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The Investigation of Rayleigh-Taylor Instability Growth Rate in Inertial Confinement Fusion

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Because the Rayleigh-Taylor instability (RTI) growth rate in inertial confinement fusion (ICF) is so important and critical for determining the required driver energy, many attempts have been made over the years to drive it analytically. In this paper, the growth rates of the acceleration and deceleration –phase Rayleigh-Taylor instability for imploding inertial confinement fusion targets are calculated analytically and numerically. Also, the effects of different physical parameters on RTI are investigated. For this purpose, we have calculated the growth rate, growth factor and ablation mode for outer surface and also inner surface of fuel pellet.

Keywords: inertial confinement fusion, hydrodynamic stability, Rayleigh-Taylor instability, growth rate, growth factor

1. Introduction

In inertial confinement fusion, laser beams irradiate a small spherical capsule, causing it to implode. The compression of the capsule during the implosion creates conditions for nuclear fusion, a reaction where deuterium (D) and Tritium (T), two isotopes of hydrogen, collide and fuse, forming a larger Helium-4 atom as well as a neutron, thereby converting mass into kinetic energy.

The laser pulse is designed to drive multiple shocks through the shell and to accelerate it to the implosion velocity required for ignition. The time interval corresponding to the shell acceleration is commonly referred to as the acceleration phase. The shocks set the shell on the desired adiabatic and merge into a single shock before reaching the shell's center. Such a single shock is reflected off the center and impulsively slows down the incoming shell. Additional shocks may be reflected off the shell and its center until the lower density material enclosed by the shell (the so-called hot spot) reaches a sufficiently large pressure to slow down the shell in a continuous (not implosive) manner. Such a continuous slowing down of the shell up to the stagnation point occurs over a period of a few hundred picoseconds and is referred to as continuous deceleration phase. During the deceleration phase, the hot spot pressure, density and temperature increase until reaching the ignition conditions determined by temperatures and areal densities exceeding 10 KeV and 0.3 gr/Cm^2, respectively. It is well known that the shell's outer surface is unstable to the Rayleigh-Taylor instability during the acceleration phase; however, because of mass ablation, the instability growth rates are significantly reduced [1].

The thickness of ICF shells is chosen to prevent the shell from breaking up when the RT bubble amplitude equals the shell thickness. Even when the shell integrity is preserved during the acceleration phase, the hot-spot ignition can be quenched by the deceleration–phase instability. The latter is the instability of the shell's inner surface that occurs when the shell is decelerated by high pressure building up inside the hot spot. The deceleration RT causes the cold shell material to penetrate and cool the hot spot, preventing it from achieving ignition conditions. Typical seeds for the deceleration-phase RT are the surface nonuniformities that feed through the shell from the outer surface during the acceleration-phase instability.

A major problem associated with ICF is the Rayleigh-Taylor instability. The stability of an ICF implosion is one of the primary factors determining the target gain and whether the target will be successfully ignite. Peak fuel compression, hot spot formation, symmetry of the core and ultimately neutron yield are all affected by hydrodynamic instabilities. In the startup phase, the shock-driven Richtmyer-Meshkov instability [2], is present at the ablation surface. It determines the seeds of the ablative RT instability that lead to significant perturbation growth during both the acceleration and deceleration phases of implosion and thus has the most damaging role for the distortion of the shell and reduction of compression symmetry. The schematic view of the inner surface of the shell is shown in Fig.1.

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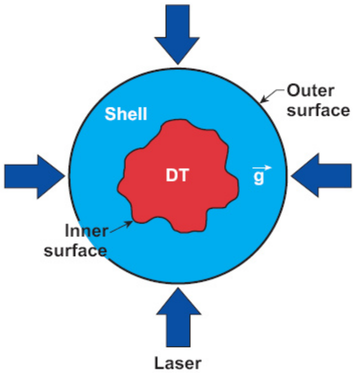

Fig. 1 Schematic view of the inner surface of the shell.

2. Basic Equations

The Rayleigh-Taylor instability occurs when a light fluid accelerates heavier one, or in other words, when the density gradient and effective fluid acceleration have opposing orientations. In fact, in a gravitational field, where a heavy fluid with density ρ_1 is on top of a lighter fluid with density ρ_2, the interface is in unstable equilibrium and any perturbation will grow until the fluids exchange their places (the potential energy is minimized).This happens at the ablation front during the acceleration phase of an ICF implosion when the hot ablating plasma is pushing the much denser (and colder) shocked shell material inwards. The R-T instability also occurs when the shell starts to decelerate, slightly before compression. During this phase, the hot and low density core is decelerating the denser and colder shell and perturbations on the inner shell interface grow. The R-T instability can also be present before the start of shell acceleration. This occurs for target designs where higher density ablator material (such as CH) is over coated on the outside of the DT capsule. Mathematically, R-T instability when is produced that the pressure gradient and density gradient are unparallel with together. In other words, we have the following relation:

