

Visualization of energy transfer to magnetic energy by rolling-up vortices with uniform background magnetic field

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For the purpose of clarifying mechanism of local structure formations in magnetohydrodynamic (MHD) turbulence, energy transfers among various scales and positions of magnetic/kinetic energies in the course of roll-up processes of vortices are studied by direct numerical simulations (DNS) and orthonormal wavelet analysis. In the previous study information on scales provided by the wavelet analysis are used to study scale-to-scale energy transfers[1]. It has been found that large scale flow structures directly excite magnetic fields with various scales. In the present study energy exchange between the kinetic and magnetic field energies are examined from the points of views of local analysis of scale-location-to-scale-location energy transfers. Cone representation analysis we have developed in Ref.[2] is extended to treat magnetic induction process. To depict the energy transfer cones with the isosurfaces of the vorticity and current distributions simultaneously, spatial features of intense energy transfer and its relation to the dominant field structures are clearly visualized.

Keywords: MHD turbulence, wavelet analysis, decomposition of energy transfer, visualization of dynamics.

1. Introduction

Turbulent motions of plasmas are considered to play key roles in magnetic confinement system. For example, the plasma and energy transports across the magnetic field lines are closely related to turbulence[3]. In terms of wavelet analysis, Kishida et al. have found that local interaction dominates the transfer process in fully developed turbulence of a neutral fluid[4]. For MHD fluids, Alexakis et al. have carefully conducted DNS studies and investigated turbulent energy transfer processes between the velocity and magnetic fields and between larger and smaller scales [5] and similar analysis is carried out for Hall-MHD case [6]. They found that nonlocal interaction is important for the energy supplying process from the kinetic energy to the magnetic one.

In our previous study we studied the wavelet scale-to-scale interaction of nonlinear and induction terms. It has been found that large scale flow structures directly excites magnetic fields with various scales, in other words, non-local interaction dominates excitation process[1]. Though we only use the spatial scale information of wavelets in that work, wavelet analysis strongly suggests that coherent structures, i.e., rolling-up of large scale vortices are relevant to the magnetic induction process.

In the present study we will attempt to use the location information of wavelets and to analyze the relation between the coherent structures and magnetic induction process. One of the advantages of the wavelet analysis, compared with the Fourier one, is that it captures the information of spatial scale and location simultaneously. Another advantage of the analysis with discrete wavelet transform, compared with the band-pass filter analysis such as Gabor transform, is that it provides exact mode expansion of the

basic equations without any redundancy or insufficiency of modes. Thus wavelet representation of the basic equations is suitable for discussing the dynamics quantitatively in terms of mode interactions as well as in relation to the geometrical features of fields. Therefore wavelet analysis is a useful tool for the analysis of each snapshot of the fields in a time series. For rolling-up process of vortices in a neutral fluid, we have already clarified that the nonlinear energy transfer occurs actively around the large scale vortices with the aid of dynamics visualization technique which we called “cone representation”[2]. In the analysis of MHD dynamics, similar visualization technique for understanding dynamical process is needed.

2. Basic equations

Incompressible MHD equations in dimensionless form are described as

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \nabla p + \mathbf{j} \times \mathbf{B} + \nu \nabla^2 \mathbf{u}, \quad (1)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \quad (2)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (3)$$

where \mathbf{B} is the magnetic field (normalized by a representative value B_0), $\mathbf{j} = \nabla \times \mathbf{B}$ is the current (normalized by B_0/L_0 ; L_0 is the characteristic length), \mathbf{u} is the velocity (normalized by the Alfvén speed $V_A = B_0/\sqrt{\mu_0 n_i M_i}$; μ_0 is the permeability of vacuum, M_i is the ion mass and n_i is the ion number density, which is assumed to be constant for simplicity), ν is the viscosity and η is the resistivity (normalized by $V_A L_0$), and p is the pressure (normalized by B_0^2/μ_0). The pressure p is given as the solution of the Poisson equation which comes from the divergence of eq.(1).

Numerical methods, simulation parameters, the initial condition of DNS and the snapshot of the velocity and

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magnetic fields we will analyze in the present study are same as those in Ref.[1].

