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Bootstrap transition to high beta equilibrium in helical system

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It is shown theoretically and computationally that helical magnetic field, produced by continuous winding helical coils and without the toroidal coil, can sustain MHD stable high beta plasma. Pressure driven toroidal current (bootstrap current) cancels the external magnetic field and reduces the MHD potential energy, depending on the plasma beta values. Ramp-up of heating power input induces bootstrap transition to higher beta plasmas with flat-top pressure profiles. Helical pitch parameter dependence of MHD stability is analyzed.

Keywords: high beta plasma, helical system, bootstrap current, bootstrap transition, LHD

1. Introduction

The existence of the MHD stable high beta core plasma lead the way for the realization of economic fusion power systems. The LHD experimental results of achieving average beta value 5% without the beta collapse suggests the possibility of the helical equilibriums with ultrahigh beta MHD stable core plasmas.

MHD stability of plasma is determined by the MHD potential energy($= W$), which is the sum of plasma thermal energy($= W_T$) and the magnetic filed energy($= W_B$),

$$W = \int \mathrm{d}V \left(\frac{3}{2} P + \frac{1}{2\mu_0} \boldsymbol{B}^2 \right) = W_T + W_B \,. \quad (1)$$

The integration domain is extended not only to the plasma volume but also to the outer region of external coils.

The diamagnetic current reduces the externally applied magnetic field($= \boldsymbol{B}_{\rm ext}$) at the plasma volume($= V_{\rm p}$). The pressure equilibrium relation and the definition of the plasma beta value($= \beta$) lead the expression for variation of the MHD potential energy δW as follows,

$$\frac{\delta W}{W_0} \equiv \frac{W - W_0}{W_0} = \frac{\beta}{2(\beta+1)} > 0 \,, \quad (2)$$

where P_0 and $\boldsymbol{B}_0$ are the pressure and magnetic field in the plasma region, and $\beta \equiv P_0 / \left(\frac{\boldsymbol{B}_0^2}{2\mu_0} \right)$,

$$P_0 + \frac{\boldsymbol{B}_0^2}{2\mu_0} = \frac{\boldsymbol{B}_{\rm ext}^2}{2\mu_0},$$

$$W_0 = V_{\rm p} \frac{\boldsymbol{B}_{\rm ext}^2}{2\mu_0}, \quad W = V_{\rm p} \left\{ \frac{3}{2} P_0 + \frac{\boldsymbol{B}_0^2}{2\mu_0} \right\} .$$

Therefore, the diamagnetic current cannot conduct to the MHD stable configuration as shown in eq.(2).

On the other hand, the toroidal current driven by plasma pressure (the bootstrap current) produces magnetic field outside the plasma column, which diminish the magnetic field outside of external coils. When the MHD potential energy reduction

$$\delta W < 0 \quad (3)$$

is caused by the cancellation of the magnetic field, the bootstrap transitions to MHD stable high beta equilibrium become possible.

Bootstrap current is usually created by the banana orbit particles, which comes from the toroidal effect. The density gradient of banana particles provides the bootstrap current[1]. In a tokamak the bootstrap current $J_{\rm bs}$ is simply given by

$$J_{\rm bs} \approx \epsilon^{1/2} \frac{1}{B_\theta} \frac{\mathrm{d}P}{\mathrm{d}\rho} \quad (4)$$

where $\epsilon = a/R$ is the inverse aspect ratio and ρ is the minor radius of the torus. R.J.Bickerton, J.W.Connor and J.B.Taylor have pointed out that in low collision frequency regime, the enhanced diffusion is associated with the currents which flow parallel to the magnetic field; not only are these current much larger than the Pfirsch-Sclüter currents, but they are also unidirectional. They have proposed a bootstrap tokamak which operates in steady state by diffusion driven net toroidal current without any external driving force [2].

In the straight helical system, we have $\epsilon = 0$, and then $J_{\rm bs}$ described in eq.(4) reduce to zero. However, the bootstrap currents described in the paper have still finite value even in a straight helical system.

