EMTP Modeling of Air-Cored Transformer Windings Under High-Frequency Transient

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A systematic methodology is presented for establishing the high frequency circuit model of these transformers windings under VFTO based on transfer functions in this paper. Firstly, the voltage transfer functions are measured by network analyzer. Secondly, the voltage transfer functions are fitted with rational functions by vector fitting and then order-reduced. Lastly, the resultant voltage transfer functions are synthesized by lumped elements. The simulation results from the circuit are in good agreement with the measured. This shows that the method is correct.

Keywords: Vector fitting, voltage transfer function, lumped elements.

1. Introduction

Accurate representation of physical systems by a transfer function is often needed in various fields of engineering for purposes of analysis, design, or simulation. In many cases where sufficient information about the structure of the system is not available, frequency response measurements can be used as a convenient data base for estimation of the transfer function parameters. System identification by terminal characteristics may also be useful for the lumped parameter modeling of distributed systems, reduction of model order, and simplification of complex systems.

In the pulsed power engineering area, wide frequency range modeling of transformers and reactors by frequency domain external measurements is sometimes required for the study of electromagnetic transients. Besides, the "black-box" representation of such equipment in the study of electromagnetic transients, the components must be represented in a wide frequency range. This introduces numerical difficulties with most of the available methods. Considerable effort has been devoted to the development of methods for transfer function synthesis from frequency response observations. The frequency domain identification problem is based on the estimation of a rational complex function, with real coefficients, to fit a given set of complex data. The nonlinear nature of the problem has yielded different formulations and solution methods.

In order to perform transient studies using digital simulation, frequency response data is required to model all the different components or parts of a pulsed power system, whose characteristic is known in the frequency domain [1-3]. Therefore, fitting techniques are necessary to develop computational models. Analysis of the frequency range considering accuracy, the shape of the frequency response, the mathematical model form and the possibility of time-domain implementation are used to decide which fitting technique is most appropriate [4]. Throughout the years, many techniques have been proposed to fit the frequency response to rational functions to ensure accurate computational models for transient studies. Most techniques use linear fitting routines [1-4] but nonlinear procedures have also been considered [5]. The high frequency model of the transformer winding is indispensable in analyzing the transient particularly caused by the very fast transients which occur in its operation.

There has been a great deal of research on transformer modeling using a variety of models. In general, there are two main categories of models. One is the terminal or black box model, which provides the terminal characteristics of a transformer and is not necessarily related to a transformer internal condition and physical configuration. This type of model mainly describes the terminal and characteristics, and can be constructed by various methods, [e.g., mathematical equations or network analysis (poles and zeros)]. The other type of transformer model is the physical model; it can either model all parts of the transformer in great detail or can be constructed according to gross physical components such as the winding layers. These types of models use network equivalent parameters (resistances, inductances and capacitances) and focus on the frequency range of interest

In this paper, to prepare an EMTP model of air-cored transformer windings, the voltage transfer functions are calculated from the measurements which are done by network analyzer. Secondly, the voltage transfer functions are fitted with rational functions by vector fitting codes and

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then the rational transfer functions are order-reduced. In this way, it is possible to integrate the detailed model of pulsed transformers into the circuit analysis softwares like EMTP. The estimated voltages on the transformer windings using the proposed model of this paper are in good agreement with the measurements.

2. Winding modelling

Mainly there are three methods to simulate such a transformer consisting two windings, Lumped Circuit Model, Multiconductor Transmission Line (MTL) Model and Full-Wave Solution. These methods will be used in conditions where the internal behavior of the windings is requisite. Black box model of equipment such as a transformer is used when the output transient is needed. To achieve such a model, a frequency response analysis of such equipment is needed and via this survey, the transfer function will be obtained. In Fig. 1, the lumped circuit model of the pulsed air-cored transformer is depicted. The lumped element of such a transformer is obtained via finite element method (FEM).

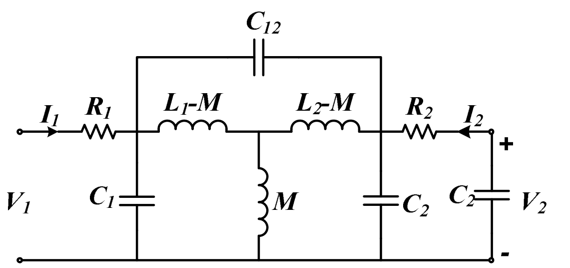


Fig.1 Lumped circuit model of the air-cored transformer

3. Vector Fitting by Pole Relocation

Consider the rational function approximation

$$f(s) \approx \sum_{n=1}^{N} \frac{c_n}{s - a_n} + d + sh \qquad (1)$$

The residues c_n and poles a_n are either real quantities or come in complex conjugate pairs, while d and h are real. The problem at hand is to estimate all coefficients in (1) so that a least squares approximation of *f(s)* is obtained over a given frequency interval. We note that (1) is a nonlinear problem in terms of the unknowns, because the unknowns *a,* appear in the denominator. Vector fitting solves the problem (1) sequentially as a linear problem in two stages, both times with known poles.