3. Wavelet Decomposition And Scale-location Spectrum

Orthonormal divergence-free wavelet $\psi_{j\vec{l}\sigma}(\vec{x})$ is given by the unitary transform of complex helical waves and has four kinds of parameters, $j, \vec{l} = (l_x, l_y, l_z), \epsilon, \sigma$ each of which implies spatial scale, location, anisotropy in wave number space and helicity, respectively[7]. Though the choice of wavelet is arbitrary in this method, we will use Meyer's wavelet here.

In the present study we use the scale and location information of wavelets and reduces the anisotropy and helicity information by taking summation with respect to ϵ and σ :

$$f_{j\vec{l}}(\vec{x}, t) := \sum_{\epsilon, \sigma} \tilde{f}_{j\epsilon\vec{l}\sigma}(t) \psi_{j\epsilon\vec{l}\sigma}(\vec{x}) \quad (4)$$

where f stands for \mathbf{u} and \mathbf{B} and $\tilde{f}_{j\epsilon\vec{l}\sigma}(t)$ is wavelet coefficient which is given by $\tilde{f}_{j\epsilon\vec{l}\sigma}(t) := \int f(\vec{x}) \cdot \psi_{j\epsilon\vec{l}\sigma}(\vec{x}) d^3\vec{x}$. In the following we call the field $f_{j\vec{l}}(\vec{x}, t)$ the *scale-location spectrum* of f . Velocity and magnetic fields are decomposed into the scale-location spectrum as follows:

$$f(\vec{x}, t) = \sum_{j, \vec{l}} f_{j\vec{l}}(\vec{x}, t). \quad (5)$$

Since the wavelet is orthogonal, the kinetic and the magnetic energies are decomposed into the sum of the energy of each scale-location spectrum, i.e.,

$$E^{(f)}(t) = \sum_{j, \vec{l}} E_{j\vec{l}}^{(f)}(t) \quad (6)$$

where $E_{j\vec{l}}^{(f)}(t) := \frac{1}{2} \int |f_{j\vec{l}}(\vec{x}, t)|^2 d^3\vec{x} = \frac{1}{2} \sum_{\epsilon, \sigma} |\tilde{f}_{j\epsilon\vec{l}\sigma}(t)|^2$.

Substituting wavelet scale-location expansion of \mathbf{u} and \mathbf{B} into the basic equations Eqs.(1) and (2) and taking inner product with each of the scale-location spectrum $\mathbf{u}_{j\vec{l}}$ and $\mathbf{B}_{k\vec{m}}$, respectively, one obtains the energy budget equations for the scale-location spectra of the kinetic and magnetic energies as follows:

$$\begin{aligned} \frac{\partial E_{j\vec{l}}^{(u)}}{\partial t} &= \sum_{k\vec{m}} \langle \mathbf{u}_{j\vec{l}} | \mathbf{u}_{k\vec{m}} \rangle_{NL} + \sum_{k\vec{m}} \langle \mathbf{u}_{j\vec{l}} | \mathbf{B}_{k\vec{m}} \rangle_{Lor} \\ &+ \sum_{k, \vec{m}} \langle \mathbf{u}_{j\vec{l}} | \mathbf{B}_0 | \mathbf{B}_{k\vec{m}} \rangle_{Lor} + \nu \sum_{k, \vec{m}} \langle \mathbf{u}_{j\vec{l}} | \nabla^2 \mathbf{u}_{k\vec{m}} \rangle, \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{\partial E_{k\vec{m}}^{(B)}}{\partial t} &= \sum_{j\vec{l}} \langle \mathbf{B}_{k\vec{m}} | \mathbf{B}_{j\vec{l}} \rangle_{ind} + \sum_{j\vec{l}} \langle \mathbf{B}_{k\vec{m}} | \mathbf{B}_0 | \mathbf{u}_{j\vec{l}} \rangle_{ind} \\ &+ \eta \sum_{j\vec{l}} \langle \mathbf{B}_{k\vec{m}} | \nabla^2 \mathbf{B}_{j\vec{l}} \rangle \end{aligned} \quad (8)$$

where $\mathbf{B}_0 = (0, 0, 0.1)$ is the uniform background magnetic

field and the brackets are defined as follows:

