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Analysis of Disk AC MHD Generator Performance by Finite Element Method

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This paper studies the possibility to generate alternating current (AC) electrical power by using disk MHD generator. Its phenomenon is performed by the finite element method (FEM). The configuration of generator consists of a channel, an insulator and stators. The shape of channel is a flat disk, the liquid metal used as conductor flows inside the channel. As a channel wall, an insulator is used to separate the metal fluid and stator. The stator winding is designed as the coils on top and bottom of a channel. Under this condition, it can produce a magnetic field by means of time harmonics function. The interaction between the metal fluid and the electromagnetic wave can be explained by Maxwell's equations and Ohm's law. As a result, the distribution of magnetic flux density throughout the channel is evidently shown in two-dimensional space. The active power is limited by slip value and electrical conductivity. The optimized value of active and reactive power is suggested by small slip value as slip $\boldsymbol{s}\leq$-0.1. In addition, mechanical power and power dissipation in the generator are reported in term of 1-$\boldsymbol{s}$ and $\boldsymbol{s}$ respectively. Electrical efficiency is explained in term of 1/(1-$\boldsymbol{s}$).

Keywords: MHD Generator   an AC MHD Generator   MHD induction generator

1. Introduction

Recently, Magnetohydrodynamics (MHD) generator is effectively utilized to generate electrical energy because of its simple structure and could directly converts thermal energy into electrical energy. An MHD generator can produce high power density with low environmental issues. On the other hand, an AC MHD generator is one type of MHD generator that generates an AC power without an inverter. It performs under the interaction between traveling wave and electrically conducting fluid. The stator winding is served as a step-up transformer by increasing terminal voltage and also reducing armature current. As a result, operating characteristics and design considerations of an AC MHD generator have been proposed by W.D. Jackson and E.S. Pierson in 1965 [1,2]. An annular and a linear type of the generator are considered under the condition of metal fluid and magnetic field. The metal fluid is eutectic sodium potassium (Nak). It has high electrical conductivity. W.D. Jackson and E.S. Pierson suggested that the performance of an AC MHD generator depends on magnetic Reynolds number based on the wavelength and the value of slip. Afterward, the theory and experiment of a linear AC MHD generator using liquid metal has been proposed by T.C. Wang and S.J. Dudzinsky in 1967 [3,4]. with different approach for calculate. However, most of all structures are considered with double-side exciting winding to generate AC power. In addition, the quasi-one-dimensional technique also applied to investigate it.

This paper proposes the method to generate AC power by the disk MHD generator. The performance of the generator is explained by finite element method. A single-side exciting winding of MHD generator is considered. Its configuration contains a channel, an insulator and stators. The shape of channel is a flat disk. The liquid metal is acted as a conductor flowing inside the channel. Channel wall acted as an insulator separates metal fluid and stator coils. The stator winding is designed as the coils on top and bottom of channel. In this condition, top stator winding can produce a magnetic field by means of time harmonics function in the same direction with metal fluid. On the contrary, bottom stator winding is used as an inductive coil. The behavior of interaction between the metal fluid and electromagnetic wave can be explained by Maxwell's equations and Ohm's law based on the method of finite-elements. The second order of triangular finite elements is utilized to calculate the magnetic vector potential. The distribution of magnetic vector potential and magnetic flux density throughout channel is evidently shown in two-dimension space. Active power on top and the bottom of stator surface is real part of the surface integrating Poynting's

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vector over the fluid surface. While reactive power is defined by imaginary part of integrating Poynting's vector over the fluid surface. Finally, mechanical power, power dissipation and electrical efficiency are considered by induction machine equations.

2. Basic of an AC MHD generation

The principle of an AC MHD generator operation is the same as that of a polyphase induction generator. In an induction generator, a rotating magnetic field (B) is generated by a distributed polyphase winding. The rotating field induces a voltage (E) in conductors imbedded in the periphery of the cylindrical rotor. Hence, the current density in the conductor depends on electrical conductivity (σ) of the conductor in term of σE. In addition, induced voltage in conductor can be changed with varying velocity (U) of conductor. Consequently, the induced voltage in the conductor is explained by cross product of velocity of rotor and magnetic flux density ($U\times B$). The current density in conductor is replaced in terms of $\sigma(U\times B)$. Therefore, the induced voltage in the conductor due to moving conductor in time varied field is replaced by $E_{ind}=E+U\times B$. The current density in conductor is explained by Ohm law in term of $J_{ind}=\sigma(E+U\times B)$. The energy in the conductor is transferred into electrical load by connecting the conductor.

