Monopole Drift-Wave Vortices in a Viscous Magnetized Plasma

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Monopole nonlinear drift-wave vortices in a viscous magnetized plasma are investigated by applying a time-dependent finite element method of two-dimensional space to Hasegawa-Mima equation with viscosity term. Under a periodic boundary condition in the direction of drift-wave propagation and a natural boundary condition transverse to the propagation, the monopole drift-wave vortices of positive potential tilt in the clockwise direction. The tilting angle depends on the value of initial localized potential.

Keywords: Hasegawa-Mima equation, time-dependent finite element method, monopole drift-wave vortices, wake release, boundary condition.

1. Introduction

Hasegawa-Mima equation [1] is an important equation in case of discussing plasma transport in a magnetized plasma. This equation includes the drift-wave propagation term and the nonlinear term leading to vortex formation. Hasegawa-Mima equation with viscosity term is written as follows.

\[
\frac{\partial}{\partial T}(\Phi - \nabla^2_\perp \Phi) + V_\star \frac{\partial \Phi}{\partial Y} = (\frac{\partial \Phi}{\partial X} \frac{\partial}{\partial Y} - \frac{\partial \Phi}{\partial Y} \frac{\partial}{\partial X}) \nabla^2_\perp \Phi - N \nabla^2_\perp (\nabla^2_\perp \Phi)
\]

Here, normalization takes place as follows.

\[
\omega_c t \rightarrow T, \quad \frac{x}{\rho_s} \rightarrow X, \quad \frac{e\phi}{kT_e} \rightarrow \Phi
\]

\[
v_\star = -\frac{\partial}{\partial \left( \frac{z}{\rho_s} \right)} \ln n_0(x) \rightarrow V_\star
\]

Time \( t \) is normalized by a reciprocal number of angular cyclotron frequency \( \omega_c \). Spaces \( x \) and \( y \) are normalized by ion-acoustic Larmor radius \( \rho_s \). \( v_\star \) is drift-wave propagation velocity in the positive \( y \) direction and is normalized by ion acoustic speed \( C_s \). Electric potential \( \phi \) is normalized by electron temperature \( T_e \). \( N \) is normalized viscosity coefficient.

2. Application of Time-dependent Finite Element Method

Time-dependent finite element method is applicable to Hasegawa-Mima equation in linear case without any problem. Figure 1 is an example of linear monopole drift-wave propagation without viscosity. The linear monopole drift-wave releases a wake in the backward direction because of dispersion effect of linear drift wave. In this article, normalized drift-wave speed is kept at a constant value of \( V_\star = 0.1 \), and we use a periodic boundary condition in the direction of drift-wave propagation and a natural boundary con-
3. Dynamics of Nonlinear Monopole Drift-Wave Vortices

Figure 2 shows the propagation of a nonlinear drift-wave vortex in case of the initial monopole positive potential $\Phi_0=20$. The monopole drift-wave vortex of positive potential tilts in the clockwise direction. The tilting angle $\theta=22^\circ$ at $T = 400$ in this case. $\theta$ grows initially and decreases very slowly. The release of a wake from the monopole is suppressed compared with the linear case because of the vortex rotation.

The tilting angle $\theta$ dependence on the value of initial localized potential $\Phi_0$ at a time $T = 400$. $V^\ast=0.1$. $N=0.1$. $\theta$ depends on the value of initial localized potential $\Phi_0$.

In case of taking a nonlinear drift-wave monopole into account, the time-dependent finite element method gives rise to unstable calculation, where $N=0$. The viscosity term with $N=0.1$, however, stabilizes the calculation as shown in Fig. 2. Nonlinear monopole drift-wave vortices have been investigated in case with no viscosity [2, 3, 4]. W. Horton showed only calculated figures of the tilting angle $\theta=90^\circ$ [2]. J.S. Hesthaven et al. [3] and J.J. Rasmussen et al. [4] did not discuss the dependence of the tilting angle $\theta$ on the amplitude of monopole vortices. Here, we show their dynamics in case with finite viscosity and discuss the dependence of $\theta$ on the initial monopole potential $\Phi_0$ of monopole vortices.
tilt in the clockwise and counterclockwise directions respectively. The graph of tilting angle $\theta$ versus the value of initial localized potential $\Phi_0$ at a time $T = 400$ is shown in Fig. 4.

![Graph of tilting angle dependence on initial localized potential](image)

**Fig. 4** The graph of tilting angle $\theta$ dependence on the value of initial localized potential $\Phi_0$ at a time $T = 400$. $V_* = 0.1$. $N = 0.1$.

### 4. Conclusions

In summary, a time-dependent finite element method of two-dimensional space is successfully applied to Hasegawa-Mima equation with viscosity term. The viscosity term is necessary in order to suppress calculation instability. The positive and negative nonlinear monopole vortices in a viscous magnetized plasma tilt in the clockwise and counterclockwise directions respectively.