$$\vec{\nabla} P \vec{\nabla} \rho \prec 0 \qquad (1)$$

This relation is well known as general relation of hydrodynamic instability and is very important for determining of R-T instability at two phases in ICF. At first, this instability is produces on outer surface of fuel capsule in acceleration phase; because in this phase regarding pressure and density, there are two regions. One region of cold fuel with high density and low pressure and another region is the corona around the cold fuel with high pressure and low density. Therefore, the relation (1) is satisfied and instability is occurred. At deceleration phase, this instability is also occurred on inner surface of fuel capsule; because in comparison to its around fuel the pressure is higher and density is lower in inside of hot spot; therefore the general condition of instability is satisfied. It is important note that these two instabilities didn't occur with together.

In the ablation phase, the energy is deposited in narrow low-density region in the plasma, where a high-pressure is created directly next to the high-density layer that becomes accelerated inward. Therefore, there is an additional flow of material across the ablation surface from the high-density region into the low-density plasma.

The growth rate of classical R-T instability, with assuming discontinuity in density of two fluids is given by:

$$\sigma = \sqrt{Aak} \qquad (2)$$

where k is the wave number of the Fourier component, a is the acceleration and A is the Atwood number .The A parameter is related to density of two fluids with following relation:

$$A = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \qquad (3)$$

Betti, et al. have determined the following relations for determining growth rate in ICF [3]:

$$\sigma = \alpha_l(F,\nu)\sqrt{aK} - \beta_l(F,\nu)KU_a \qquad (4)$$

$$\sigma = \alpha_2(F,\nu)\sqrt{\frac{\alpha K}{1+KL_{min}}} - \beta_2(F,\nu)KU_a \qquad (5)$$

These relations are well known as "Takabe relation" and "generalized Takabe relation" respectively. In these relations, the coefficients $\alpha_1, \alpha_2, \beta_1 and \beta_2$ are functions of dimensionless numbers of the Froude number ($F = \frac{U_a^2}{aL_0}$, U_a is the ablation velocity and L_0 is the ablation –front thickness) and the effective power index for thermal conduction, ν. These coefficients are determined by Betti's diagrams [3].

The gradient length, L, or the scale length of disturbance is given by:

$$L = \frac{\rho}{\vec{\nabla}\rho} \qquad (6)$$

and is expressed the continuity of fuel density. The minimum value of gradient length is given by:

$$L_{min} = \min\left(\frac{\vec{\nabla}\rho}{\rho}\right)^{-1} \qquad (7)$$

The relations (4) and (5) are the basic relations of linear theory of Rayleigh-Taylor instability in ICF.

The growth factor related to l mode on outer surface of fuel capsule in acceleration phase is given by [4]:

$$G_1^{out} = \exp(\int_0^{t_0} \sigma_l^{out}(t)dt) \quad (8)$$

where $\sigma_l^{out}(t)$ is growth rate on outer surface.

Also, the growth factor related to l mode on inner surface of capsule in deceleration phase is given by:

$$G_l^{in} = \exp(\int_{t_{dec}}^{t_{dec}+\Delta t_{dec}} \sigma_l^{in}(t)dt) \quad (9)$$

where $\sigma_l^{in}(t)$ is growth rate on inner surface. The integrating is from starting time of deceleration until ignition time.

The transport growth factor (G_l^{fed}) related to l mode, is transported instabilities from outer surface of fuel capsule to its inner surface and we can estimate with following relation:

$$G_l^{fed} \approx \exp(-\frac{l\Delta R}{R}) \quad (10)$$

where $\frac{\Delta R}{R}$ is the ratio of fuel capsule thickness at time of internal implosion to its main radius, and its reverse ($A_{if} = \frac{R}{\Delta R}$) is well known as "flight aspect ratio". Finally, total growth factor is calculated from multiplying of mentioned three growth factors:

$$G_T = G_l^{out} G_l^{fed} G_l^{in} \quad (11)$$

3. Calculation of Growth Rate

To determine growth rate at spherical geometry, instead of K at relation (5), we substitute [5]:

$$K = \frac{l+1}{R} \cong \frac{l}{R}$$

Then, we have following relation:

$$\sigma = \alpha_2 \sqrt{\frac{al/R}{1+L_{min} l/R}} - \beta_2 \frac{l}{R} u_a \quad (12)$$

With assuming constant values of acceleration for outer surface of fuel capsule at time of fuel acceleration, from $R = R_0$ to $R = R_0/3$, when occurs internal implosion, we will have:

$$t_0 = \sqrt{\frac{4R_0}{3a}} \quad (13)$$

Now, according to Atzeni's and et al. parameterization method [4], we have parameterized the L_{min} and U_a as following:

$$L_{min} = f_1 \Delta R \quad (14)$$

$$u_a = f_2 \frac{\Delta R(t_0)}{t_0} \quad (15)$$

where f_1 and f_2 are numerical constants.