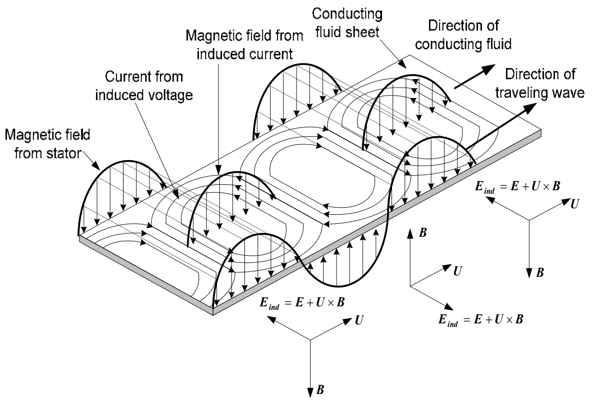

Fig.1 Basic of AC MHD generation.

Figure 1 shows the basic of linear AC MHD generator. The stator of AC MHD generator similarly acts as in the general induction generator. However, its windings are distributed in a flat magnetic structure. Therefore, the motion of the field is a straight line rather than rotating as in a generator. The rotor is replaced by a layer of metal fluid confined in an envelope of rectangular cross section of the channel. Induced current in the metal fluid depends on the current density in terms of $J_{ind}=\sigma(E+U\times B)$. Therefore, the induced current generates the magnetic field in term of time harmonics function. Also, energy from the interaction between magnetic field and metal fluid can be extracted by inductive coils.

3. Analysis model

The MHD schematic of the single-side disk AC MHD generator is shown in Fig.2. The configuration of disk channel is symmetry. Also, right hand side of disk channel is investigated and applied to make the model of AC MHD generator. The model used for numerical simulation is illustrated in Fig.3. The area of metal fluid (I), channel wall (II) and stator (III) is designed in the Cylindrical coordinate system. The metal fluid flows along r-direction with constant velocity (U_f). Besides, the electrical conductivity and permeability are replaced by conductivity $\sigma=\sigma_f$ and permeability $\mu=\mu_f$, respectively. Channel walls act as insulators separate the metal fluid and the stator. Also, the electrical conductivity and permeability are replaced by conductivity $\sigma=\sigma_i$ and permeability $\mu=\mu_i$, respectively. Electrical conductivity and permeability of stator are defined by $\sigma=\sigma_c$ and $\mu=\mu_c$, respectively. The top stator winding is connected with three phase system which produces traveling wave velocity (U_s) along r-direction. In this paper, the top stator winding is replaced by surface current density. On the other hand, energy from interaction between magnetic field and metal fluid is transferred to electrical load by inductive coils that installed at bottom stator.

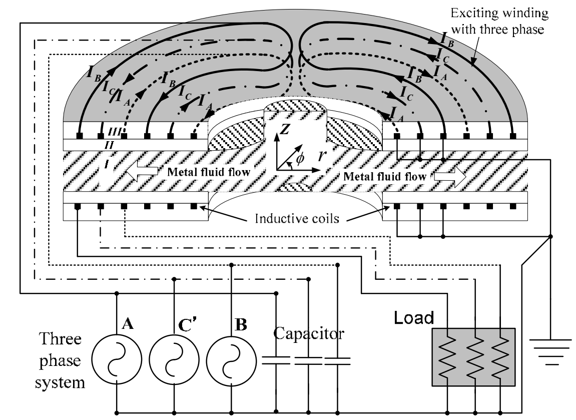

Fig.2 Configuration of the disk AC MHD generator.

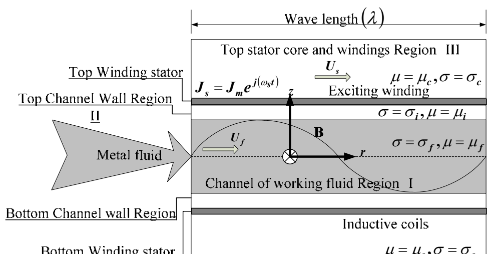

Fig.3 The model of an AC MHD generator.