Pfirsch-Sclüter currents are derived from the following relation

$$0 = \boldsymbol{\nabla} \cdot \left(\frac{\boldsymbol{B} \times \boldsymbol{\nabla} P}{\boldsymbol{B}^2} \right) + \boldsymbol{\nabla} \cdot \left(\frac{\boldsymbol{B}}{|\boldsymbol{B}|} J_\parallel \right) . \quad (5)$$

which shows the condition of the no charge accumulation induced by the the imbalance of the net outward force due to the larger surface area at the larger major radius. For the case of usual axisymmetric system

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with concentric magnetic surfaces with the rotational transform ι,

$$B_\phi = \frac{R}{R+\rho\cos\theta} B_0 \,, \quad \frac{\iota}{2\pi} = \frac{R\,B_\theta}{\rho B_\phi} \,, \tag{6}$$

the eq.(5) can be reduced to

$$\frac{\iota B_0}{2\pi R}\frac{\partial}{\partial\theta}\left(\frac{J_\parallel}{B_\phi}\right) \simeq -\frac{\mathrm{d}P}{\mathrm{d}\rho}\frac{2}{RB_0}\sin\theta \,, \tag{7}$$

and can be integrated as

$$J_\parallel \simeq C\cdot B_\phi + \frac{4\pi}{\iota B_0}\frac{\mathrm{d}P}{\mathrm{d}\rho}\cos\theta \,, \tag{8}$$

where C is the integration constant of the differential equation (7). If we set $C = 0$, the expression eq.(8) is the expression for the Pfirsch-Sclüter currents J_{PS} [3],

$$J_{\mathrm{PS}} = \frac{4\pi}{\iota B_0}\frac{\mathrm{d}P}{\mathrm{d}\rho}\cos\theta \,. \tag{9}$$

The integration constant C is determined from not eq.(5) but from the force balance condition in the ρ direction. In axisymmetric torus, MHD equation $\nabla P = \boldsymbol{J}\times\boldsymbol{B}$ determines the plasma current $\boldsymbol{J}$ as follows [4],

$$\boldsymbol{J} = P'(\Psi)\, r\, \boldsymbol{e}_\phi + \frac{I'(\Psi)}{2\pi}\boldsymbol{B} \,. \tag{10}$$

Then the pressure driven equilibrium current, the first term of the RHS of the eq.(10), is expressed as

$$J_\parallel = \frac{B_\phi}{|\boldsymbol{B}|}P'(\Psi)r \simeq \frac{1}{RB_\theta}\frac{\mathrm{d}P}{\mathrm{d}\rho}(R+\rho\cos\theta) \,, \tag{11}$$

where we have used the simplified expression

$$P'(\Psi) = \frac{\mathrm{d}P}{\mathrm{d}\rho}\left\{\frac{\mathrm{d}\Psi}{\mathrm{d}\rho}\right\}^{-1} \simeq \frac{1}{RB_\theta}\frac{\mathrm{d}P}{\mathrm{d}\rho} \,. \tag{12}$$

The expression eq.(11) determines the integration constant C as

$$C \simeq \frac{1}{B_0 B_\theta}\frac{\mathrm{d}P}{\mathrm{d}\rho} \,. \tag{13}$$

Then the expression eq.(8) becomes as

$$\begin{aligned} J_\parallel &= \frac{1}{B_\theta}\frac{\mathrm{d}P}{\mathrm{d}\rho}\frac{R}{R+\rho\cos\theta} + \frac{4\pi}{\iota B_0}\frac{\mathrm{d}P}{\mathrm{d}\rho}\cos\theta \\ &\simeq \frac{1}{B_\theta}\frac{\mathrm{d}P}{\mathrm{d}\rho} + \frac{2\pi}{\iota B_0}\frac{\mathrm{d}P}{\mathrm{d}\rho}\cos\theta \,, \end{aligned} \tag{14}$$

where we have used the expression for the rotational transform eq.(6). The first term of the RHS of eq.(14) is net toroidal currents similar to the diamagnetic current produced by the poloidal magnetic field B_θ and the plasma pressure gradient in the ρ direction. Since this currents produce magnetic field mainly outside the plasma column without producing the magnetic field inside the plasma column, we call this pressure driven toroidal net currents is also one of the bootstrap currents. The second term of the RHS of term eq.(14) prevent the net charge accumulation due to the imbalance of the net outward force. This expression is just same to the current profile given by the second term of the eq.(11), but the factor 2 smaller compared to the traditional expression of the Pfirsch-Sclüter currents eq.(9). The net charge accumulation du to the pressure imbalance is partially prevented by the pressure driven toroidal net current.

