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Estimation of Very Fast Transient Voltage Distribution in air-cored Pulsed Transformer Windings Based on FDTD Method

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To study the voltage distribution in transformer windings under VFTO, a model for transformer windings has been developed based on Multi-conductor Transmission-Lines (MTLs) theory and the computing equations of the model based on Finite- Difference Time-Domain method (FDTD) is deduced. The Vector Fitting method is adopted to treat the frequency-dependent parameter by expanding the time-domain form of internal impedance of the transformer windings into a sum of finite exponential functions which are easy for the time-domain convolution. For validation, an actual transformer winding model has been studied. The calculated results are compared with the measured result and a satisfactory result is obtained.

Keywords: Very Fast Transient Overvoltages (VFTO), Multi-conductor Transmission-Lines (MTLs), Finite-Difference Time-Domain method (FDTD), Voltage distribution.

1. Introduction

In a nanosecond periodically pulsed generator, the magnitude of output pulse of a tesla transformer's secondary winding is even higher than several hundred kilovolts. The reflected wave travels back and injects the secondary winding with a high amplitude, steep wave front, and high repeating frequency, which comprises high-order harmonics that can cause internal resonance and lead to an extremely uneven distribution of turn-to-ground voltage and interturn voltage. This often results in partial discharge and interturn insulation failure in the winding. So it is necessary to study the very fast transient overvoltage (VFTO) in a tesla transformer's secondary winding. The VFTOs depend not only on the incident wave's style, but also on the structure of winding. Most of the related studies emphasize the wave propagation in windings of ac machines and power transformers. Seldom are done either on the simulation or experiment of transient behavior of a tesla transformer's taper winding. There are three methods to simulate the transient behavior of the secondary winding of an air-cored pulse transformer. The first method uses the lumped circuit model and is accurate enough when the winding's length is far less than the wavelength. This model consists of lumped element, including self-inductance, shunt capacitance and resistance, mutual inductance and capacitance. Electromagnetic transient program (EMTP) or frequency-domain method can achieve the exact response of the whole network after linking all these elements. Initial works [1-3] showed its successful application in predicting VFTO or partial discharge in windings of power transformer and ac machines. The other method utilize MTL model and is more accurate while the pulse has a shorter rise or fall time or the coils are long enough to compare with the wavelength. In the situation, each coil should be taken as a distributed line. The voltage distribution along each coil can be calculated by solving the telegraphist's equations with given boundary conditions. The most recent research [4-6] on the wave's propagation in windings used the MTL model combined with the single transmission line (STL) model.

The third model is Full-wave solution which is a direct field solution and is hard to obtain due to the complexity of model and lots of degrees of freedom.

Other related topics can also be found. The study of the voltage distribution in transformer windings under VFTO is of essential interested for insulation design [7, 8]. An accurate simulation model is the most fundamental conditions for the research of the voltage distribution in transformer windings. Because the frequency of VFTO is very high, the lump parameter model couldn't satisfy the computation request. The multi-conductor transmission lines model with distribution parameter is an effective method to solve the problem [9-11].

To study the voltage distribution in transformer windings under VFTO, a model for transformer windings has been developed based on Multi-conductor Transmission-Lines (MTLs) theory and the computing equations of the model based on Finite-Difference Time-Domain method (FDTD) is deduced. The Vector

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Fitting method is adopted to treat the frequency-dependent parameter by expanding the time-domain form of internal impedance of the transformer windings into a sum of finite exponential functions which are easy for the time-domain convolution. For validation, an actual transformer winding model has been studied. The calculated results are compared with the measured result and a satisfactory result is obtained.

2. MTL Model

An n-conductor uniform lossy transmission line system is constructed by splitting the winding along its axial plane and spreading n turns into parallel lines. This modeling assumes that the average perimeter of coils is far longer than the winding's axial length. Thus, by an initial capacitive coupling, the voltage distribution will be established instantaneously across the whole winding. The lengths of adjacent lines are almost the same because of the winding's small taper angle (5°). As for any two distant lines, though they are quite different in length, their electromagnetic coupling is weak enough to be neglected, compared with two adjacent ones. So the taper winding in this case can be treated as an equal-length MTL model shown in Fig. 2.