The phenomenon of AC MHD generator is explained by Maxwell's equation. It can be evaluated by the following formulas, while a displacement current density is neglected:

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t \tag{1}$$
$$\nabla \times \mathbf{H} = \mathbf{J} \tag{2}$$
$$\nabla \cdot \mathbf{B} = 0 \tag{3}$$
$$\nabla \cdot \mathbf{J} = 0 \tag{4}$$

where

E is electric field intensity
H is magnetic field intensity
B is magnetic flux density
J is current density

By Ohm's law neglecting Hall effect, the current density equation is given by

$$\mathbf{J} = \sigma\left(\mathbf{E} + \boldsymbol{U}_f \times \mathbf{B}\right). \tag{5}$$

The relationship between magnetic flux density and vector magnetic potential is explained by

$$\mathbf{B} = \nabla \times \mathbf{A}. \tag{6}$$

The components of magnetic flux density (**B**) in the Cylindrical coordinates, are replaced by $\mathbf{B}_r=-\partial\mathbf{A}/\partial z$, $\mathbf{B}_z=-\partial(r\mathbf{A})/r\partial \boldsymbol{r}$ and $B_\phi=0$. Also, the electric field intensity is expressed as

$$\mathbf{E} = \frac{-\partial \mathbf{A}}{\partial t}, \tag{7}$$

where **A** is magnetic vector potential. Substituting (6) and (7) into (5), then a current density is defined by Ohm's law as follow

$$\nabla \times \nabla \times \mathbf{A} = \mu_0 \sigma\left(\frac{-\partial \mathbf{A}}{\partial t} + \boldsymbol{U}_f \times \nabla \times \mathbf{A}\right). \tag{8}$$

From (8), D is displacement current as zero when $\nabla \cdot$ D, and $\nabla(\nabla \cdot \mathbf{A})=0$, is obtained that

$$\nabla^2 \mathbf{A} - \mu_0 \sigma\left(\frac{\partial \mathbf{A}}{\partial t} - \boldsymbol{U}_f \times \nabla \times \mathbf{A}\right) = 0. \tag{9}$$

4. Finite element formulation

The AC MHD equation is evaluated in finite element method. The magnetic vector potential (**A**) is defined by $\mathbf{A}(r,z,t)=rA_m(z)\mathbf{exp}(j\omega_s t)\boldsymbol{a}_\phi$ where r is radius of disk generator. A_m is magnitude of magnetic vector potential, $\boldsymbol{a}_\phi$ is unit vector along ϕ-direction, ω_s is angular velocity. In addition, an electromagnetic source generating by the stator winding is replaced in term of $\boldsymbol{J}_s=J_m\mathbf{exp}(j\omega_s t)\boldsymbol{a}_\phi$. Also, equation (9) can be described by

$$\nabla^2 \boldsymbol{A}_m - \left(\boldsymbol{j}\mu\sigma\omega_s + \frac{\mu\sigma\boldsymbol{U}_f}{r}\right)\boldsymbol{A}_m + \left(\frac{1}{r} - \mu\sigma\boldsymbol{U}_f\right)\frac{\partial \boldsymbol{A}_m}{\partial \boldsymbol{r}} - \boldsymbol{J}_m = 0, \tag{10}$$

where $\omega_s=2\pi f$, f is the electromagnetic wave frequency, j is imaginary part of complex number and J_m is an amplitude current density at stator winding. The relationship between angular velocity and wave velocity is defined as

$$\boldsymbol{U}_s = \frac{\omega_s \lambda}{2\pi}. \tag{11}$$

where λ is wavelength. Also, the relationship between wave velocity ($\boldsymbol{U}_s$) and metal fluid velocity ($\boldsymbol{U}_f$) is represented by the slip value ($\boldsymbol{s}$) as

$$\boldsymbol{s} = \frac{\boldsymbol{U}_s - \boldsymbol{U}_f}{\boldsymbol{U}_s}. \tag{12}$$

