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Estimation of Minimum Size of Electrodes for Face-to-Face Double Probe using Two-Dimensional Particle-in-Cell Simulation

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To optimize the "Face-to-face Double Probe (FDP)" method, which is proposed by the authors to measure the Mach number of a plasma flow [Saitou and Tsushima, Jpn. J. Appl. Phys. **40**, L1387 (2001)], performances of an FDP have been investigated focusing on a characteristic length of a pair of electrodes using a two-dimensional particle-in-cell simulation. Resulting from the simulation, both the characteristic length needed for a good FDP performance and a correction factor needed for the FDP with small electrodes are presented.

Keywords: plasma flow, Mach number, Mach probe, face-to-face double probe, particle-in-cell simulation

1. Introduction

A plasma flow velocity or its Mach number is one of important parameters to investigate phenomena observed in plasmas. In addition to the well known methods such as the Mach probe [1] or the directional probe [2, 3], the authors have proposed a new probe named "Face-to-face Double Probe" (FDP) [4–6]. The structure of an FDP is simple since it just consists of a pair of electrodes, which are face to face each other, with a variable power supply and an ammeter as illustrated in Fig. 1. Evaluation of the flow Mach number using the FDP is also simple because an analysis given in the next section suggests that a voltage without a net current between the electrodes is proportional to the Mach number under an adequate approximation. The simplicity of the FDP method contributes to reliability, and might break a new ground in its application such as an instrument for a rocket or a satellite.

Furthermore, the FDP method is superior to the conventional Mach probe (CMP) method from a view point of a spatial resolution. For the FDP, the spatial resolution is determined by a distance between the electrodes because an expansion of the presheath related to its measurement is restricted in the distance. On the contrary, in the case of the CMP, the presheath expands far away along a magnetic field line as seen in Fig. 2, where the cases of the CMP and the FDP are illustrated at the top and the bottom, respectively, with the presheath symbolized by a gradation from black to white. It may be worth noting that the good spatial resolution of the FDP comes from its structure. Considering the characteristics mentioned above, the FDP method is expected to be applicable to Mach number measurements under various conditions.

Performances of the FDP have been analyzed on the basis of the one-dimensional fluid model with a strong magnetic field case of $a\lambda_D \gg \rho_i$, where a is the characteristic length of the FDP electrode normalized by the electron Debye length, λ_D, and ρ_i is the characteristic radius of the ion gyro-motion [4, 5]. In fact, the FDP method has been applied to the boundary plasma of tokamaks [7] and, under such strong magnetic fields, the one-dimensionality is realized. It is, however, easily found that this condition makes the probe quite large when the magnetic field is weak, i.e., the size of the electrode, $a\lambda_D$, is larger than $\rho_i \sim 10$ (mm) for a singly ionized argon plasma with $T_i \sim 1$ (eV) under the magnetic field of 0.1 T, for example. Existence of such a large electrode disturbs the plasma surrounding the probe by itself. The size of the electrode has to be as small as possible to decrease such

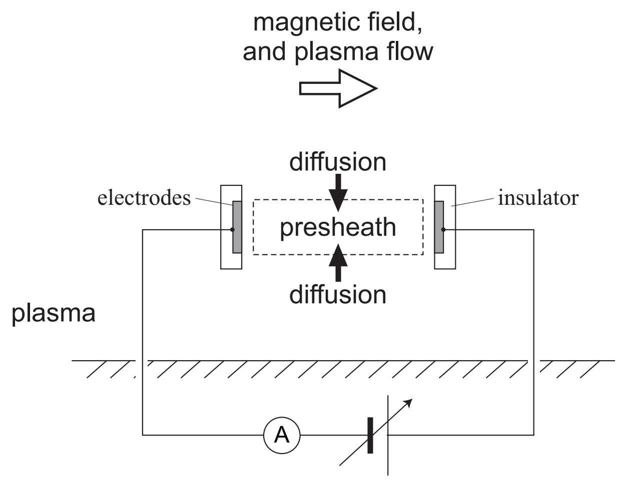

Fig. 1 Schematic drawing of the FDP and its measurement circuit.

