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Acoustic Black Hole in Plasma Flow– Theory: Observation of a Classical Analogue to the Hawking Radiation

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A black hole is an astrophysical object and from which no light ray can escape due to its extreme strong gravity. An analogue system of black hole can be realized in a laboratory system using a transonic flow. On such a flow, no sound wave can escape from the supersonic region into the subsonic region and the sonic point corresponds to the event horizon of the black hole. We propose an analogue model of the black hole using a plasma flow. Creating a well of the plasma potential by extracting ions, one dimensional transonic plasma flow can be realized. We find the flow becomes a sonic analogue of the black hole if the depth of the well is about the electron temperature. We estimate the expected temperature of the acoustic black hole and the signal of the classical counter part to the Hawking radiation.

Keywords: acoustic black hole, Hawking radiation, quasi-normal oscillation, plasma, ion acoustic wave

1. Introduction

Experiments in general relativity are very difficult because generating gravitational fields requires huge amount of masses. Instead of trying to do direct experiments, there is the possibility of simulating aspects of general relativity by developing "analogue model" of general relativity [1]. The simplest analogue model is that of sound wave in a flowing fluid; we can simulate the wave propagation in the black hole spacetime by using the fluid with a transonic flow.

It is well known that the black hole is not black in the quantum level and can emit particles with thermal spectrum (Hawking radiation [2]) of which temperature is inversely proportional to the black hole mass: $k_B T_H = c^2\hbar/(8\pi GM)$. This emission of particles occurs as the definition of the vacuum state changes in curved spacetimes. By investigating the quantum effect of the analogue black hole, we can simulate the Hawking radiation in the laboratory.

As one of the analogue models of the black hole, our group is now performing an experiment using a plasma fluid. In the transonic flow of the plasma fluid, ion sound waves obey the wave equation of which structure is analogous to the wave equation in the black hole spacetime. The goal of our research is to simulate the classical analogue of the Hawking radiation in the plasma fluid. For this purpose, we consider a theoretical model corresponding to our plasma experiment and check the configuration and setting of our experiment.

2. Sonic Black Hole in the Plasma Flow

Fig. 1 shows a setting of our plasma experiment. The fast plasma flow is produced in a weak magnetic field. The parameters characterizing our plasma experiment are

$$
\begin{aligned}
&\text{ion density:} && n \sim 10^9\ (\mathrm{cm}^{-3}),\\
&\text{electron temperature:} && T_e \sim 10\ (\mathrm{eV}),\\
&\text{ion sound velocity:} && c_s \sim 10^6\ (\mathrm{cm/sec}).
\end{aligned}
$$

The flow velocity along the magnetic field is controlled by enhancing the radial ion flow to the electrode placed at periphery of the plasma.

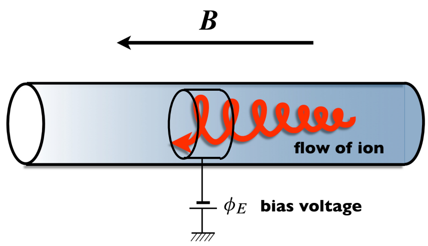

Fig. 1 Schematic of our plasma experiment.

The resulting one dimensional flow of ions along the magnetic filed obeys the following equation:

$$
\frac{\partial}{\partial z}\left[e^{\phi_E(z)}e^{-\alpha M^2/2}M\right] = 0, \tag{1}
$$

where ϕ_E represents the bias voltage at the electrode, $M = u/c_s$ is the Mach number of the plasma flow and a constant $\alpha < 1$ is the phenomenological parameter determined by our experimental setting. The Fig. 2 show the structure of flows determined by Eq. (1).

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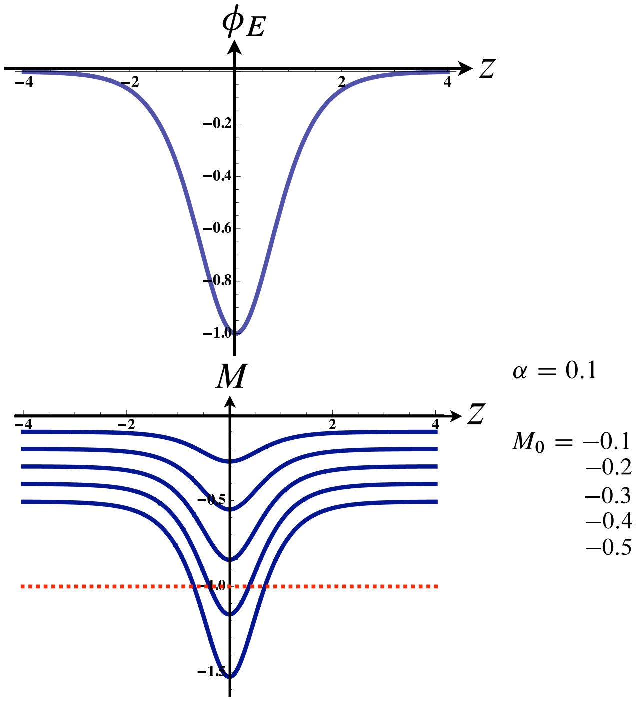

Fig. 2 The top panel is the assumed spatial distribution of the bias voltage at the electrode. The lower panel shows the Mach number as a function of a spatial coordinate z. The dotted line is $M = -1$.