The appearance of different frequencies in exciting winding and metal fluid is explained by $\omega_f=\boldsymbol{s}\omega_s$. Also, substituting (11) and (12) into (10), then a magnetic vector potential is defined by

$$\nabla^2 \boldsymbol{A}_m - \left(\boldsymbol{j}\mu\sigma\boldsymbol{s}(\omega_s) + \frac{\mu\sigma\boldsymbol{s}(1-\boldsymbol{s})\omega_s}{r}\right)\boldsymbol{A}_m + \left(\frac{1}{r} - \mu\sigma\boldsymbol{s}(1-\boldsymbol{s})\omega_s\right)\frac{\partial \boldsymbol{A}_m}{\partial \boldsymbol{r}} - \boldsymbol{J}_m = 0. \tag{13}$$

From (13), the magnetic vector potential is a function of the slip value. The standard algorithm [5] of finite element analysis is referred in this paper.

5. Power flow in AC MHD generator

The power flow in AC MHD generator is evaluated by using electrical power. The mechanical power is the pressure gradient through the channel by applying electromagnetic force. Furthermore, an electrical power is calculated by integrating Poynting's vector over the fluid surface (Γ). The active power is evaluated by

$$P = -\mathbf{Re}\left(\frac{1}{2}\int_{\Gamma} \boldsymbol{E}_z \boldsymbol{H}^* \boldsymbol{d\Gamma}\right), \tag{14}$$

where $\boldsymbol{d\Gamma}$ is channel surface and $\boldsymbol{H}^*$ is complex conjugate of magnetic field intensity. The reactive power of generator is defined by imaginary part of integrating Poynting's vector over the fluid surface as

$$Q = -\mathbf{Im}\left(\frac{1}{2}\int_{\Gamma} \boldsymbol{E}_z \boldsymbol{H}^* \boldsymbol{d\Gamma}\right). \tag{15}$$

The mechanical power is the relationship between electrical power and slip. It is calculated by

$$P_m = -\mathbf{Re}\left(\frac{1}{2}\int_{\Gamma} \boldsymbol{E}_z \boldsymbol{H}^* \boldsymbol{d\Gamma}\right)(1-\boldsymbol{s}). \tag{16}$$

Moreover, the power dissipation from generator can be defined by

$$P_d = -\mathbf{Re}\left(\frac{1}{2}\int_{\Gamma} \boldsymbol{E}_z \boldsymbol{H}^* \boldsymbol{d\Gamma}\right)\boldsymbol{s}, \tag{17}$$

where P_m is the mechanical power in fluid based on electromagnetic force. P_d is the power dissipation in the generator. Thus, electrical efficiency of induction generator is given by

$$\eta = P/P_m = 1/(1-\boldsymbol{s}). \tag{18}$$

Therefore, the electrical efficiency (η) is determined by only the fluid electrical losses in channel.

6. Numerical results and discussions

The AC MHD generator configuration is designed by conditions of constant fluid velocity throughout the channel illustrated in Fig. 4. The electrical conductivity and permeability are replaced by electrical conductivity of a eutectic sodium potassium as $\sigma_f = 1\times10^6$S/m and a permeability of free space as $\mu_f = \mu_0 = 4\pi\times10^{-7}$H/m, respectively. The channel walls are electrical insulator, conductivity and permeability are defined by $\sigma_i = 0$ and $\mu_i = \mu_0$, respectively. Electrical conductivity and permeability of stators as laminated steel are defined by $\sigma_c = 0$ and $\mu_c = 2000\mu_0$ respectively. The stator on top is an exciting winding with traveling wave $\boldsymbol{J}_s = 11.52\mathbf{exp}(j\omega_s t)$ A/m^2 along r-direction with $\lambda = 0.154$m. Physical parameters in the simulation model are listed in Table 1. This model is divided into 792 elements and 1665 triangular element nodes. The second-order is considered in this paper. In case of boundary condition, the magnetic vector potential at the edges of bottom and top stator is defined by zero (**A**=0)

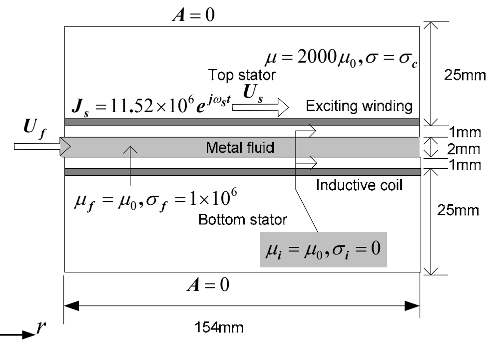

Fig.4 Configuration of AC MHD generator consideration.