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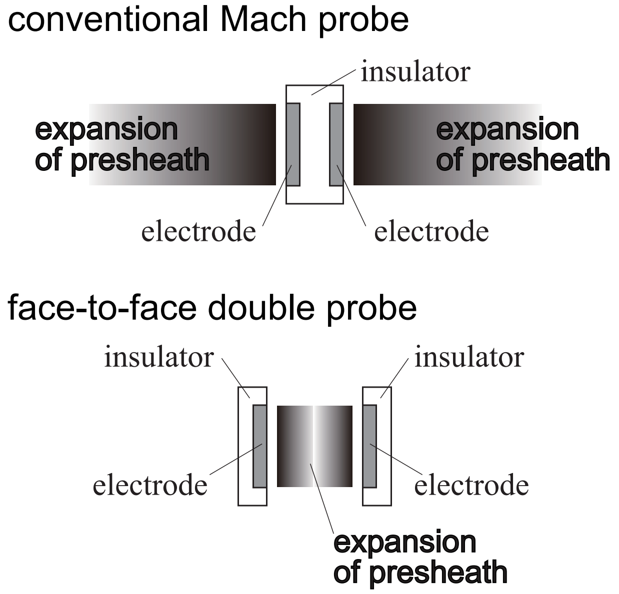

Fig. 2 Comparison of presheath expansion in case of the conventional Mach probe (top) and the FDP (bottom). The expansion of the presheath is shown as gradation from black to white.

a disturbance to the plasma. In order to optimize the size of the FDP electrode even under the weak magnetic field, where $a\lambda_D < \rho_i$, the performance of the FDP has been investigated using a two-dimensional particle-in-cell (PIC) simulation [8] in this paper.

2. Fluid Model

By the steady state fluid model [9],

$$\frac{d\tilde{n}}{d\xi} = 1 - \tilde{n}, \tag{1}$$

$$\tilde{n}M\frac{dM}{d\xi} + \frac{d\tilde{n}}{d\xi} = (1+\alpha)(1-\tilde{n})(M_\infty - M), \tag{2}$$

the behavior of the plasma in the presheath of an FDP as well as a CMP can be analyzed, where $\tilde{n}$ is the density normalized to the background plasma density n_0, M and M_∞ the Mach numbers of ions inside and outside the electrodes, respectively, $\xi = x/\lambda_D$ the normalized spatial coordinate, and $\alpha = \eta/nm_iD$ the normalized viscosity with the ion mass m_i and the diffusivity D. After tedious calculation, we obtain a $V - I$ characteristics between the electrodes such that

$$\begin{aligned}\frac{eV_0}{k_BT_e} =& \ln\left[\frac{2+\alpha+(1+\alpha)M_\infty}{2+\alpha-(1+\alpha)M_\infty}\right]^\delta \\ &+\frac{\alpha M_\infty}{\sqrt{q}}\left\{\tan^{-1}\left[\frac{(1+\alpha)(2-M_\infty)}{\sqrt{q}}\right]\right. \\ &\left.+\tan^{-1}\left[\frac{(1+\alpha)(2+M_\infty)}{\sqrt{q}}\right]\right\},\end{aligned} \tag{3}$$

where e is the elementary charge, V_0 the voltage when a net current flowing through the electrodes is 0, k_B the Boltzmann constant, and T_e the electron temperature of the plasma between the electrodes, respectively. Parameters δ and q are given as follows:

$$\delta = \frac{2+\alpha}{2(2+\alpha)}, \tag{4}$$

$$q = 4(1+\alpha) - (1+\alpha)^2 M_\infty^2. \tag{5}$$

For $-1 < M_\infty < 1$, the solution (3) can be approximated by

$$M_\infty \simeq \frac{1}{1+\dfrac{\alpha}{\sqrt{1+\alpha}}\tan^{-1}\sqrt{1+\alpha}}\frac{eV_0}{k_BT_e} \tag{6}$$

$$\propto \frac{eV_0}{k_BT_e}. \tag{7}$$

It is found that the Mach number is proportional to the voltage V_0 as long as the electron temperature is constant. In applying this method to experiments, the Mach number can be estimated by measuring the voltage V_0 and the electron temperature T_e. Figure 3 in Ref. [4] is helpful to see the relation between V_0 and M_∞ in more detail. In the following sections, a notation M is used instead of M_∞.