Stage #1: Pole identification

Specify a set of starting poles $\overline{a}_n$ in (1), and multiply *f(s)* with an unknown function σ(s). In addition we introduce a rational approximation for σ(s). This gives the augmented problem:

$$\begin{bmatrix} \sigma(s) f(s) \\ \sigma(s) \end{bmatrix} \approx \begin{bmatrix} \displaystyle\sum_{n=1}^{N} \frac{c_n}{s - \overline{a}_n} + d + sh \\ \displaystyle\sum_{n=1}^{N} \frac{\widetilde{c}_n}{s - \overline{a}_n} + 1 \end{bmatrix} \qquad (2)$$

Note that in (2) the rational approximation for σ(s) has the same poles as the approximation for σ(s)f(s). Also, note that the ambiguity in the solution for σ(s) has been removed by forcing σ(s) to approach unity at very high frequencies. Multiplying the second row in (2) with *f(s)* yields the following relation:

$$\left(\sum_{n=1}^{N} \frac{c_n}{s - \overline{a}_n} + d + sh \right) \approx \left(\sum_{n=1}^{N} \frac{\widetilde{c}_n}{s - \overline{a}_n} + 1 \right) f(s) \qquad (3)$$

Or

$$(\sigma f)_{fit}(s) \approx \sigma_{fit}(s) f(s) \qquad (4)$$

Equation (3) is linear in its unknowns c_n, d, h, $\widetilde{c}_n$. Writing (3) for several frequency points gives the overdetermined linear problem

$$Ax = b \qquad (5)$$

where the unknowns are in the solution vector *x.* Equation (5) is solved as a least squares problem. Details about the formulation of the linear equations are shown in [1]. A rational function approximation for *f(s)* can now be readily obtained from (3). This becomes evident if each sum of partial fractions in (3) is written as a fraction:

$$(\sigma f)_{fit}(s) = h \frac{\prod_{n=1}^{N+1} (s - z_n)}{\prod_{n=1}^{N} (s - \overline{a}_n)}, \sigma_{fit}(s) = \frac{\prod_{n=1}^{N} (s - \widetilde{z}_n)}{\prod_{n=1}^{N} (s - \overline{a}_n)} \qquad (6)$$

From (6) we get

$$f(s) = \frac{(\sigma f)_{fit}(s)}{\sigma_{fit}(s)} = h \frac{\prod_{n=1}^{N+1} (s - z_n)}{\prod_{n=1}^{N} (s - \widetilde{z}_n)} \qquad (7)$$

Equation (7) shows that *the poles of f(s) become equal to the zeros of* σ_{fit}(s) (Note that the starting poles cancel in the division process because we use the same starting poles for $(\sigma f)_{fit}$ and for σ_{fit}(s)). Thus, by calculating the zeros of σ_{fit}(s) we get a good set of poles for fitting the original function *f(s).* The calculation of zeros from the representation by partial fractions (4) is straightforward, as shown in [3]. On occasion, some of the new poles may be unstable. This problem is overcome by inverting the sign of their real parts.

Stage #2: residue identification

In principle we could now calculate the residues for *f(s)* directly from (7). However, a more accurate result is in general obtained by solving the original problem in (1) with the zeros of σ(s) as new poles a_n for *f(s)*. This again gives an overdetermined linear problem of form Ax = b where the solution vector *x* contains the unknowns c_n, *d* and *h*.