With substituting relations (14) and (15) to relation (12) and then substituting obtained relation to relation (8) and calculation of integral (for above limit of integral we use from relation(13)), finally we will determine the following relation for growth factor on outer surface of fuel capsule:

$$G_l^{out} = \exp[\alpha_2 (\frac{4/3\, l}{1+lf_1 \frac{\Delta R}{R}})^{1/2} - \beta_2 f_2 l \frac{\Delta R}{R}] \quad (16)$$

With substituting G_l^{out} =1 and after mathematical calculations, we have obtained the cut-off mode (l_{cut}) parameter:

$$l_{cut} = \frac{1}{2f_1} \frac{R}{\Delta R} (\sqrt{1 + \frac{16}{3} \frac{f_1 \alpha_2^2}{f_2^2 \beta_2^2} \frac{R}{\Delta R}} - 1) \quad (17)$$

where the modes with $l \succ l_{cut}$ are stable.

For calculation of growth factor on inner surface of fuel capsule, we suppose that the radius of hot spot can reduce from R_{dec} to R_h with constant acceleration. Therefore, we will have:

$$(R_{dec} - R_h) = 1/2\, a(\Delta t_{dec})^2 \quad (18)$$

Now, we can parameterize the reduction of the hot spot radius by:

$$R_{dec} - R_h = f_3 R_h \quad (19)$$

And also, the parameters of gradient length (L_m) and cut-off velocity on inner surface of fuel capsule ($U_{a,in}$) will be:

$$L_{in} = f_4 R_h \quad (20)$$

and

$$u_{ain} = f_5 \frac{R_h}{\Delta t_{dec}} \quad (21)$$

With substituting the relations (18)-(21) to relation (9) and integrating, we can obtain:

$$G_l^{in} = \exp[\alpha_2 (\frac{2 f_3 l}{1 + f_4 l})^{1/2} - \beta_{in} f_5 l] \quad (22)$$

Also, with substituting $G_l^{in} = 1$,we can obtain the cut-off mode for inner surface of fuel capsule:

$$L_{cut} = \frac{1}{2 f_4} [\sqrt{1 + \frac{8 f_3 f_4 \alpha_2^2}{\beta_{in}^2 f_5^2}} - 1] \quad (23)$$

In Fig.2, we have shown the variations of growth factor on outer surface of fuel capsule versus mode number for A_{if} = 10. As we see from this figure, increasing βf_2 leads to decreasing growth factor and as a result leads to decreasing instability. Therefore, increasing βf_2 is well known as a positive factor in hydrodynamic stability.

In Fig. 3, we sketched the variations of growth factor on outer surface of fuel capsule versus mode number for A_{if} =30. From comparison of this figure with Fig.2, we have concluded that increasing the flight aspect ratio parameter will cause to increasing instability; therefore its increasing is a negative factor in hydrodynamic stability.

In Fig. 4, we have shown the variations of G_1^{out} versus mode number for different values of A_{if} =10, 20, 30. We have concluded that increasing the A_{if} parameter will cause to increasing instability; therefore its increasing is a negative factor.

In Fig. 5, we have shown the variations of growth factor on outer surface of fuel capsule versus βf_2 and A_{if} parameters in a three dimensional diagram. Because this obtained curve is an increment curve, we found that the influence of A_{if} in instability is more than βf_2; in other wise, this curve should be a decrement curve.

In Fig. 6, we have sketched the total growth factor (relation (11)) and the growth factors on outer and inner surface of fuel capsule and also the variations of $G_1^{out} G_1^{fed}$ for a specific target. As it is clear from this figure, the smaller value of $G_1^{out} G_1^{fed}$ in comparison to G_l^{out}, show the transferred instability from outer surface of fuel capsule in acceleration phase to inner surface during the path is weakened.

4. Summary

In final conclusion with calculation of growth factors of cut-off modes and their sketches, we have determined range and peak of critical modes. The knowing of this parameter is very important, in regarding to technology. Also, we have concluded, the ratio of thickness to radius of fuel capsule has an important role in instability and the smaller value of this parameter is better. Also, increasing of βf_2 parameter will cause to decreasing, and increasing of A_{if} parameter will cause to increasing of RT instability. Our results have shown the influence of A_{if} is more than βf_2 in reach to hydrodynamic stability in ICF.