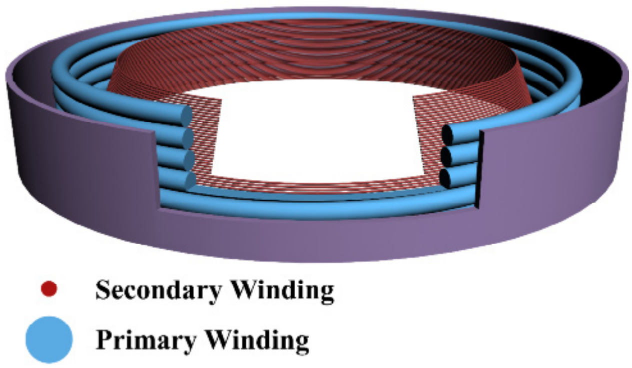

Fig.1 Typical configuration of a tesla transformer

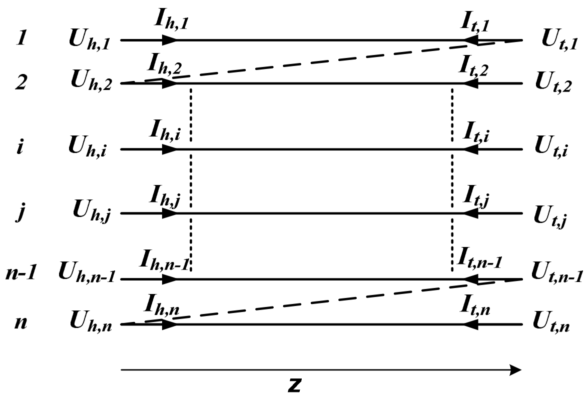

Fig.2 MTL model of the tapered secondary winding.

In Fig.2, u_{ih}, u_{it}, i_{ih} and i_{it} are the head and tail end's voltages and currents of the ith line, respectively. Arrows indicate the positive direction of currents.

When every turn in the winding is represented as a transmission line, then the propagation phenomena in transformer winding can be fully described by making use of the telegraphist's equations in Laplace domain

$$\frac{d}{dz}V(z,s) = -[Z_i + sL]I(z,s)$$
$$\frac{d}{dz}I(z,s) = -[G + sC]V(z,s) \qquad (1)$$

where V and I are vectors of the line voltages and line currents, respectively; s is the Laplace transform variable; The position along the line is denoted as z; L and C are the per-unit length external inductance matrix and capacitance matrix; the internal impedance contains both resistance and internal inductance as Z_i=R(s)+sL_i(s); G will be neglected because it is too small.

3. Electrical Parameters Evaluation

3.1. Capacitance Coefficient Matrix

Based on the electromagnetic theory, total static electric energy W_e of an n-conductor system in region V can be given by

$$W_e = \frac{1}{2}\int_V E \cdot D dV = \frac{1}{2}\sum_{i=1}^{n}\left(C_{ii}u_i^2 + \sum_{\substack{j=1 \\ j\neq i}}^{n} C_{ij}u_i u_j \right) \qquad (2)$$

Where E and D are electrical field intensity and electric flux density, u_i and u_j are the voltage to ground of the ith and jth conductors. Also, C_{ii} is a self capacitance of the ith conductor and C_{ij} is a mutual capacitance and C_{ij}=C_{ji}.

To evaluate the elements of C by field method, the following procedure have been used. To calculate K_{ii}, a unit potential was applied on the ith conductor, while the others were set to zero potential. Due to an approximate axisymmetrical structure of the model, a 2-D static electric field FE (Finite Element) analysis can calculate W_e. Then by (2), the diagonal elements of C are obtained.

3.2. Inductance Matrix

Elements in L are evaluated by the following definitions:

$$L_{ii} = \frac{\psi_{ii}}{I_i} \qquad (3)$$

where L_{ii} and Ii are the self-inductance and current of the ith coil and ψ_{ii} represent the self flux linkages of ith coil.

Fig.3 depicts the actual FE model of the taper secondary winding.