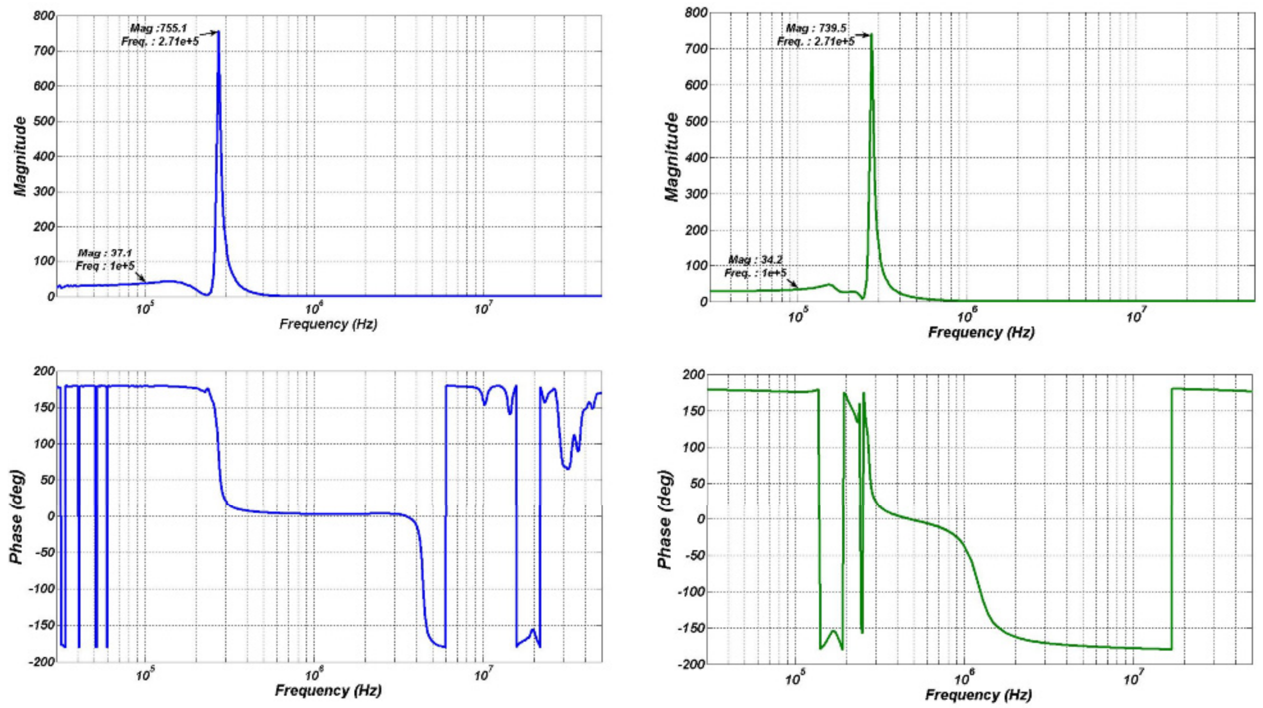
4. Results and discussion

The computation of the amplitude and phase spectrum of the air-cored transformer winding takes place with 1000 measuring values, recorded in the range of 25 kHz to 50 MHz via Agilent 4395A, a network spectrum impedance analyzer. The resultant transfer function includes 8 poles and 8 zeros which are listed in table I. In Fig. 2, the resultant magnitude and phase of the voltage transfer function are depicted and compared. As it can be seen, there is a very acceptable correspondence between the experimental and simulation results. The investigations show, that a computation of the transfer function of a transformer is possible with the presented method even in frequency ranges, which can only be approximated inadequately with methods used until now based on equivalent circuits and corresponding impedance values. Comparison between output results of this transformer is displayed in Fig. 3 which declares good similarities between them that shows the capabilities of the proposed method.

Also, the magnitude of the transfer function via lumped circuit model is depicted in Fig.4 which has a great correspondence with experimental and computed transfer functions.

Table I. zeros and poles of the fitted transfer function

Zeros	1e+6 *[0.0773 - 1.6673i, -0.0440 - 1.7040i, -0.1265 - 1.5661i, 0.1138 - 1.0128i, 0.0773 + 1.6673i, -0.0440 + 1.7040i, -0.1265 + 1.5661i, 0.1138 + 1.0128i]
Poles	1e+6 *[0.0783 + 1.6693i, 0.0773 - 1.6673i, -0.0441 + 1.7040i, -0.0441 - 1.7040i, -0.1265 + 1.5661i, -0.1265 - 1.5661i, 0.1137 + 1.0128i, 0.1137 - 1.0138i]



a) Experimental results ***b) Results via vector fitting procedure***

Fig.2. Obtained magnitude and phase of the Transfer Function, a). By experiment, b). By vector fitting

5. Conclusions

To compute the transfer function, an analytical procedure is used which is able to detect the system parameters. A verification of the suitability of the used method takes place by the calculation of the transfer functions of a two winding air-cored transformer up to frequencies of 50 MHz.

The method is suitable for an automatic calculation of transformer transfer functions without basic restrictions of the considered frequency ranges. As is described, a methodology for the estimation of a transfer function from frequency response measurements is presented and discussed that shows the accuracies of this method.