In the paper, we have developed a new numerical scheme to solve the equilibrium based on the Biot-Savart' law. Since this scheme can compute the magnetic field at the region including inside and outside of helical coils, MHD potential energy W can be calculated, simultaneously.

We have analyzed the equilibrium and the stability of straight helical systems, to verify based on the first principle that the LHD type helical magnetic field configuration can sustain MHD stable high beta core plasmas.

Straight helical coil system and the rotating helical coordinate system are explained in §2. New numerical scheme to solve MHD equilibrium and plasma model are explained in §3. To analyze the role of bootstrap current for the MHD stability, a new criterion for the MHD potential energy is developed in §4. Bootstrap transition from peaked pressure profile to flat-top pressure profile is analyzed in §5. In §6, it is shown that small value of the helical pitch parameters is favorable for the high beta plasma confinement. §7 is devoted to the summary and discussions.

2. Straight Helical System

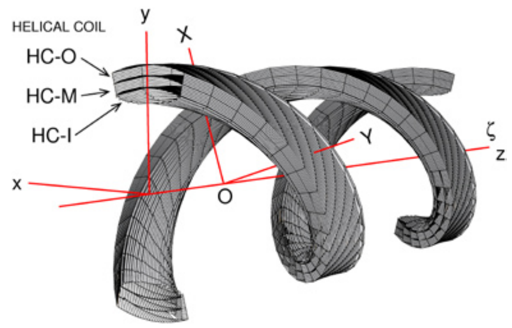

Fig. 1 Straight helical coils composed of 3 layers(Hc-O, Hc-M, Hc-I) and the rotating helical coordinate system(X, Y, ζ). Helical pitch parameter ($=\gamma$)is controlled by the distribution of coil currents among these three layers.

Straight helical system has symmetry and MHD equations are possible to be solved without approximations. Figure 1 shows an example of straight helical coils and the rotating helical coordinate system (X, Y, ζ), which rotates in synchronization with them. X, Y and ζ are for the directions of the long axis, short axis and magnetic axis of magnetic surfaces, respectively. Under the rotating helical coordinate system,

ζ dependency disappears in a helical magnetic field.

An important index of a helical coil system is a helical pitch parameter($=\gamma$), which is defined by the product of the radius of current distribution center($= a_c$) and the axial wave number of the helical coil system($= k$),

$$\gamma \equiv a_c \times k \tag{15}$$

We have assumed in the following analysis that the size and the configuration of the helical coils are similar to those of the LHD[5]. Table 1 shows typical examples of the coil currents and of the magnetic field parameters.

Table 1 Coil currents and helical pitch parameters.

γ	$\lvert B_{ax}\rvert$	coil current(kA × turns)		
	(T)	Hc-O	Hc-M	Hc-I
1.3817	2.538	33 × 150	0 × 150	0 × 150
1.2538	2.538	11 × 150	11 × 150	11 × 150
1.1221	2.538	0 × 150	0 × 150	33 × 150

3. Equilibrium Analysis

Ideal MHD equation $\boldsymbol{\nabla} P = \boldsymbol{J} \times \boldsymbol{B}$ can be integrated along the magnetic field $\boldsymbol{B}$ and the plasma current $\boldsymbol{J}$. Arbitrary functions $P(\Psi)$ and $I(\Psi)$ are introduced and plasma current is expressed as follows,

$$\boldsymbol{J} = \frac{1}{\mu_0} I'(\Psi)\boldsymbol{B} + P'(\Psi)\begin{pmatrix} -kY \\ kX \\ 1 \end{pmatrix}, \tag{16}$$

where $P(\Psi)$ is the plasma pressure distribution. The first term of the right-hand side of eq.(16) is the driven current term, which is independent to the plasma pressure. X and Y components of the second term of eq.(16) are the diamagnetic currents, which produce magnetic field on the inside of the plasma column, mainly. The ζ components of the second term of eq.(16) is the bootstrap currents, which produce magnetic field on the outside of the plasma column, mainly.