Table 1. Parameter for consideration.

Parameters	Value
Power line frequency (Hz)	50
Permeability of metal fluid (H/m)	$4\pi\times10^{-7}$
Permeability of iron core (H/m)	$2000\times 4\pi\times10^{-7}$
Permeability of insulator (H/m)	$4\pi\times10^{-7}$
Electrical conductivity of metal fluid (S/m)	1×10^6
Current density (A/m^2)	11.52x10^6
Wave length (m)	0.154
Wave velocity (m/s)	7.7
Magnetic Reynolds number	0.24

The numerical simulation is investigated the model of AC MHD generator which is produced AC power from interaction between traveling wave and metal fluid. The power flow in AC MHD generator is evaluated under the ratio of the metal fluid velocity and the electromagnetic wave velocity which is the slip value in term of $\boldsymbol{s} = (U_s - U_f)/U_s$. In this paper, the range of the slip value is defined by $-1 \le \boldsymbol{s} \le 1$. In case of the negative slip value, the velocity of metal fluid is higher than two time of the velocity of electromagnetic wave. The maximum active power of -2.54kW at $\boldsymbol{s}$=-0.6 with $\sigma_f = 1\times10^6$S/m is shown in Fig.5. The negative active power is expressed by generating electrical power in the MHD channel. On the contrary, if the velocity of traveling wave is higher than metal fluid velocity, the slip value is positive. This phenomenon acts as an accelerator. The metal fluid is accelerated by electromagnetic force. The maximum power used for propulsion is 1.77kW at slip $\boldsymbol{s}$=0.7 with electrical conductivity $\sigma_f = 1\times10^6$S/m. Nevertheless, in the range of the slip value $-0.1 \le \boldsymbol{s} \le 0.1$, this area has low active power; the generator is acted as accelerator. As a result, the AC MHD generator is designed with small slip value $s \leq -0.1$. In the same way, if generator is acted as an accelerator. It is designed with high slip value $s \geq 0.1$.

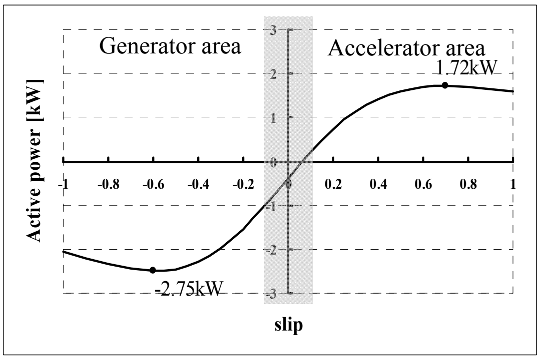

Fig.5 The relationship between active power and slip.

Figures 6 and 7 show magnetic flux density distribution at s=-0.6 and s=0.7, respectively. The magnetic field not only distributes in the channel but also induces high magnetic flux density at the metal surface.

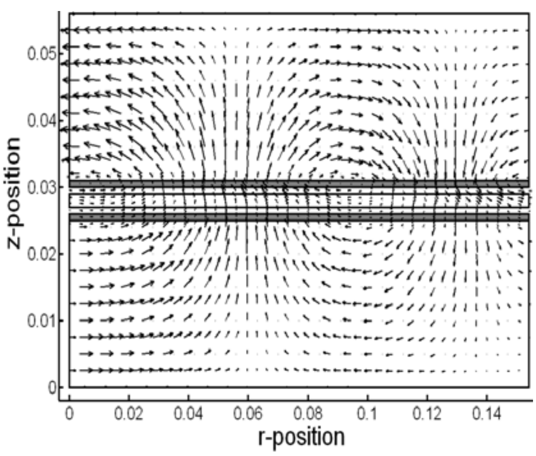

Fig.6 Magnetic flux density distribution at s=-0.6.