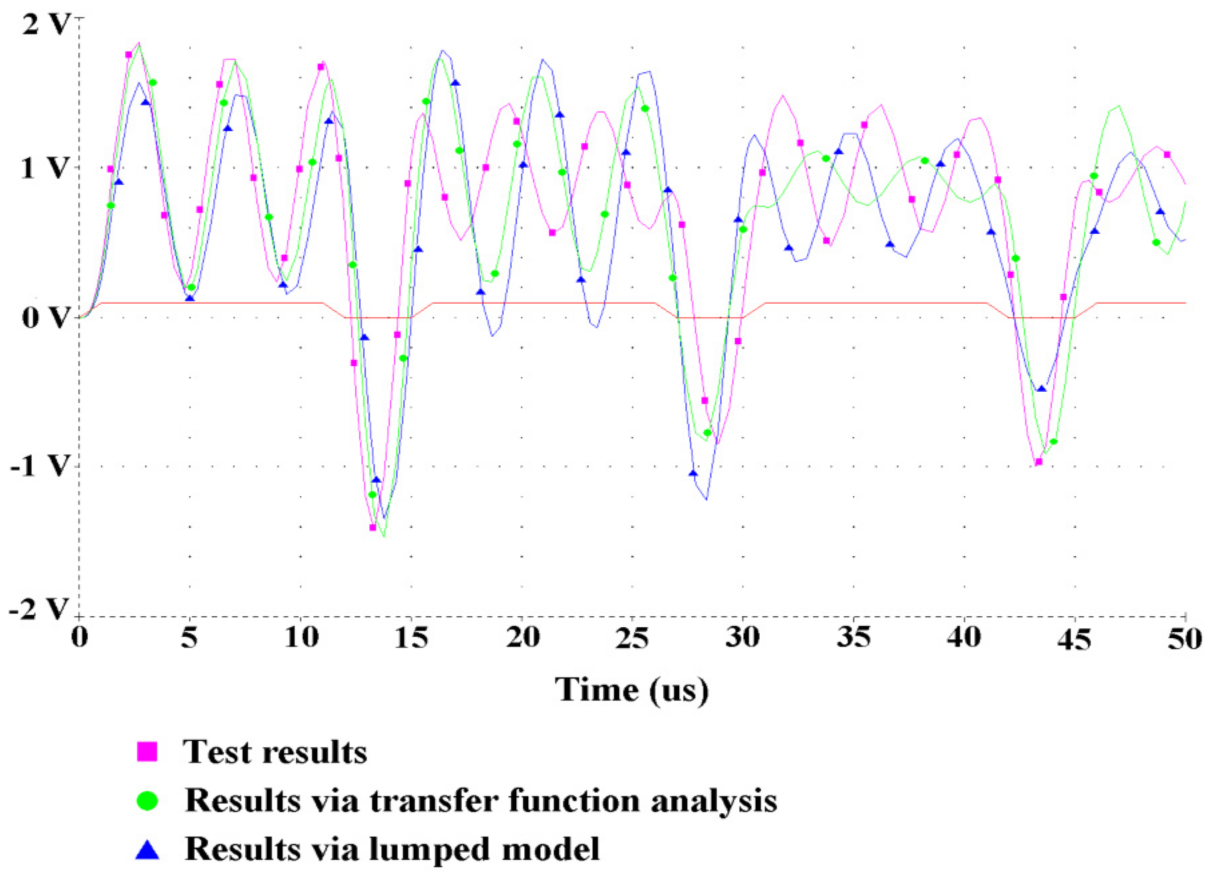


Fig.3 Output results via test results, transfer function analysis and lumped model simulation

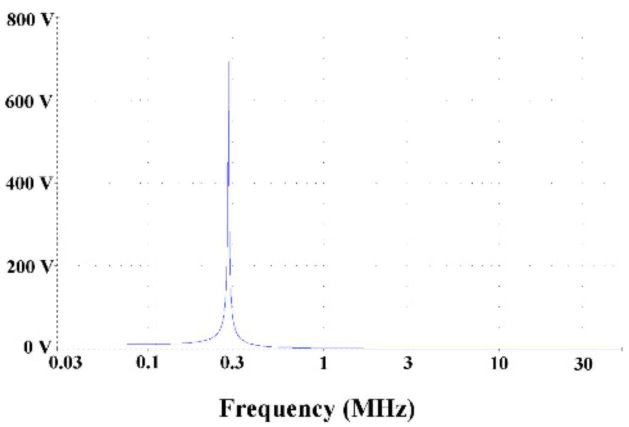


Fig.4 Magnitude of the transfer function via lumped circuit model

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