$$\langle \mathbf{u}_{j\vec{l}} | \mathbf{u}_{k\vec{m}} \rangle_{NL} := - \int \mathbf{u}_{j\vec{l}} \cdot ((\mathbf{u} \cdot \nabla) \mathbf{u}_{k\vec{m}}) d^3\vec{x}, \quad (9)$$

$$\langle \mathbf{u}_{j\vec{l}} | \mathbf{B}_{k\vec{m}} \rangle_{Lor} := \int \mathbf{u}_{j\vec{l}} \cdot ((\nabla \times \mathbf{B}_{k\vec{m}}) \times \mathbf{B}) d^3\vec{x}, \quad (10)$$

$$\langle \mathbf{u}_{j\vec{l}} | \nabla^2 \mathbf{u}_{k\vec{m}} \rangle := \int \mathbf{u}_{j\vec{l}} \cdot (\nabla^2 \mathbf{u}_{k\vec{m}}) d^3\vec{x}, \quad (11)$$

$$\langle \mathbf{B}_{k\vec{m}} | \mathbf{B}_{j\vec{l}} \rangle_{ind} := \int \mathbf{B}_{k\vec{m}} \cdot \nabla \times (\mathbf{u}_{j\vec{l}} \times \mathbf{B}) d^3\vec{x}, \quad (12)$$

$$\langle \mathbf{B}_{k\vec{m}} | \nabla^2 \mathbf{B}_{j\vec{l}} \rangle := \int \mathbf{B}_{k\vec{m}} \cdot \nabla^2 \mathbf{B}_{j\vec{l}} d^3\vec{x}. \quad (13)$$

The pressure term vanishes because each of the scale-location spectra is divergence-free.

The reason for the definition of the Lorentz force and induction terms are discussed in detail in the next section.

4. Mathematical Foundation of Mode Decomposition of Energy Transfer between The Kinetic and Magnetic Energies

In this section we discuss the foundations of mode decomposition of energy transfer due to the magnetic induction and the Lorentz force in order to justify the cone representation of mode interaction, which is a visualization technique we propose in this article.

Our discussion is based on the variational principle. It is considered in detail that the work done on the magnetic field by the fluid motion. The basics of variational principle for MHD fluids is discussed by Newcomb[8].

Since the magnetic field, exactly speaking, the magnetic flux density field is a differential 2-form[9], it is quite natural to evaluate the transfer between the kinetic and magnetic energies due to magnetic induction in terms of the variation of differential 2-form.

In three dimensional space, assuming that the magnetic field $\mathbf{B} = \epsilon_{ijk} B^i dx^j \wedge dx^k$ is a frozen-in 2-form, one obtains the variation of magnetic field due to the virtual displacement $\vec{x} \rightarrow \vec{x} + \delta\vec{x} = \vec{x} + \epsilon \xi(\vec{x})$ as

$$\begin{aligned} \delta\mathbf{B} &= \epsilon L_{\xi} \mathbf{B} \\ &= \frac{\epsilon}{2} \epsilon_{ijk} (\xi^m \frac{\partial B^i}{\partial x^m} - B^m \frac{\partial \xi^i}{\partial x^m} + B^i \frac{\partial \xi^m}{\partial x^m}) dx^j \wedge dx^k \end{aligned} \quad (14)$$

where $\epsilon, \xi := \xi^i(\partial/\partial x^i)$ and L are small parameter of variation, displacement vector field of $O(1)$ and the Lie differentiation operator, respectively¹. Therefore, the virtual work done on the magnetic field is given by

$$\begin{aligned} \delta E^{(B)} &= \delta \int \frac{1}{2\mu} |\mathbf{B}|^2 d^3\vec{x} = \epsilon \int \frac{1}{\mu} \mathbf{B} \cdot L_{\xi} \mathbf{B} d^3\vec{x} \\ &= \epsilon \int \frac{1}{\mu} \mathbf{B} \cdot (\nabla \times (\xi \times \mathbf{B})) d^3\vec{x}. \end{aligned} \quad (15)$$

¹In the present study Einstein's summation convention is used. Since the magnetic field is divergence-free ($\partial B^m/\partial x^m = 0$), the Lie derivative of magnetic field 2-form can be rewritten as $L_{\xi} \mathbf{B} = \nabla \times (\xi \times \mathbf{B})$.

