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Tune Depression of Ion Plasmas Observed in a Linear Paul Trap

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The collective motion of an ion plasma trapped in a linear Paul trap is equivalent with that of a charged-particle beam propagating through a linear transport line. Namely, the dynamics of the space-charge-dominated beam can be studied by using the ion plasma. At Hiroshima University, a linear Paul trap system "S-POD (Simulator for Particle Orbit Dynamics)" has been developed to investigate various collective effects in beams. In beam physics, tune depression, which is a depression rate of a frequency of a beam motion due to the space-charge effect, is used for an index of strength of the space-charge self-field. In this paper, we evaluate the tune depression of the ion plasma trapped in our linear Paul trap system by two different methods. The tune depression of the ion plasma in the linear Paul trap was larger than that of an ordinary beam in an existing accelerator. It is an advantage to use the ion plasma for the experimental study of beam dynamics.

Keywords: non-neutral plasma, space-charge-dominated beam, linear Paul trap

1. Introduction

Charged-particle beams have been used in a wide range of fields in physics, engineering, life science, medical science, etc. Because the ordinary beam has very high temperature and low density, the influence of space-charge on beam dynamics can be neglected. However, the densification of beams advances rapidly with the progress of various technologies in recent year. In many cases, the space-charge effect deteriorates the quality of the beam. [1] Therefore, it is necessary to study the dynamics of space-charge dominated beams systematically.

The typical beam transport line obtains effective focusing-force by combination of focusing and defocusing-force induced by the quadrupole magnetic field. In a linear Paul traps, transverse confinement is provided by an rf voltage applied to cylindrical quadrupole electrodes (see Fig. 1). Therefore, the effect of the external field on a beam traveling in the transport channel corresponds to that on the ion plasma confined in the linear Paul trap. The collective motion of the space-charge-dominated beam is also equivalent with that of the ion plasma. [2, 3] Namely, we can study the dynamics of space-charge-dominated beams by using the ion plasma. An experiment utilizing the same idea is in progress at Princeton University. [4, 5] Another experiment using a compact electron storage ring is carried out at Maryland University. [6, 7] At Hiroshima University, a tabletop system "S-POD (Simulator for Particle Orbit Dynamics)" was constructed to investigate various collective effects in beams. [8, 9]

The transverse equation of motion of the confined ions with mass m_i and charge Q is described in the Mathieu equation when the self-field of the ion plasma is neglected. The stability region of the Mathieu equation is determined by the Mathieu parameters $a = 8QU/(m_i r_0^2 \omega^2)$ and $q = 4QV/(m_i r_0^2 \omega^2)$. Here, U is a dc voltage component of quadrupole potential field, V and $\omega/(2\pi)$ are an rf voltage component and its frequency of that, and r_0 is distance from trap axis (z-axis) to the quadrupole electrodes. The first stability region is $0 < q < 0.91$ at $a = 0$.

The ions confined in a linear Paul trap oscillate at an rf frequency (micromotion) and at a low frequency (secular motion) in the transverse direction. The secular frequency is decided by pseudopotential generated by the external rf potential. When q is small, the secular frequency $\omega_s/2\pi$ is given by $\omega_s = \omega\sqrt{a + q^2/2}/2$, approximately. The ratio ω_s/ω correspond to the *betatron bare tune* ν_0 in beam physics. When the secular motion resonates with external field changing periodically, the motion of the ion becomes unstable. The first stability region of Mathieu equation accords with $0 < \nu_0 < 0.5$. The tune is an important parameter to characterize the stability of beams.

In practice, the secular frequency depends on an effective potential composed of the pseudopotential and the self-field of an ion plasma. The frequency is depressed in a dense plasma because the Coulomb force among the ions cancels out the external field. In the beam physics, the depression rate (*incoherent tune depression*) is used for an index of the strength of the space-charge self-field. Similarly, the tune depression for collective motions (*coherent tune depression*) is determined.

A purpose of this paper is to measure the tune

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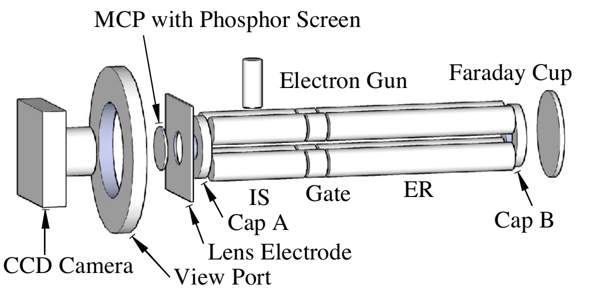

Fig. 1 Schematic view of the multisection linear Paul trap.

depression of ion plasmas confined in our linear Paul trap designed for study of beam physics. It is important to determine the tune depression for systematic experiments of beam physics. The tune depression is determined by two different methods. One is a method using the shift of a resonance condition. The other one is a method to evaluate the tune depression from the transverse density distribution of the ion plasma. The tune depressions evaluated by two different methods were consistent. These are larger than that of an ordinary beam in an existing accelerator. It indicates an advantage of the experimental study of beam dynamics using the ion plasma.