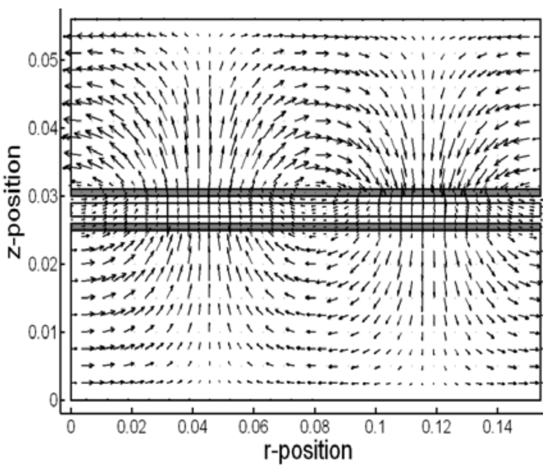

Fig.7 Magnetic flux density distribution at s=0.7.

Figure 8 shows the relationship between reactive power and slip. The optimized range of slip is evaluated from the reactive power in application. As a result, the maximum reactive power as 39kVAR is around the slip value at the origin. This slip value should not be used generator and accelerator. For maximum active power, maximum reactive power in case of generator and accelerator is 24.22kVAR and 29.32kVAR, respectively. Also, the reactive power should be compensated by connecting a capacitor at the input terminal.

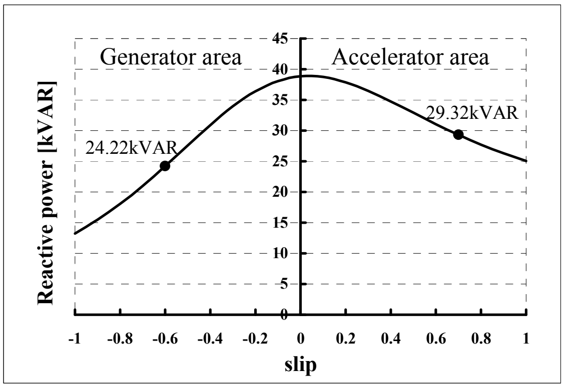

Fig.8 The relationship between reactive power and slip.

Figure 9 shows the relationship between mechanical power and slip. The power used to moving the metal fluid into the channel is defined by mechanical power. It is calculated by the induction machine equations based on the slip. In case of positive mechanical power, the phenomenon acts as generator, the mechanical power is increases with 1-s time of the active power. The mechanical power is 4kW with s=0.6. However, In case of negative mechanical power, the phenomenon acts as an accelerator. Metal fluid is also accelerated by the electromagnetic force. The mechanical power is reduces with 1-s time of the active power. The power is -0.52kW with s=0.7. It suggests that the mechanical power of the generator is higher than the accelerator with constant active power at metal surface.

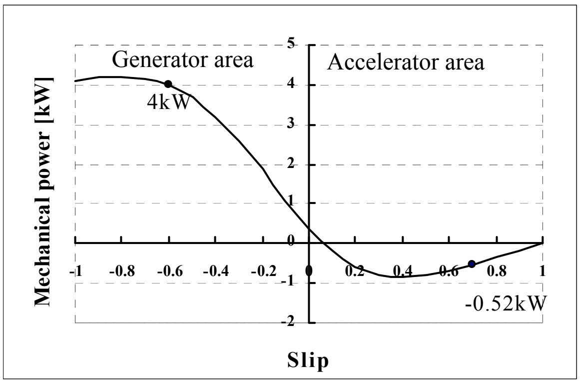

Fig.9 The relationship between mechanical power and slip.

Loss in the generator is explained by the power dissipation and slip in Fig. 10. The power dissipation is evaluated by product of slip value with an active power. In case of generator, the maximum power dissipation increases with small slip value. Since, high magnetic flux density appears at the surface of metal fluid near exciting winding. Power dissipation is 1.5kW with s=-0.6. In case of an accelerator, the maximum power dissipation is increases with increasingly value of slip. The maximum power dissipation is 1.2kW with s=0.7.