3. PIC Simulation Model

A two-dimensional PIC simulation is used to investigate plasma behaviors in the space between the FDP electrodes and in the envelope around the space. In Fig. 3, a schematic drawing of the simulation region is shown and the simulation region is symmetric against $Y = 0$, where the spatial variables are normalized by λ_D, i.e., $X = x/\lambda_D$ and $Y = y/\lambda_D$ with the spatial steps of $dX = dY = 0.1$. The temporal variable, t, is normalized by the ion plasma angular frequency, $\tau = \omega_{pi}t$, and the temporal step of $d\tau = 0.01$ is used. A ratio of the ion mass to the electron mass is $m_i/m_e = 200$ and a ratio of the ion temperature to the electron temperature is $T_i/T_e = 1$. The two electrodes with the characteristic length, a, are located at the simulation boundaries of $X = 0$ and $0 \le Y \le a$; and $X = 6.4$ and $0 \le Y \le a$. The other boundaries except for $Y = 0$, i.e., $X = 0$ and $a < Y \le 51.2$; $X = 6.4$ and $a < Y \le 51.2$; and $0 \le X \le 6.4$ and $Y = 51.2$, are free boundaries. A potential of each electrode is a floating potential. The outside region between the electrodes, $0 \le X \le 6.4$ and $Y > a$, the plasma flows with $M = 0.2$ along the X-axis and the plasma is assumed to diffuse into the space between the electrodes. Here, in the whole region, a magnetic field with the normalized magnitude of $B_n = |e|B/\omega_{pi}m_i = 5.645$ along the X-axis is assumed and the Bohm diffusion is taken into account. The size of the electrode is in a range of $0.2 < a < 25.6$, or $0.33 < a\lambda_D/\rho_i < 4.4$ since $\rho_i/\lambda_D \sim 0.6$ in the simulation.

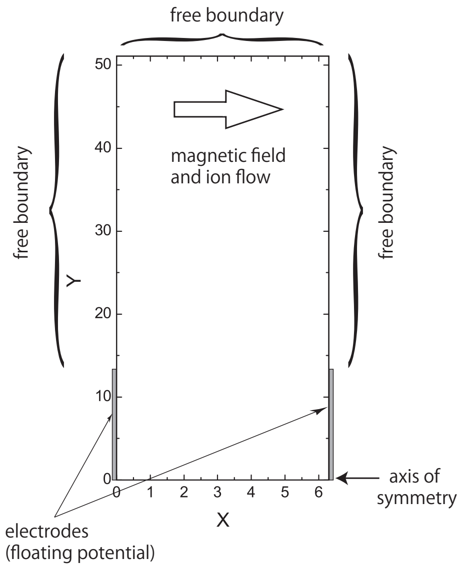

Fig. 3 Simulation region and its boundaries. The potential of the electrodes is floating.

4. Results and Discussion

A spatial distribution of the two-dimensional electrostatic potential normalized as $\Phi = e\varphi/k_BT_e$ was calculated for several values of the electrode length, a. The results in the cases of $a =$, 0.2, 0.8, 3.2, and 12.8 are shown in Fig. 4. In the region outside the space between the electrodes, $0 \le X \le 6.4$ and $Y > a$, contour lines of Φ are almost parallel to the X-axis. In the region between the electrodes, $0 \le X \le 6.4$ and $Y < a$, the contour line of the potential is convex to the lower side and the convexity is clearer for larger a. For all the cases, the potential distribution is asymmetry between the left-hand and the right-hand sides. Thus, this asymmetrical potential distribution due to the plasma flow makes a difference between the floating potentials of the electrodes. In the present case, the plasma flows from left to right, the potential of the right-hand side electrode is expected to be higher than that of the left-hand side one, i.e., $\Phi_R > \Phi_L$, as expected from the fluid model. The dependence of the potential difference, $\Delta\Phi = \Phi_R - \Phi_L$, on a is shown in Fig. 5.

The lowest limit of the saturation of $\Delta\Phi$ is determined by an intersection of two lines, A and B, which are depicted in Fig. 5. The lowest value of a is 5.3 with the potential difference $\Delta\Phi = \Delta\Phi_s = 0.135$ at the intersection. For $a \ge 5.3$, the potential difference is almost saturated around a value of $\Delta\Phi \sim 0.15$, although the line, A, has a small gradient. In this case, since the line A is approximated by the equation of $\Delta\Phi = (0.0022a + 0.12)$, the Mach number is given as follows:

$$M \simeq \frac{1}{0.016a + 0.89} \frac{e\Delta\Phi}{k_BT_e} \tag{8}$$

from Eq. (6) with $\alpha = 1$ as long as the electron temperature between the electrodes is the same as the electron temperature of the flowing plasma. The simulation result almost coincides with the result obtained from the fluid model.

For $0 < a < 5.3$, $\Delta\Phi$ gradually increases with increasing a. Since the line B is approximated by the equation of $\Delta\Phi = 0.025a$, the corrected equation for $0 < a < 5.3$ is give by

$$M \simeq \frac{5.4}{a} \frac{e\Delta\Phi}{k_BT_e} \tag{9}$$

from Eq. (6) with $\alpha = 1$ as long as the electron temperature between the electrodes is the same as the electron temperature of the flowing plasma, as well. Using this correction factor, it is considered that the FDP method is applicable to measure the Mach number of the plasma flow even when $0 < a < 5.3$.