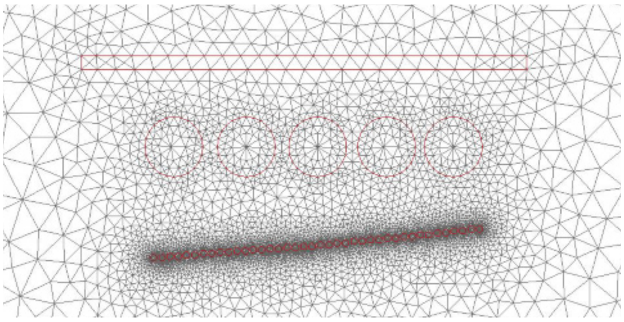

Fig.3 Mesh view

4. The FDTD Method

The transmission lines are divided into NZ sections and each of length is Δz, similarly, the total solution time is divided into NT segments of length Δt. To insure second-order accuracy of the discretization, interlacing the NZ+1 voltage points and the NZ current point, each voltage and adjacent current points is separated by Δz/2. In addition, the time points are also interlaced, and each voltage time point and adjacent current time point are separated by Δt/2. Then the voltage recursion relations can be obtained from (7) using second-order central differences.

A common way of approximating the internal impedance term with the Laplace transform variable is as

$$Z_i(s) = A + B\sqrt{s} \qquad (4)$$

Thus A represents the dc per-unit-length resistance matrix. The component $B\sqrt{s}$ represents the high frequency per-unit-length resistance matrix as well as the high frequency per-unit-length internal inductive reactance matrix.

The quantities A and B for a wire of radius r_w are

$$A = r_{dc} = \frac{1}{\sigma \pi r_w^2}, \qquad B = \frac{1}{2\pi r_w}\sqrt{\frac{\propto}{\sigma}} \qquad (5)$$

where σ and μ are the electrical conductivity and magnetic permeability of the conductor, respectively.

Next considering the current recursion relations, Z_i needs to be changed because it is frequency-dependent parameter. In the Laplace domain, it is expressed as follows by using the vector fitting method

$$Z_0(m) = \int_m^{m+1} \frac{1}{\sqrt{s}} ds \cong \sum_{i=1}^{N} a_i e^{m\alpha_i} \qquad (6)$$

The voltage recursion relations can be obtained from the following formula using second-order central differences

$$V_k^{n+1} = V_k^n - \frac{\Delta t}{\Delta z} C^{-1}\left(I_k^{n+1/2} - I_{k-1}^{n+1/2}\right) \quad k = 2,\cdots,N\Delta z \quad (7)$$

The current relation can be found as follows

$$I_k^{n+3/2} = F^{-1}\left(L\frac{\Delta z}{\Delta t} - A\frac{\Delta z}{2} + B\frac{\Delta z}{\sqrt{\pi \Delta t}} Z_0(0)\right) I_k^{n+1/2} - F^{-1} B \frac{\Delta z}{\sqrt{\pi\Delta t}} \sum_{i=1}^{n} \Psi_i^n - F^{-1}\left(V_{k+1}^{n+1} - V_k^{n+1}\right)$$
$$k = 1,\cdots,N\Delta z \qquad (8)$$

where

$$\Psi_i^n = a_i e^{\alpha_i}\left[I_k^{n+1/2} - I_k^{n-1/2}\right] + e^{\alpha_i}\Psi_i^{n-1} \qquad (9)$$

$$F = \left(L\frac{\Delta z}{\Delta t} + A\frac{\Delta z}{2} + B\frac{\Delta z}{\sqrt{\pi\Delta t}} Z_0(0)\right) \qquad (10)$$

where n represents the time point; k represents the position point.

5. Simulation and Test Results

The basic parameters of the transformer winding used in the experiment are described in Table I. In the experiment, VFTO is simulated by the voltage signal waveforms of 100 kHz pulse.

Table I: basic parameters of windings

Winding	Secondary	Primary
Turn number	40	5
Radius of conductor(mm)	0.15	1.4
Winding diameter(mm)	80	85

Fig.4 shows the output voltages of turns 30th, 24th, 13th and 7th. In this figure, the solid line is the calculated waveform using the method in this paper and dashed line is the measured one.