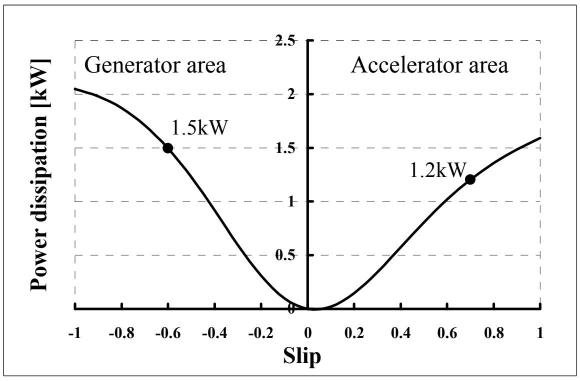

Fig.10 The relationship between power dissipation and slip.

The electrical efficiency is evaluated only under the fluid electrical losses. Figure 11 shows the electrical efficiency of AC MHD generator and accelerator depend on value of 1/(1-s) and (1-s), respectively. In case of generator, the maximum active power with -2.54kW at s=-0.6 and $\sigma_f=1\times10^6$S/m is shown in Fig. 6. At this point, electrical efficiency is 62.5%. In case of accelerator, the maximum active power with 1.72kW at s=0.7 and $\sigma_f=1\times10^6$S/m is shown in Fig. 5. At this point, electrical efficiency is 30%.

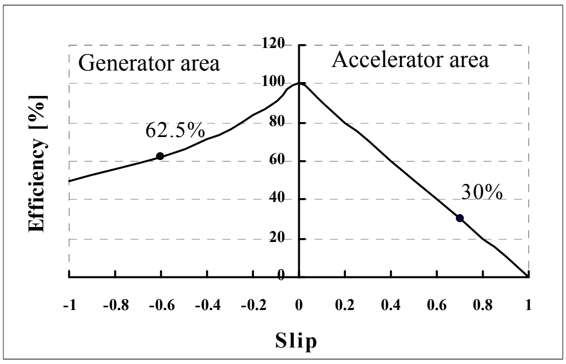

Fig.11 The relationship between electrical efficiency and slip.

The relationship between absolute maximum active power and electrical conductivity of the metal fluid is shown in Fig. 12. The optimum value of the electrical conductivity is defined by $\sigma_f=5\times10^6$S/m for active power 2.7kW. However, electrical conductivity is less than $\sigma_f=5\times10^5$S/m, a maximum active power decreases. As a result, the generator has high power dissipation. Moreover, an electrical conductivity is higher than $\sigma_f=5\times10^7$S/m, a maximum active power decreases. Thus, an electromagnetic induction appears on the metal surface. However, a maximum reactive power is constant

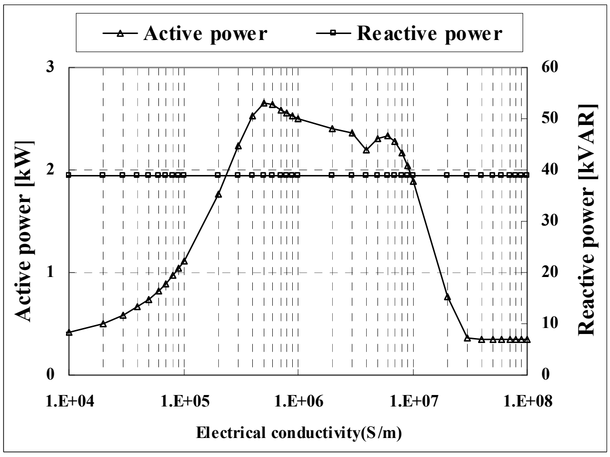

Fig.12 The relationship between active power, reactive power and electrical conductivity.

7. Conclusions

This paper has proposed the possibility to produce an AC power by disk MHD generator with a single-side exciting. Its performance is evaluated by finite element method. The result confirms that the distribution of magnetic field appears at the metal surface and high magnetic flux density. The active power of interaction between traveling magnetic field and metal fluid is limited by the slip value and electrical conductivity of the metal fluid. The optimized value of an active and reactive power of the generator is suggested by small slip value as $s\leq$-0.1. Maximum power in this system is suggested with active power (-2.54kW), reactive power (24.22kVAR), mechanical power (4kW), power dissipation (1.5kW) and efficiency (62.5%). In addition, active power is function of electrical conductivity. However, AC MHD generator is adjusted to accelerator by $s\geq$0.1. Maximum power in this system is performed by active power (1.72kW), reactive power (29.32kVAR), mechanical power (-0.52kW), power dissipation (1.2kW) and efficiency (30%).

8. References

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