As for the distance, d, of the FDP electrodes to be used, the condition of $d \gg \lambda_D$ should hold, because the fluid model is used to analyze the plasma between the electrodes.

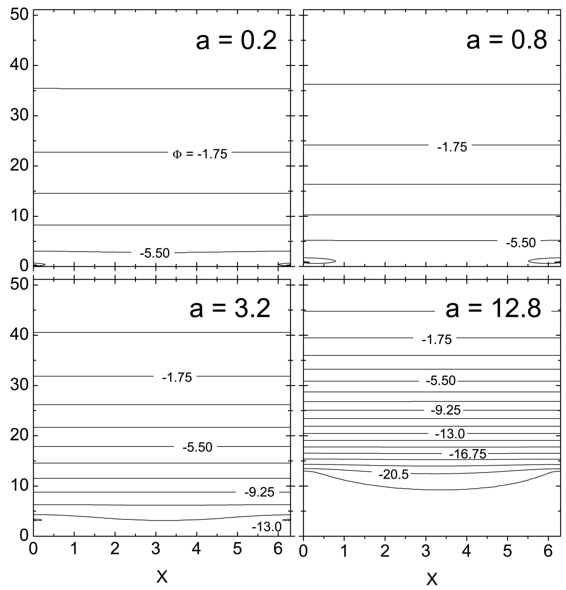

Fig. 4 Typical example of obtained electrostatic potential distribution for $a = 0.2$, 0.8, 3.2, and 12.8, where a is the length of the electrode. The potential at a left top is assumed to be 0.

5. Summary

The studies of the FDP were performed using the two-dimensional PIC simulation. The difference between the electrostatic potentials of the floating electrodes saturates and almost coincides with the value given by the fluid model when the electrode length, a, is greater than 5.3 and the Mach number is given by Eq. (8). The potential difference increases with

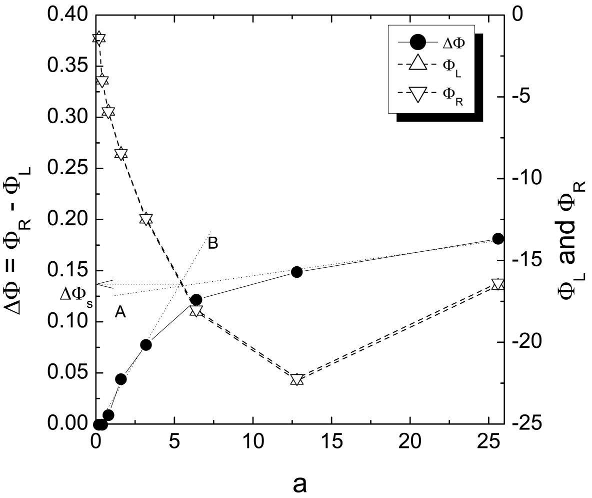

Fig. 5 Dependence of the potential difference on the length of the electrode (closed circles), the left hand side, and the right hand side electrodes (open normal and inverse triangles), respectively. The dotted lines, A and B, are approximation lines for the potential difference.

increasing the value of a for $0 < a < 5.3$. If the electrode length lies within this range, the corrected equation, Eq. (9), should be adopted and it might be possible to estimate the adequate Mach number as long as the electron temperature does not change. This may suggest that the FDP method can be used to estimate the Mach number of the plasma flow even when $0 < a < 5.3$ with the corrected equation reduced from Fig. 5. In cases of other Mach numbers, the correction factors appeared in Eqs. (8) and (9) may be different. To determine the factors, PIC simulations for several Mach numbers should be performed. To convert the potential difference to the Mach number, it is required to estimate the electron temperature of the plasma between the electrodes. The electron temperature has to be estimated in the simulation, which is one of our future tasks.

To apply the FDP method to laboratory experiments or space plasma observations, the method may be used under the condition that a plasma flows obliquely and some amount of charged particles directly enter into the electrodes, since the electrodes are not always placed along the stream line. It is easy to suspect that the directly incoming charged particles change the electrode potential. In addition, the detailed study of the distance between the electrodes has not yet been done, although it should be sufficiently larger than the Debye length from the analysis of the fluid model. These are our future tasks for further improvement of the FDP method.

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