Substituting the velocity field \mathbf{u} into ξ of this expression, we obtain the rate of change of the magnetic energy due to the MHD fluid motion.

It should be remarked that one of the advantages of using Lie derivative is that it is based on the Lagrangian specification of fluid motion so that the mathematical expressions are applicable, for example, to the compressible MHD case as well. The other advantage is that the expression Eq.(14) is *physical*, i.e., it is *invariant under the arbitrary change of local coordinate system*, which may be lost if the terms are split into two or more parts, for example, $\xi^m \frac{\partial B^i}{\partial x^m}$ and $-B^m \frac{\partial \xi^i}{\partial x^m} + B^i \frac{\partial \xi^m}{\partial x^m}$. This invariance allows us to apply the formula to general curved coordinate systems, which are often used in the numerical analysis of fusion plasmas.

Since the integral $\delta_{\mathbf{u}} E^{(B)} := \int \frac{1}{\mu} \mathbf{B} \cdot (\nabla \times (\mathbf{u} \times \mathbf{B})) d^3 \vec{x}$ is linear with respect to the velocity field, it is able to be decomposed with respect to the scale-location spectrum of \mathbf{u} as follows:

$$\delta_{\mathbf{u}} E^{(B)} = \sum_{j, \vec{l}} \delta_{\mathbf{u}_{j\vec{l}}} E^{(B)} \quad (16)$$

where the components are given by

$$\delta_{\mathbf{u}_{j\vec{l}}} E^{(B)} := \frac{1}{\mu} \int \mathbf{B} \cdot (\nabla \times (\mathbf{u}_{j\vec{l}} \times \mathbf{B})) d^3 \vec{x}. \quad (17)$$

This integral implies *the work per unit time done by the $\mathbf{u}_{j\vec{l}}$ components of fluid motion on the magnetic potential energy*.

One of the advantages of this definition is that integration by parts naturally gives the mode expansion of the energy transfer due to the Lorentz force:

$$\delta_{\mathbf{u}_{j\vec{l}}} E^{(B)} = -\frac{1}{\mu} \int \mathbf{u}_{j\vec{l}} \cdot ((\nabla \times \mathbf{B}) \times \mathbf{B}) d^3 \vec{x}. \quad (18)$$

This integral is the minus of the inner product of the Lorentz force term $\mathbf{j} \times \mathbf{B}$ with the $\mathbf{u}_{j\vec{l}}$ components of fluid motion. That is, the integral implies *the energy loss per unit time of the (j, \vec{l}) components of kinetic energy $E^{(u)}$ by the Lorentz force*:

$$\delta_{\mathbf{u}_{j\vec{l}}} E^{(B)} = -(\delta E_{j\vec{l}}^{(u)})_{\text{Lorentz force}}. \quad (19)$$

In order to evaluate the rate of change of the (k, \vec{m}) component of magnetic potential energy $E^{(B)}$, the work per unit time $\delta_{\mathbf{u}_{j\vec{l}}} E^{(B)}$ is decomposed as

$$\delta_{\mathbf{u}_{j\vec{l}}} E^{(B)} = \sum_{k, \vec{m}} \delta_{\mathbf{u}_{j\vec{l}}} E_{k\vec{m}}^{(B)} \quad (20)$$

where the components are given by

$$\delta_{\mathbf{u}_{j\vec{l}}} E_{k\vec{m}}^{(B)} := \frac{1}{\mu} \int \mathbf{B}_{k\vec{m}} \cdot (\nabla \times (\mathbf{u}_{j\vec{l}} \times \mathbf{B})) d^3 \vec{x}. \quad (21)$$