Magnetic field $\boldsymbol{B}$, vector potential $\boldsymbol{A}$ and the magnetic flux function Ψ can be calculated by Biot-Savart' law as follows,

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int d^3\boldsymbol{r}' \left\{ \frac{(\boldsymbol{r}'-\boldsymbol{r}) \times \boldsymbol{J}^{s}(\boldsymbol{r}')}{|\boldsymbol{r}'-\boldsymbol{r}|^3} \right\}, \tag{17}$$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int d^3\boldsymbol{r}' \left\{ \frac{\boldsymbol{J}^{s}(\boldsymbol{r}')}{|\boldsymbol{r}'-\boldsymbol{r}|} - \frac{\boldsymbol{J}^{s}(\boldsymbol{r}')}{|\boldsymbol{r}'|} \right\}, \tag{18}$$

$$\Psi(\boldsymbol{r}) = A_\zeta + k\,(XA_Y - YA_X), \tag{19}$$

$$\boldsymbol{J}^{s}(\boldsymbol{r}') \equiv \boldsymbol{J}(\boldsymbol{r}') + \boldsymbol{J}^{c}(\boldsymbol{r}'),$$

where $\boldsymbol{J}^{c}(\boldsymbol{r}')$ is the current density of helical coils.

Because the flux function Ψ is a linear function of the vector potential $\boldsymbol{A}$, Ψ is given by the sum of contribution from the plasma current($= \Psi^{p}(\Psi)$) and contribution from the helical coil currents($= \Psi^{c}$),

$$\Psi = \Psi^{p}(\Psi) + \Psi^{c}. \tag{20}$$

Plasma equilibrium computations is reduced to solve eq.(20) self-consistently for Ψ. We have confirmed that relaxation scheme and Newton iteration method are possible to solve eq.(20), treating the separatrix position of Ψ as an unknown parameters.

In the following, we assume that the driven current is not present ($I'(\Psi) = 0$). The pressure profile $P(\Psi)$ is assumed to be

$$P(\Psi) = \beta_{ax}\frac{\boldsymbol{B}_{ax}^2}{2\mu_0}\frac{\sum_{i=1}^{3} P_i \exp\left\{-D\left(\frac{\Psi}{\Psi_s}\right)^i\right\}}{P_1+P_2+P_3} \tag{21}$$

where β_{ax}, P_1, P_2, P_3 and D are some constants. $\boldsymbol{B}_{ax}$ and Ψ_s are the magnetic field on the magnetic axis and the value of Ψ at the separatrix, respectively.

When $P_1 = 1$ and $P_2 = P_3 = 0$, the pressure profile become a peaked profile and the bootstrap currents distribute in a similar profile, also. When $P_1 = 0$, the pressure profile become a flat-top profile and the bootstrap currents become zero on the magnetic axis. The bootstrap currents distribute in a surface current type profile.

4. Stability Analysis

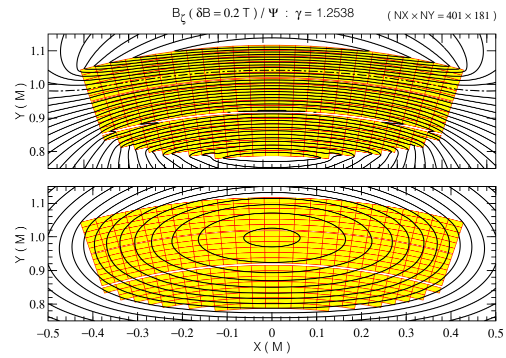

Fig. 2 The field intensity distribution(B_ζ: up) and the magnetic flux function(Ψ: down), in a helical coil.

Stability analysis is based on the energy conservation law for the ideal MHD equations[6],

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho\boldsymbol{v}^2 + \frac{3}{2}P + \frac{1}{2\mu_0}\boldsymbol{B}^2\right) = -\boldsymbol{\nabla}\left[\left(\frac{1}{2}\rho\boldsymbol{v}^2 + \frac{5}{2}P\right)\boldsymbol{v} + \frac{1}{\mu_0}\boldsymbol{E}\times\boldsymbol{B}\right], \tag{22}$$

where $\boldsymbol{v}$, ρ and $\boldsymbol{E}$ are fluid velocity, mass density and electric field, respectively.

The conservation lows eq.(22) shows that W,

$$W = \int dV \left(\frac{3}{2}P + \frac{1}{2\mu_0}\boldsymbol{B}^2\right) \equiv W_T + W_B \tag{23}$$

is the MHD potential energy, and W minimum configuration is an MHD stable equilibrium. To analyze the role of the bootstrap current to the MHD stability, the integral domain of eq.(23) should be extended to the region outside of the helical coils. For the computation of the MHD potential energy W, we have developed a magnetic field calculation code which can compute accurately the field intensity inside the helical coils. Numerical example of field intensity distributions inside the helical coil is shown in Fig.(2).