Comparing the measuring and simulating results in Fig.4, we can see their waveforms are relatively close. These results have also indicated that the model and the method are effective and accurate.

6. Conclusion

In this paper, the voltage distribution in transformer windings under VFTO has been studied. The paper develops a model of transformer windings based on MTLs theory and deduces the computing equation for the MTLs model based on FDTD method. The Vector Fitting method is adopted to treat the frequency-dependent parameter by expending the time domain form of internal impedance into a sum of finite exponential functions which are easy for the time-domain convolution. The measured and simulated results are compared; the fact that their waveforms are relatively close has indicated that the model and the method used in this paper are effective and accurate.

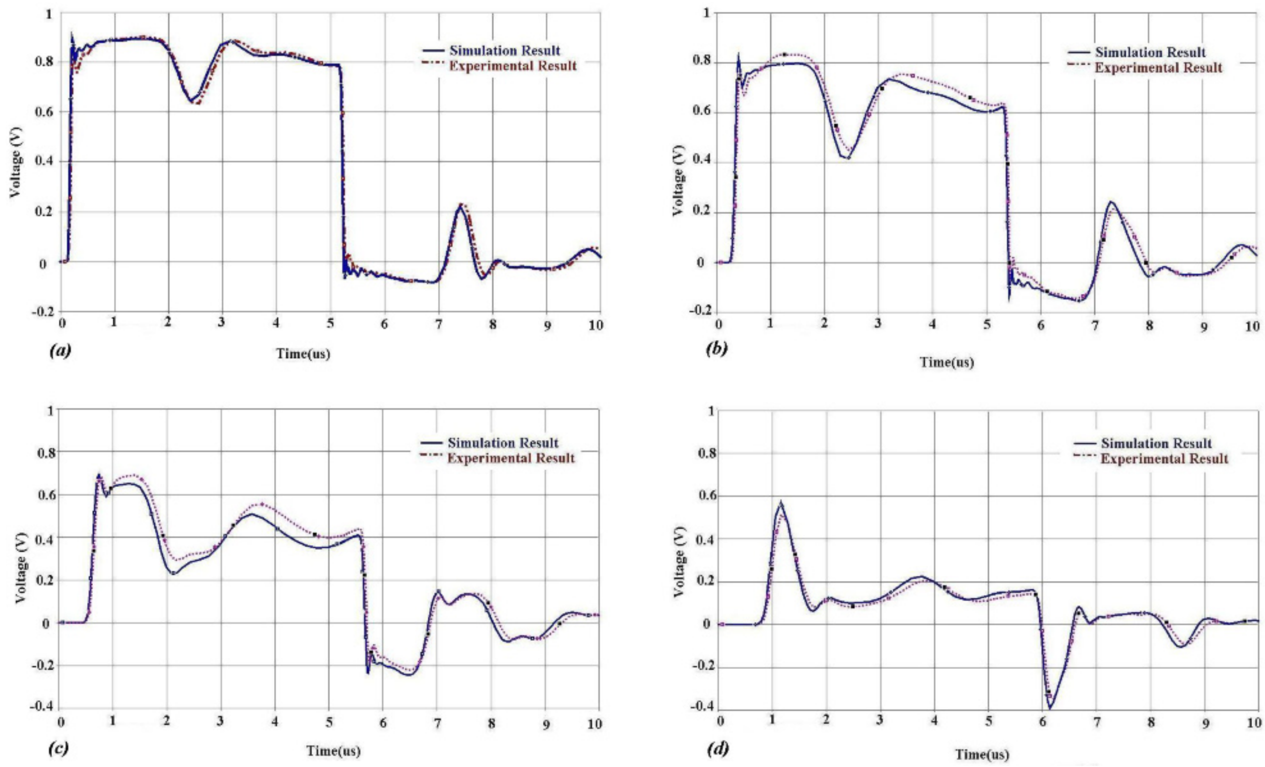

Fig.4 Comparison between numerical and measured values. Voltage to ground of turn 30, 24, 13 and 7

7. References

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