This integral has its counterpart in the Lorentz force term of the energy budget equation of $E_{j\vec{l}}^{(u)}$:

$$\delta_{\mathbf{u}_{j\vec{l}}} E_{k\vec{m}}^{(B)} = -\delta_{\mathbf{B}_{k\vec{m}}} E_{j\vec{l}}^{(u)} \quad (22)$$

where $\delta_{\mathbf{B}_{k\vec{m}}} E_{j\vec{l}}^{(u)} := \frac{1}{\mu} \int \mathbf{u}_{j\vec{l}} \cdot ((\nabla \times \mathbf{B}_{k\vec{m}}) \times \mathbf{B}) d^3 \vec{x}$. This is the reason why we have defined the mode interaction terms as Eqs.(10) and (12), which satisfy detailed energy balance relation $\langle \mathbf{u}_{j\vec{l}} | \mathbf{B}_{k\vec{m}} \rangle_{\text{Lor}} = -\langle \mathbf{B}_{k\vec{m}} | \mathbf{u}_{j\vec{l}} \rangle_{\text{ind}}$.

It should be emphasized here that, though we discussed here in terms of the wavelet scale-location spectrum, the discussion given here is also applicable to any orthogonal decomposition of the velocity and magnetic fields, for example, their spherical shell decomposition in the wave number space.

5. Colored cone representation of the scale-location-to-scale-location energy transfer

In the previous section we derived the energy transfer between the kinetic and magnetic energies in terms of wavelet scale-location spectrum. Since each of the obtained integral contains the information on the locations of interacting wavelet modes, visualization of the ‘‘energy flow’’ from one wavelet site to another one become possible. By superimposing some graphical object which depicts information of locations and magnitude of ‘‘energy flow’’ on some other physical quantities, for example, the isosurfaces of the enstrophy or current density distribution, one can intuitively grasp the spatial features of dynamics given by Eqs.(1) and (2) and its relation to vorticity or current.

We have developed ‘‘cone representation’’ of wavelet scale-location-to-scale-location mode interaction in order to analyze the nonlinear energy transfer in rolling-up motion of a neutral fluid [2]. The basic idea of the method is now applied to the mode interaction associated with magnetic induction process.

A cone simultaneously depicts the following four kinds of information which the scale-location-to-scale-location energy transfer integral $\langle \mathbf{B}_{k\vec{m}} | \mathbf{u}_{j\vec{l}} \rangle_{\text{ind}}$ has: the ‘‘position’’ of the energy donating mode, that of the energy receiving mode, the ‘‘direction’’ of energy transfer and the amplitude of the energy transfer (see Fig.1).

The center of base of a cone is assigned to the position of energy donating mode. (In the following, the ‘‘position’’ of the scale-location spectrum is defined by modulus maximum of wavelet.) On the other hand, its vertex is assigned to that of energy receiving mode. Thus the cone indicates the ‘‘direction of energy flow’’ from donating to receiving modes as a whole.

The magnitude of ‘‘energy flow’’ is represented by its base radius and color. The base radius of a cone is determined to be proportional to the absolute value of the transfer integral. That is, it is normalized by the maximum value of ensemble and the wavelet grid interval size as follows:

$$r = R \left(T_{l, \vec{m}}^{(j, k)} \right) \times \frac{L}{2^{\max\{j, k\}}} \quad (23)$$

where R is an appropriate function and $T_{l, \vec{m}}^{(j, k)}$ is normalized

transfer defined by

$$T_{\vec{l}, \vec{m}}^{(j,k)} = \frac{|\langle \mathbf{B}_{k\vec{m}} | \mathbf{B} | \mathbf{u}_{j\vec{l}} \rangle_{ind}|}{\max \left\{ |\langle \mathbf{B}_{k\vec{m}} | \mathbf{B} | \mathbf{u}_{j\vec{l}} \rangle_{ind}| \right\}} \quad (24)$$

and has its value between $0 \leq T_{\vec{l}, \vec{m}}^{(j,k)} \leq 1$. In the figures 3 and 4, the functional form of R is $R(x) = x$.

The activity of induction is represented by the colors of cones. If magnetic field acquires energy by magnetic induction, cones have such colors that are gradating from red to yellow. If magnetic field loses energy, on the other hand, the colors gradating from blue to green is painted (see Fig.2).