For the initial flow velocity $M_0 = -0.4$, we have sonic points for the bias voltage $\phi_0 = -10$ (eV) and the transonic flow can be realized.

By introducing the velocity potential φ, the perturbation of the flow velocity is given by $\delta u = \partial_z \varphi$ and φ obeys the following wave equation for the ion acoustic wave:

$$-(\partial_t + \partial_z u) n (\partial_t + u \partial_z)\varphi + c_s^2 \partial_z (n \partial_z \varphi) = 0, \quad (2)$$

where u and n are the background flow velocity and the background ion density, respectively and we have assumed the wavelength of the acoustic wave is sufficiently larger than the Debye length $\lambda_D = \sqrt{\epsilon_0 T_e/(Ze^2 n)} \sim 10^{-2}$ (cm). We can formally rewrite this wave equation to the Klein-Gordon equation in the curved spacetime.

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu}\left(\sqrt{-g}\, g^{\mu\nu} \frac{\partial \varphi}{\partial x^\nu}\right) = 0, \; x^\mu = (t, x, y, z) \quad (3)$$

where the acoustic metric $g_{\mu\nu}$ is defined by

$$g_{\mu\nu} = n \begin{pmatrix} -(c_s^2 - u^2) & 0 & 0 & -u \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -u & 0 & 0 & 1 \end{pmatrix}. \quad (4)$$

When the background flow has a sonic point, the curved spacetime represented by this metric has the similar feature of the black hole spacetime in the general relativity and the sonic point corresponds to the black hole event horizon. Therefore, the propagation of the sound wave in the transonic flow is equivalent to the propagation of the wave in the black hole spacetime with the metric (4). Hence, by quantizing this system, we can simulate the Hawking radiation in the laboratory by observing phonon emissions.

3. Classical Counterpart to the Hawking Radiation

Although we cannot observe the Hawking radiation via classical plasma experiment, it is possible to detect sound waves with Planckian distribution from the sonic point. In the course of the formation of a sonic point, the wavelength of the sound wave propagating towards upstream of the flow is stretched and the resulting power spectrum for such waves becomes

$$|\varphi_\omega|^2 \propto \frac{1}{e^{\omega/T_{\mathrm{BH}}} - 1} \quad (5)$$

where the “temperature” of the acoustic black hole is given by

$$T_{\mathrm{BH}} = \frac{c_s}{2\pi} \left|\frac{dM}{dz}\right|_{\text{sonic point}}. \quad (6)$$

The power spectrum (5) is the Planckian with the temperature T_{BH} and we call this as the classical counterpart to the Hawking radiation [3]. For our plasma experiment, the temperature is given by

$$2\pi T_{\mathrm{BH}} \sim \frac{c_s|\phi_0|}{2d} \sim 100 \text{ (kHz)} \quad (7)$$

where ϕ_0 is the bias voltage at the electrode and d is the size of the electrode.

4. Quasi-normal Oscillation of the Acoustic Black Hole

We can also observe the quasi-normal oscillation [4] of acoustic black hole. This oscillation is completely determined by parameters which characterize black holes. We can transform the wave equation (2) to the Schrödinger type equation as follows

$$\left(-\frac{d^2}{dz^{*2}} + V_{\mathrm{eff}}(z^*)\right)\tilde{\varphi}_\omega = \left(\frac{\omega}{c_s}\right)^2 \tilde{\varphi}_\omega, \quad (8)$$

$$z^* = \int \frac{dz}{1 - M^2}, \quad (9)$$

$$V_{\mathrm{eff}} = \frac{1}{2}\frac{d^2\phi_E}{dz^{*2}} + \frac{1}{4}\left(\frac{d\phi_E}{dz^*}\right)^2. \quad (10)$$

The quasi-normal mode is determined by the solution of the wave equation (8) with the boundary condition

$$\text{pure ingoing at } z^* \to -\infty, \quad (11)$$
$$\text{pure outgoing at } z^* \to +\infty.$$

The lowest frequency of this oscillation for our plasma experiment is estimated to be

$$\mathrm{Re}[\omega] \approx 0.3 \times \frac{c_s|\phi_0|}{d} \sim 100 \text{ (kHz)}. \quad (12)$$

5. Summary

We have considered the theoretical model of the acoustic black hole for the plasma flow. Using plasma fluid and ion acoustic wave, it is possible to reduce the unwanted signal and noise which make difficult to detect the signal of the Hawking radiation and the quasi-normal oscillation of the acoustic black hole. We will report the present status and the result of our plasma experiment in a separate publication [5].

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