MHD stability of an equilibrium is evaluated by the amount of the decreased MHD potential energy δW,

$$\delta W = W - W_0,\ W_0 = \int \mathrm{d}V \left(\frac{1}{2\mu_0} \boldsymbol{B}_{\mathrm{vac}}^2 \right) \quad (24)$$

where $\boldsymbol{B}_{\mathrm{vac}}$ is the vacuum magnetic field and W_0 is the MHD potential energy of the vacuum state. When $\delta W < 0$, transition to vacuum state is energetically-prohibited and beta collapse of core plasma will not occur. Core plasma develops to a minimum state of W under the imposed constraints.

5. Bootstrap Transition to Flat-Top Pressure Profile

In first, we compare the equilibrium and its stability, for the case of peaked pressure profile and for the case of flat-top pressure profile. The helical pitch parameter is set to one of the standard value of the LHD ($\gamma = 1.2538$). Pressure distributions for both cases are assumed to be the following form,

$$\begin{aligned} P(\Psi) &= \beta_{\mathrm{ax}} \frac{\boldsymbol{B}_{\mathrm{ax}}^2}{2\mu_0} \exp\left\{ -7 \left(\frac{\Psi}{\Psi_s} \right)^i \right\} \quad (25) \\ i &= 1\,(\,2\,)\ \text{for peaked (flat-top) profile.} \end{aligned}$$

Pressure profiles and the bootstrap current distributions under the vacuum magnetic field are shown in Fig.3.

We have calculated variations of MHD potential energy($= \delta W$) and beta value at the magnetic axis ($= \beta_{\mathrm{ax}}$), increasing the stored energy W_T (Fig. 4).

Peaked pressure profile can reach stably to high beta equilibrium with relatively small amount of stored energy, until the decrease of the MHD potential energy reverse ($W_T \simeq 0.77\,\mathrm{MJ/m}$). When heating input is increased much more, peaked pressure profile equilibrium become unstable. Flat-top pressure profile, on the other hand, is more stable, and can increase the stored energy almost 3 times of the level of peaked profile case ($W_T \simeq 2.12\,\mathrm{MJ/m}$).

Bootstrap transition from peaked pressure profile to flat-top pressure profile will be occur, when sufficient heating input is present.

6. Helical Pitch Parameter Dependency of High Beta Equilibrium

Helical pitch parameter, γ defined by eq.(15) is an important index of a helical coil system. We study the pitch parameter dependency of high beta equilibrium

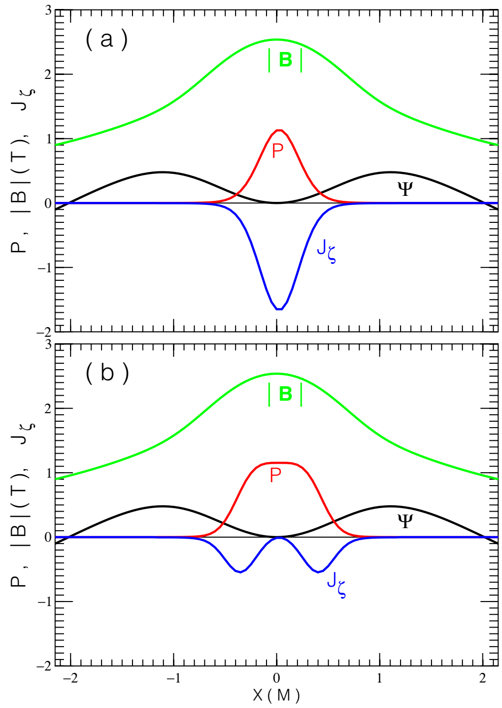

Fig. 3 Pressure P, bootstrap current J_ζ, flux functionΨ and field intensity $|\boldsymbol{B}|$ along the long axis X of the magnetic surface. (a) and (b) are for the case of peaked and flat-top pressure profiles, respectively.