When $\langle \mathbf{B}_{k\vec{m}} | \mathbf{B} | \mathbf{u}_{j\vec{l}} \rangle_{ind} > 0$, i.e., energy is transferred from $E_{j\vec{l}}^{(u)}$ to $E_{k\vec{m}}^{(B)}$, the parameters of a cone are given by

$$\text{base center: } \vec{x} = \frac{L}{2j} \left(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z + \frac{1}{2} \right), \quad (25)$$

$$\text{vertex: } \vec{x} = \frac{L}{2k} \left(m_x + \frac{1}{2}, m_y + \frac{1}{2}, m_z + \frac{1}{2} \right), \quad (26)$$

$$\text{color: } (R, G, B) = \left(1, 1 - C \left(T_{\vec{l}, \vec{m}}^{(j,k)} \right), 0 \right), \quad (27)$$

where C is color legend control function. In the figures 3 and 4, C is given by $C(x) = \min \{ 1, 3x \}$. Colors from red $(R, G, B) = (1, 0, 0)$ to yellow $(R, G, B) = (1, 1, 0)$ are assigned. The offset factor $1/2$ are due to the position of modulus maximum of Meyer's wavelet.

When $\langle \mathbf{B}_{k\vec{m}} | \mathbf{B} | \mathbf{u}_{j\vec{l}} \rangle_{ind} < 0$, i.e., energy is transferred from $E_{k\vec{m}}^{(B)}$ to $E_{j\vec{l}}^{(u)}$, the parameters of a cone are given by

$$\text{base center: } \vec{x} = \frac{L}{2k} \left(m_x + \frac{1}{2}, m_y + \frac{1}{2}, m_z + \frac{1}{2} \right), \quad (28)$$

$$\text{vertex: } \vec{x} = \frac{L}{2j} \left(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z + \frac{1}{2} \right), \quad (29)$$

$$\text{color: } (R, G, B) = \left(0, 1 - C \left(T_{\vec{l}, \vec{m}}^{(j,k)} \right), C \left(T_{\vec{l}, \vec{m}}^{(j,k)} \right) \right). \quad (30)$$

Colors from blue $(R, G, B) = (0, 0, 1)$ to green $(R, G, B) = (0, 1, 0)$ are assigned.

6. Visualization of Flow and Current Structures and Energy Transfer

In this section we will make an attempt of the application of cone representation visualization developed in the previous section to the analysis of energy transfer in an MHD system. In the present study we analyzed the same velocity and magnetic fields as those presented in Ref.[1].

In the previous study we carried out the scale-to-scale wavelet analysis of the energy transfer between the fields. The analysis was based on the following energy transfer integrals:

$$L_{jk} := \int \mathbf{u}_j \cdot (\nabla \times \mathbf{B}_k) \times \mathbf{B} d^3 \vec{x}, \quad (31)$$

$$L_{jk}^{(0)} := \int \mathbf{u}_j \cdot (\mathbf{B}_0 \cdot \nabla) \mathbf{B}_k d^3 \vec{x}, \quad (32)$$

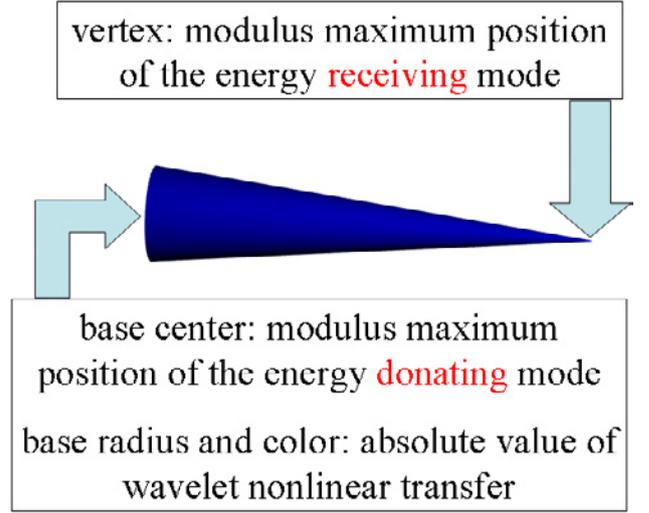


Fig. 1 Implications of each part of a cone representation of wavelet scale-location-to-scale-location energy transfer.