2. Experimental Setup

The structure of the multisection linear Paul trap system designed for the study of beam physics is schematically shown in Fig. 1. The distance from the trap axis to the electrodes is $r_0 = 5$ mm and a radius of cylindrical quadrupole electrodes is 5.75 mm. The quadrupole electrodes are separated axially into three parts, namely, Ion Source (IS), Gate, and Experiment Region (ER). The lengths of the electrodes are $L_{IS} = 50$ mm, $L_{Gate} = 9$ mm, $L_{ER} = 100$ mm. They are separated from each other by 0.5 mm. The trap has two end cap electrodes (Cap A and Cap B), 5 mm in length and 5(13) mm in inner (outer) radius, at both ends.

In this experiment, to confine Ar^+ ion plasma in the IS region, Cap A and Gate are biased to dc voltages of $U_A = U_{Gate} = 30$ V, respectively. The rf voltage and frequency applied to the quadrupole electrodes are $V < 50$ V and $\omega/2\pi = 1$ MHz, respectively. Ar gas with a pressure of $P_{Ar} \sim 10^{-6}$ Pa is introduced as an ion source into a vacuum chamber containing the ion trap where the base pressure is $P_0 \sim 2 \times 10^{-8}$ Pa. An electron beam (≤ 120 eV, ~ 0.1 mA) is injected into the IS region for an injecting period of $T_{IS} \sim 1$ s. Ar^+ ions generated in the IS region by electron impact ionization is trapped. The trapping ion number is controlled by the current of the electron beam.

In this experiment, we measure the total ion number and the transverse distribution of the plasma after 10 ms confinement in the IS region. The storage time is longer than the rf period and shorter than the $1/e$ lifetime of the confined plasma. For observing the transverse profile of the ion density, the trapped ions are extracted from the ion trap to the input surface of an microchannel plate (MCP, Hamamatsu Photonics F2222-21P) thorough the holes of Cap A and a lens electrode. The biased voltage U_{lens} on the electrode and the input surface of the MCP are -1.0 kV and -1.5 kV. The extracted ions generate secondary electrons on the input surface of the MCP. The two-dimensional (2D) distribution of the secondary electrons induced by the ions is amplified through a bundle of capillaries. The electrons reaching the output surface of the MCP are accelerated by the voltage difference of 1.5 kV between the output surface of the MCP and the phosphor screen (P43). The accelerated electrons induce a visible-light image on the phosphor screen. The luminosity distribution is detected on a 640×480 pixel array of a charge-coupled device (CCD) camera (JAI CV-A11) with a dynamic range of 8 bits.

The axial confinement potential in the IS region is square-well-like because the quadrupole electrodes shield the static field of Cap A and Gate. Since the length of the trap region is longer than its radius, the end effect is negligible. Therefore, the axial density distribution of the trapped ion plasma is almost homogeneous and a 2D approximation can be introduced. The transverse profile of the trapped plasma oscillates simultaneously with the external rf field. The period of the transverse oscillation ($2\pi/\Omega = 1$ μs) is shorter than the period required to dump all ions from the IS region (> 20 μs). Consequently, the 2D distribution of the ions projected onto the MCP becomes the time-averaged transverse distribution of the ion plasma.

3. Determination of Tune Depression

Ions trapped in the linear Paul trap oscillate in the longitudinal and the transverse direction. These oscillations exhibit various resonance phenomena on ion plasma. [10, 11] Here, we utilize a nonlinear resonance in the transverse direction to determine the tune depression. This resonance is driven by the nonlinear fields. The nonlinear fields come mainly from non-hyperbolic electrode surface and fabrication error. In the 2D model, the potential including nonlinear perturbations can be written, in the cylindrical coordinate, as

$$\phi(r,\theta,t) = (U + V\cos\omega t)\sum_{n=0}^{\infty} C_m \left(\frac{r}{r_0}\right)^m \cos m\theta, \quad (1)$$