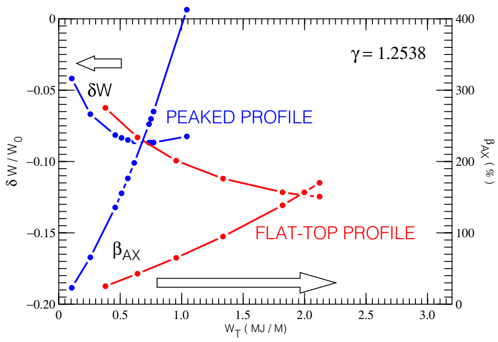

Fig. 4 Variation of MHD potential energy and beta value at the magnetic axis. Abscissa represent the plasma thermal energy stored in the magnetic surface.

in the case of flat-top pressure profile given by eq.(26).

$$P(\Psi) = \beta_{\mathrm{ax}} \frac{\boldsymbol{B}_{\mathrm{ax}}^2}{2\mu_0} \exp\left\{ -7 \left(\frac{\Psi}{\Psi_s} \right)^2 \right\} \quad (26)$$

Numerical results are summarized in Fig.5.

When γ is large, the size of the magnetic surface become large, since the axial magnetic field($= B_\zeta$) is increased relatively. However, the ability of MHD stability decreases as suggested by eq.(2). This tendency is shown by the plot of MHD potential energy δW for the case of $\gamma = 1.3817$ in Fig.5.

When γ is small, the size of the magnetic surface become small, since the axial magnetic field decreased relatively. However, the ability of MHD stability increases since the role of bootstrap current is increased. This tendency is shown by the plot of MHD potential energy δW for the case of $\gamma = 1.1221$ in Fig.5.

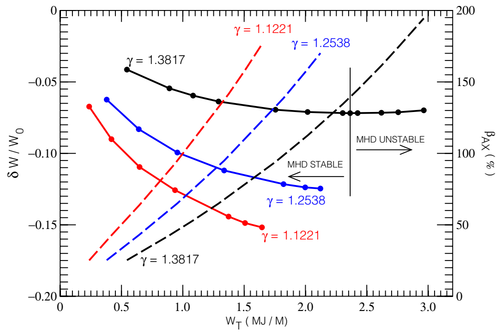

Fig. 5 Variation of MHD potential energy and beta value at the magnetic axis. Abscissa represent the plasma thermal energy stored in the magnetic surface.

Structure of vacuum magnetic field is shown in Fig.6 (a) and (b) for the case of $\gamma = 1.1221$. Clearance between helical coils and the last closed flux surface are very wide, though the plasma volume is small ($V_{\rm lcfs} \simeq 18.6\rm m^3$: reduced value for the LHD).

Next, we show the MHD stable ultrahigh beta equilibrium, which has the minimum MHD potential energy in Fig.6 (c) and (d), sustained in this magnetic field ($\gamma = 1.1221$). We have confirmed that the size of the magnetic surface become very large ($V_{\rm lcfs} \simeq 45.1\rm m^3$: reduced value for the LHD). Magnetic field intensity is very much reduced in the outer region of helical coils by sustaining of high beta plasma (compare Fig.6(b) and (d)). The reduction of total magnetic field energy decreases the MHD potential energy and realize the sustainment of MHD stable ultrahigh beta plasma. Beta value on the magnetic axis reach to $\beta_{\rm ax} \simeq 176\%$.

Figure 7 shows profiles of equilibrium quantities along the long axis of magnetic surface for the case of $\beta_{\rm ax} = 176\%$.

Figure 7(b) shows that steep pressure gradient is primarily sustained by the magnetic well, shown by the graph of the specific volume U. Remaining peripheral pressure gradient should be sustained stably by high magnetic shear, shown by the graph of the rotational transform $\iota/2\pi$.