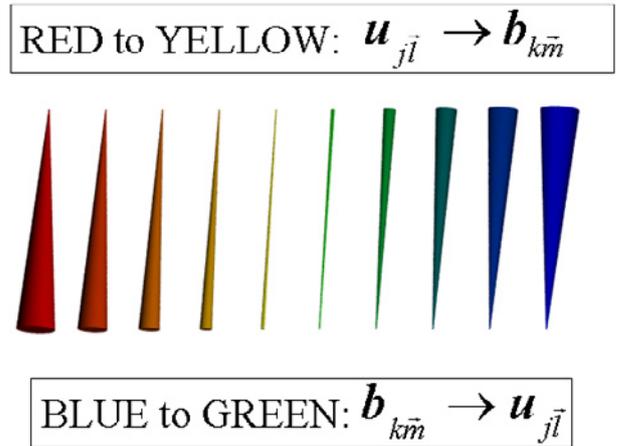


Fig. 2 Color legend of cones that represents the modulus of the integral $\langle \mathbf{B}_{k\vec{m}} | \mathbf{B} | \mathbf{u}_{j\vec{l}} \rangle_{ind}$.

where u_j and B_k are the wavelet scale spectra of u and B defined by

$$u_j := \sum_{\varepsilon, \vec{l}, \sigma} \tilde{u}_{j\varepsilon\vec{l}\sigma}(t) \psi_{j\varepsilon\vec{l}\sigma}(\vec{x}), \quad B_k := \sum_{\varepsilon, \vec{m}, \sigma} \tilde{B}_{k\varepsilon\vec{m}\sigma}(t) \psi_{k\varepsilon\vec{m}\sigma}(\vec{x}).$$

The relation between these integrals and the scale-location-to-scale-location analysis in the present study is given by the following decomposition relations:

$$L_{jk} = - \sum_{\vec{l}} \sum_{\vec{m}} \langle \mathbf{B}_{k\vec{m}} | \mathbf{B} | \mathbf{u}_{j\vec{l}} \rangle_{ind}, \quad (33)$$

$$L_{jk}^{(0)} = - \sum_{\vec{l}} \sum_{\vec{m}} \langle \mathbf{B}_{k\vec{m}} | \mathbf{B}_0 | \mathbf{u}_{j\vec{l}} \rangle_{ind}. \quad (34)$$

In figures 3 and 4, spatial distribution of intensive magnetic induction $\langle \mathbf{B}_{4\vec{m}} | \mathbf{B} | \mathbf{u}_{4\vec{l}} \rangle_{ind}$ and Alfvén waves $\langle \mathbf{B}_{4\vec{m}} | \mathbf{B}_0 | \mathbf{u}_{4\vec{l}} \rangle_{ind}$ at the time $t = 22$ are depicted. Not all the cones but the larger ones are depicted. Threshold is determined to include 50 per cent of the net enhancing/reducing

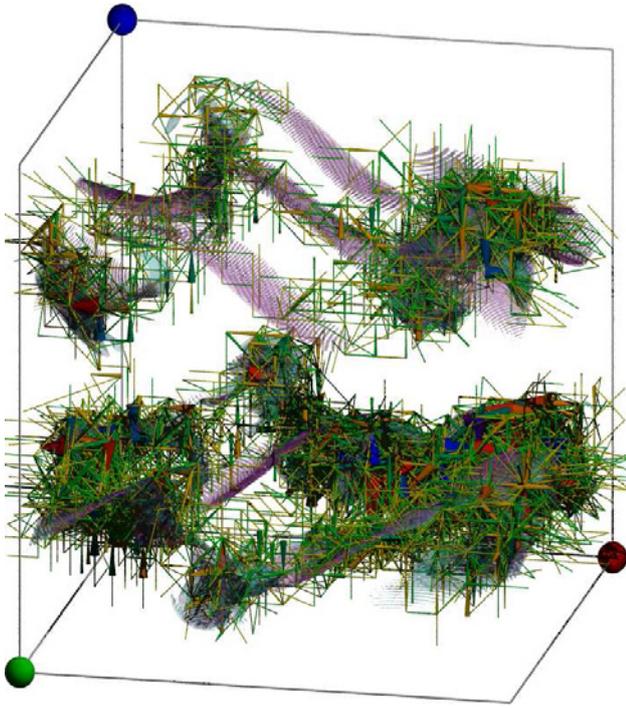


Fig. 3 Simultaneous presentation of the spatial distribution of vorticity (blue surfaces), current sheet (purple surfaces) and active magnetic induction (colored cones). Mode interaction between $\mathbf{B}_{4\bar{m}}$ and $\mathbf{u}_{4\bar{j}}$, i.e., cones correspond to the integral $\langle \mathbf{B}_{4\bar{m}} | \mathbf{B} | \mathbf{u}_{4\bar{j}} \rangle_{ind}$ are depicted.