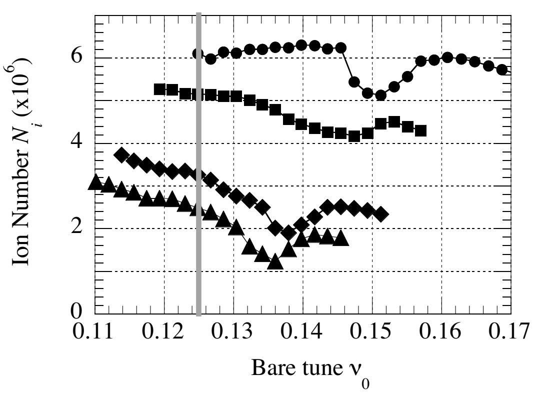

Fig. 2 Ion number N_i plotted against the bare tune ν_0. The symbols denote different current of electron beam for ionization.

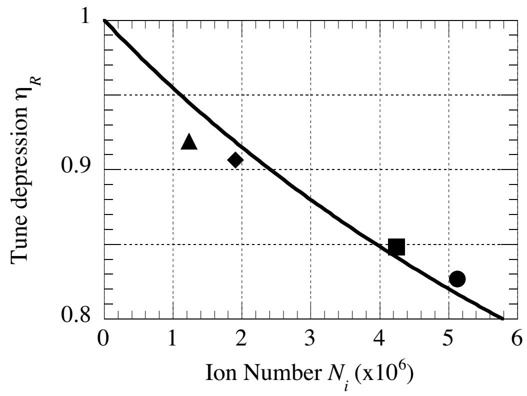

Fig. 3 Tune depression η_R estimated from the observed shift of the nonlinear resonance. Symbols corresponds to data set in Fig. 2

where C_m's are the dimensionless weight factor depending on the relative strengths of the linear and nonlinear terms and m is positive integer.

Figure 2 shows the trapped ion number N_i as a function of the bare tune ν_0 determined by the external rf potential V. The symbols denote different electron current for ionization. In the series of data shown in same symbols, the ion number generated by the electron impact is kept. However, local reduction is observed. From the theoretical analysis, it is expected that the incoherent nonlinear resonance of $m = 8$ and the coherent resonance of $m = 4$ appear in $\nu_0 \approx 0.125 = 1/8 = 1/(2 \times 4)$ when the tune depression is small. Moreover, the dips shift with the increasing of ion number. It is clear that the dip is caused by the nonlinear resonance with the tune shift due to the space-charge.

The problem that which resonance the observed dip is caused by is very interesting. However, the difference of the tune shift expected from theoretical and numerical analysis is small. [12] Therefore, the cause of the resonance is not discussion in this paper. Treatment of the incoherent tune is easier than that of the coherent tune. The following analysis is based on a theory of incoherent tune shift.

Here, we estimate the tune depression η_R from the shift of resonance condition. In each series of experiments, the resonance line has moved from $\nu_0 \approx 0.125$ to the point at the minimum ion number. Namely, the depressed tune ν at the position corresponds to 0.125. Therefore, the tune depression can be estimated from the ratio of $\eta_R = \nu/\nu_0 = 0.125/\nu_0$. Figure 3 summarizes the relationship between η_R and ion number N_i at the dip. Symbols correspond to data set in Fig. 2. η_R decreases with the increasing of N_i. The maximum value of η_R is 0.82.

According to ref. [3], η_R of the stationary plasma with a line-density N_L and a transverse plasma temperature $T_\perp$ is estimated from

$$\eta = \sqrt{1 - \frac{1}{1 + \dfrac{2}{N_l r_p}\dfrac{k_B T_\perp}{m_i c^2}}}. \tag{2}$$

where k_B is the Boltzmann constant, c is the speed of light, and r_p denotes the classical particle radius. The solid curve in Fig. 3 shows the fitting curve for experimentally determined η_R taking $T_\perp$ as a fitting parameter. Here, we suppose to the plasma length $L = 30$ mm from the typical longitudinal temperature $T_\parallel = 0.3$ eV. In this case, the fitting parameter $T_\perp$ is set to 0.25 eV. The temperature estimated from tune shift is consistent with the temperature calculated from the density distribution observed by the imaging system to show later.

The shift of the resonance condition can directly determine the tune depression but this method is usable only around some resonance conditions. In contrast, the tune depression can be estimated from the transverse density distribution without such a limitation. Figure 4(a) shows the density distribution of an ion plasma observed by the imaging system. This image corresponds to the plot denoted by the closed circle in Fig. 3. The thick curve in Fig. 4(b) shows the radial density distribution $n(r)$. Here, we also assume $L = 30$ mm. The radial density distribution is similar to the Gaussian distribution with an rms radius $b = \sqrt{\langle r^2 \rangle} = 1.35$ mm.