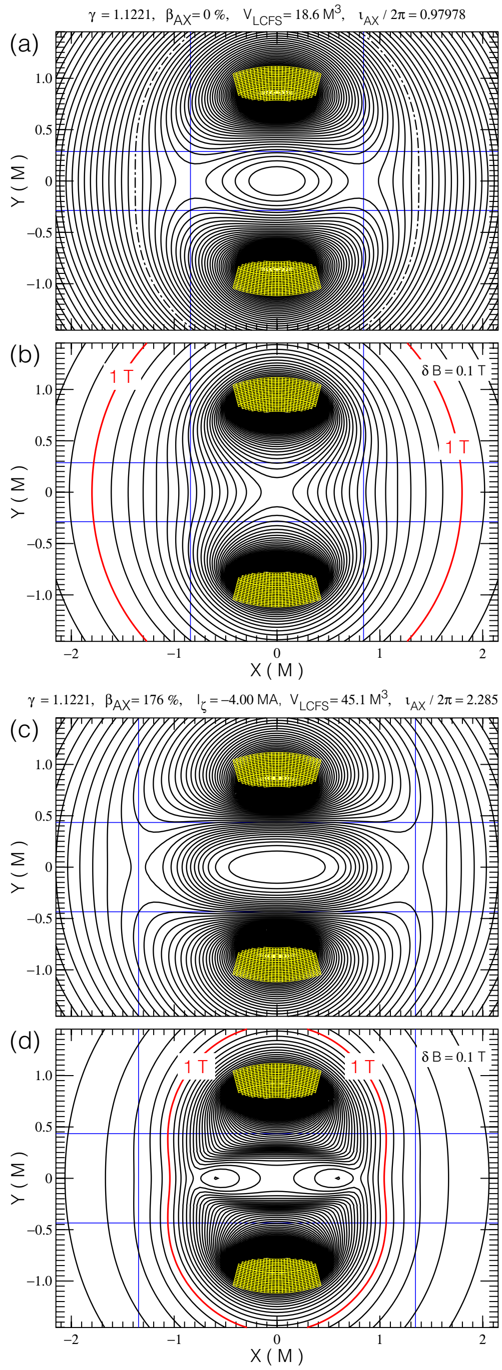

Fig. 6 Magnetic field structures for the case of $\gamma = 1.1221$. (a), (b) are for vacuum magnetic field and (c), (d) are for ultrahigh beta equilibrium. Contour plots for magnetic flux function (a, c) and magnetic field intensity (b, d) are shown together with cross sections of helical coils.

7. Summary and Discussions

We have developed a new numerical scheme to solve MHD equilibrium using Biot-Savart' law. This scheme can compute the equilibrium magnetic field of inner and outer region of the magnetic surface. Furthermore, we have developed a new numerical scheme to compute the MHD potential energy, which can evaluate the role of bootstrap current for the MHD stability of core plasma.

We have concluded that LHD type magnetic field configuration, which is produced by continuous winding helical coils and without the toroidal coil, can sustain MHD stable ultrahigh beta plasma. The MHD potential energy can be decreased by the pressure driven toroidal current (bootstrap current), even though plasma thermal energy increased. Since the MHD potential energy decreases according to the increase of the heating power input to plasma, MHD stable bootstrap transition to higher beta plasmas occur by ramp-up of heating power input.

By the comparison of the MHD potential energy, it is suggested that peaked pressure profile equilibrium make a bootstrap transition to the flat-top pressure

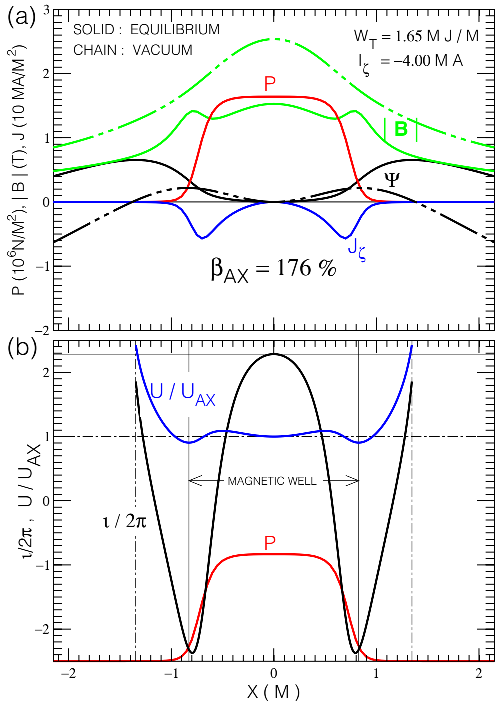

Fig. 7 (a) Pressure, bootstrap current, flux function and magnetic field intensity along X coordinate. (b) Specific volume U and rotational transform $\iota/2\pi$. Pressure profile is superimposed in this graph.

profile equilibrium, and small value of the helical pitch parameter is more favorable for the high beta plasma sustainment.

Ultrahigh beta core plasmas discussed in the paper have excellent features for the reactor system. As shown in Fig. 6(a,c), there is no drastic changes in diverter-leg of ultrahigh beta core plasma and vacuum magnetic field. Moreover, magnetic field in core plasma region retain almost the 60% of the value of the vacuum magnetic field, even for the case of $\beta_{\rm ax} = 176\%$ (see Fig.7(a)). This will be very much favorable for the alpha particle heating of core plasma. In addition, small magnetic surface volume for the vacuum magnetic field shown in Fig.6(a) may also be favorable for reactor systems, because the relatively small heating power unit become sufficient for the fusion ignition. After the ignition, high level fusion power output is expected because of large plasma volume of ultrahigh beta plasma.