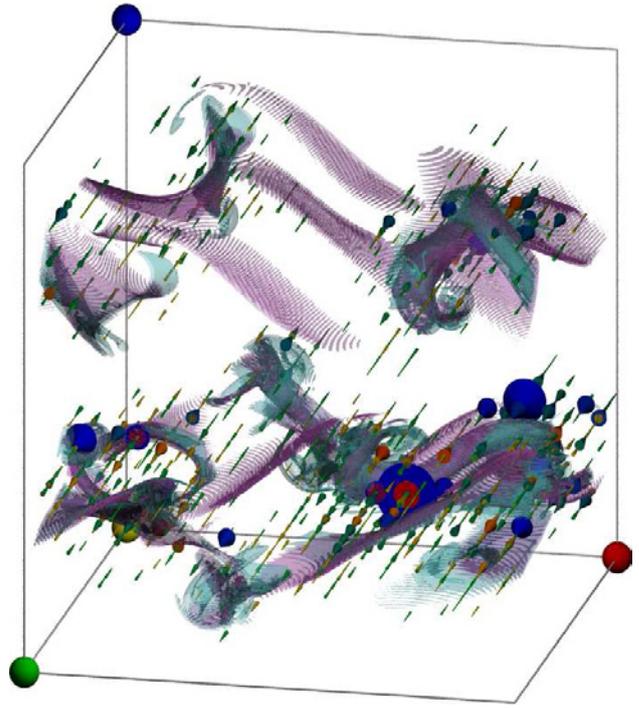


Fig. 4 Same as Fig. 3. Energy transfers due to the Alfvén wave $\langle \mathbf{B}_{4\bar{m}} | \mathbf{B}_0 | \mathbf{u}_{4\bar{j}} \rangle_{ind}$ are depicted.

magnetic energy transfer. In these pictures, spatial scale parameter of wavelets are fixed to $j = 4$, where the wavelet scale spectrum of the magnetic energy has its peak [1]. Therefore it is expected that the typical spatial distribution of magnetic induction is seen.

To understand the spatial features of flow field and magnetic field, isosurfaces of the enstrophy density field $\frac{1}{2}|\nabla \times \mathbf{u}|^2$ and the current one $\frac{1}{2}|\nabla \times \mathbf{B}|^2$ are also depicted. Threshold of isosurfaces are set to include 1 per cent volume of whole domain. As is discussed in Ref.[1], the flow has several strong coherent vortices which are developed around the initial shear layers. Strong current sheets are formed around vortices and stretching flow region between the vortices.

The cones for the magnetic induction appear around the strong vortices and the current sheets (see Fig.3). This result directly suggests that the coherent structure is the promoter of the dominant magnetic induction. The cones that represents the excitation/reduction of magnetic field are interweaving each other. Similar fluctuating feature was observed for the nonlinear interaction of rolling-up vortices in a neutral fluid [10].

It is demonstrated that the cone representation can be applicable to the analysis of magnetic induction term in the magnetic energy budget equation (8), and that simultaneous displaying with other physical quantities such as enstrophy density supports intuitive understanding of the

physical process.

Two remarks should be made. First, since the data presented here are obtained by the decomposition of L_{44} and $L_{44}^{(0)}$ scale-to-scale interactions which is one of the 81 number of components of L_{jk} and $L_{jk}^{(0)}$, the information included in these figures is a restricted portion of the whole interactions. The other is that the information of such small cones that are *not* depicted here should be compensated by some other analysis, for example, statistics of the cones. The analysis of transfers between other spatial scales and statistical analysis of the ensemble of cones are now underway.

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