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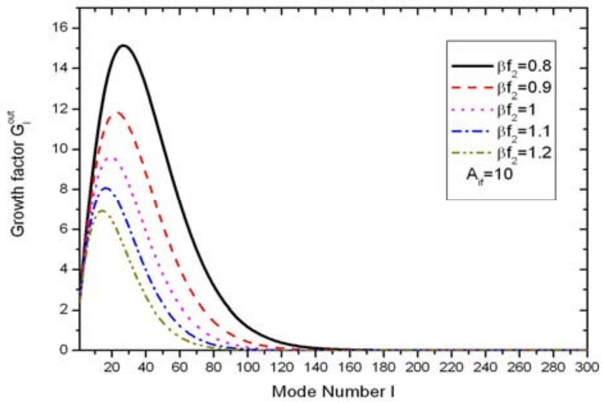

Fig.2 The variations of growth factor on outer surface of fuel capsule versus mode number for different values of βf_2 and A_{if}=10 .

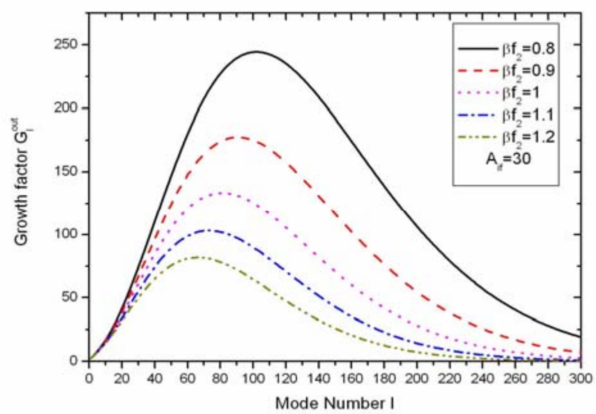

Fig3 The variations of growth factor on outer surface of fuel capsule versus mode number for different values of βf_2 and A_{if}=30 .

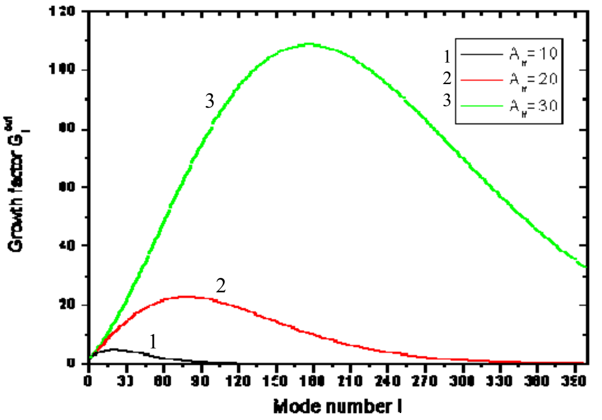

Fig.4 The variations of growth factor on outer surface of fuel capsule versus mode number for different values of A_{if} .

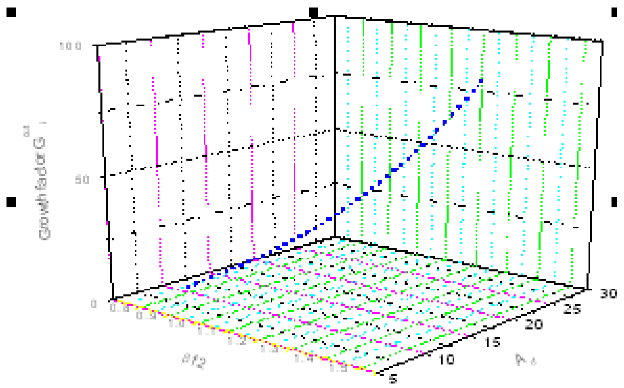

Fig.5 The three dimensional diagram of growth factor on outer surface capsule versus βf_2 and A_{if}.

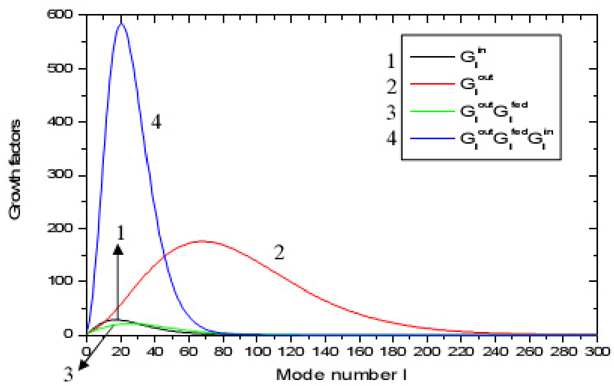

Fig.6. The variations of total growth factor and growth factors on outer and inner surface of fuel capsule versus mode number for: A_{if} =20, f_2 =0.1, β_2 f_2=0.9, f_3=0.9, f_4=0.04, $\beta_{in}f_5$=0.08, α_2=1.