The fine solid curves show the pseudopotential ϕ_{ext} generated by the rf field for transverse confinement, the self-potential ϕ_{sc} of the ion plasma, and the total potential $\phi_T = \phi_{ext} + \phi_{sc}$. The self-field of the ion plasma ϕ_{sc} is calculated from the observed density distribution $n(r)$ under the 2D approximation. Here, the boundary condition is imposed so that $\phi_{sc}(r_0) = 0$. The total potential is distorted by influence of self-field from a harmonic potential. Therefore, the tune varies

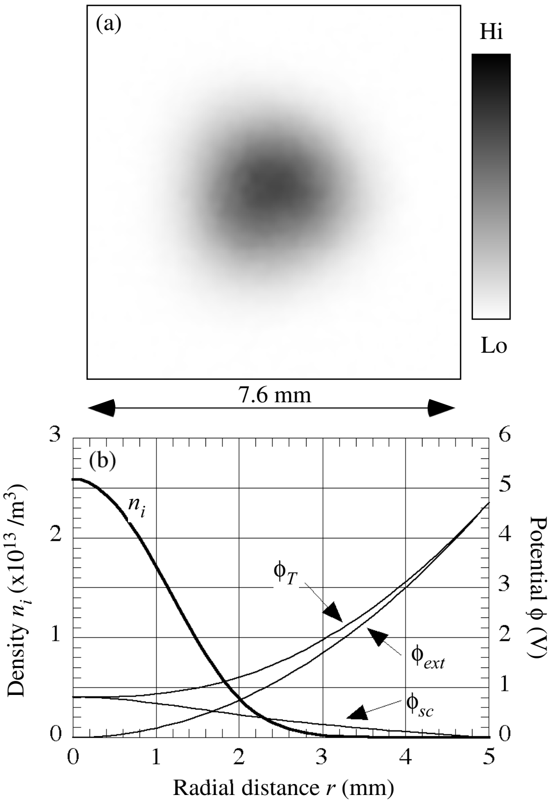

Fig. 4 (a) Density distribution of the ion plasma observed by the imaging system. (b) Radial density distribution n_i, pseudopotential ϕ_{ext}, self potential ϕ_{sc}, and the total potential ϕ_T.

with the amplitude of the radial oscillation.

Here, to compare with η_R estimated from the shift of the resonance condition, we evaluate the typical value of the tune depression η_I by using eq. (2). The transverse temperature is then necessary. According to ref. [3], the transverse temperature of the stationary plasma with the rms radius b is estimated from

$$T_\perp = \frac{m_i c^2}{k_B} \frac{\kappa_p^2 b^2 - r_p N_l}{2}, \tag{3}$$

where $\kappa_p = \omega \nu_0 / c$ is a constant depending on the characteristics of the external rf field. In this case, the temperature of the ion plasma taking into account of the self-field is estimated to be $T_\perp = 0.19$ eV and the typical tune depression η_I is evaluated as 0.78.

The correspondence between η_I and η_R is plotted in Fig. 5. The symbols accord with data in Fig. 3. The tune depressions evaluated by two different methods are consistent. This result shows the validity of the tune depression determined from the density distribution observed by the imaging system.

4. Conclusions

We determined the tune depression of the ion plasma trapped in our linear Paul trap system by two

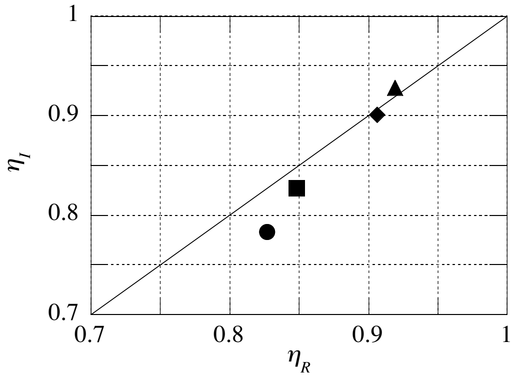

Fig. 5 Tune depression η_I estimated from the density distribution plotted against the tune depression η_R estimated from the shift of the nonlinear resonance. Symbols corresponds to data set in Fig. 2

different methods. One is a method using the shift of resonance condition. The other one is a method to evaluate the tune depression from the transverse density distribution of the ion plasma. The tune depressions evaluated by two methods were consistent. The maximum value of the tune depression observed in this experiment was ≈ 0.8. It was larger than that of an ordinary beam in an existing accelerator. It is an advantage to use the ion plasma for the experimental study of beam dynamics.

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