In this case, the solution is given by

$$E = \begin{cases} 
E_0 e^{ikz} + be^{-ikz}, & z < 0 \\
 c_1 z + d_1 , & 0 \leq z \leq L \\
 f e^{ikz} + ge^{-ikz}, & L < z < 2L \\
 c_2 z + d_2 , & 2L \leq z \leq 3L \\
 ae^{ikz}, & z > 3L 
\end{cases} \quad (7)$$

where the coefficients $a, b, c_1, c_2, d_1, d_2, f,$ and $g$ are determined from the continuity conditions of $E$ and its derivative at $z = 0, L, 2L,$ and 3$L$. By means of a moderately lengthy calculation, we obtain the wave transmittance in the case of two-layer plasma as

$$T = \frac{\mu^4}{1 + (1 + \mu^2)^2 - 4\mu \sin\left(\frac{4\mu}{\mu}\right) - 2(1 - \mu^2)\cos\left(\frac{4\mu}{\mu}\right)}. \quad (8)$$

with $\mu = 2kL$. In this case, the transmittance becomes a function of $kL$ only. In Fig.3, we show the transmittance $T$ as a function of $kL$. We see that in this case as well, the reflectionless transmission of electromagnetic waves can be possible as shown in Fig.3. This, as in the case of single-layer underdense plasma ($\omega > \omega_{pe}$), should be considered the Fabry-Perot resonance.

We note that the reflectionless transmission of electromagnetic waves due to the Fabry-Perot resonance as shown in Fig.3 can arise for over-dense plasmas with $\omega < \omega_{pe}$ [2], and is also possible for plasmas of diffusive profiles from the analogy with the reflectionless potential scattering discussed in quantum mechanics [3]. A similar method for measuring the electron density of sheet plasmas is proposed in Ref.4, though this method is not concerned with the Fabry-Perot resonance. Such reflectionless transmission due to the Fabry-Perot resonance in multi-layer plasmas can be applied to frequency filters [5,6] and interferometers [7,8] in the micro and millimeter-wave range, and also might be applied to measurements of the electron density of plasma display panel (PDP) plasmas.

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