In the present paper, we have analyzed a straight helical system, because of the simplicity. In this case, MHD equilibrium can be solved without any approximation. For the axisymmetric toroidal case, equilibrium of $\beta_{\rm ax} = \infty$ solutions have been numerically obtained[7], for the case of no toroidal magnetic field.

In toroidal helical systems chaotic field line region appears in surroundings of the magnetic surface. In this case, magnetic flux function is expressed by the adiabatic invariant of lines of force[8].

The flux function for the magnetic field $\boldsymbol{B}$ and for the plasma current $\boldsymbol{J}$, are reduced to the following[8],

$$\Psi = r\,A_\phi + p\big(-Y\,A_X + X\,A_Y\big) \qquad (27)$$

$$I = \frac{2\pi}{\mu_0}\left\{r\,B_\phi + p\big(-Y\,B_X + X\,B_Y\big)\right\} \qquad (28)$$

where p and r are pitch number and usual radial coordinate, respectively, and (X, Y, ϕ) is the toroidal rotating helical coordinate system[9].

In magnetic surface region, adiabatic invariants $\langle\Psi\rangle$ and $\langle I\rangle$ can be calculated.

$$\langle\Psi\rangle \equiv \frac{1}{2\pi}\oint \Psi {\rm d}\phi = \left\langle \Psi\frac{{\rm d}\phi}{{\rm d}\ell}\right\rangle , \qquad (29)$$

$$\langle I\rangle \equiv \frac{1}{2\pi}\oint I {\rm d}\phi = \left\langle I\frac{{\rm d}\phi}{{\rm d}\ell}\right\rangle , \qquad (30)$$

where $\langle\cdots\rangle$ represents the average along the lines of force. Numerical example of the adiabatic invariant $\langle\Psi\rangle$ is shown in Fig.8.

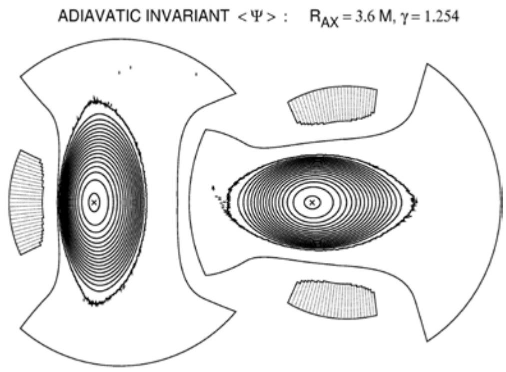

Fig. 8 Numerical example of the adiabatic invariant $\langle\Psi\rangle$ for the vacuum magnetic field of the LHD.

These adiabatic invariants satisfy the following relations,

$$0 = \boldsymbol{B}\cdot\boldsymbol{\nabla}\left(\langle\Psi\rangle\right) , \qquad (31)$$

$$0 = \boldsymbol{J}\cdot\boldsymbol{\nabla}\left(\langle I\rangle\right) . \qquad (32)$$

Then the MHD equation $\boldsymbol{\nabla} P = \boldsymbol{J}\times\boldsymbol{B}$ gives the following expression.

$$\boldsymbol{J} = \frac{I'(\Psi)}{2\pi}\boldsymbol{B} + P'(\Psi)\begin{pmatrix} -p\,Y \\ p\,X \\ r \end{pmatrix} + (\text{higher order corrections}) \qquad (33)$$

Because the plasma current $\boldsymbol{J}$ is given as a function of the magnetic vector potential $\boldsymbol{A}$, it seems that the equilibrium calculation of the toroidal helical system is possible by similar method written in §3.

The plasma volume is determined by the adiabaticity of lines of force of peripheral region of magnetic surface. The adiabaticity is controllable by the vertical field coils currents configuration. This physical situation is same for the vacuum or high beta magnetic configuration. In the LHD vacuum magnetic field, magnetic surfaces accompanied with very thin chaotic field line layers are confirmed by appropriate choices of the vertical field coils currents.

Numerical demonstration and validation of its results are future tasks.

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