J. Plasma Fusion Res. SERIES, Vol. 8 (2009)

# Time-Domain Model of a Traveling-Wave Tube

Anass Aïssi[1/2], Frédéric André[2] and Fabrice Doveil[1]

[1]*Turbulence Plasma PIIM, UMR6633 CNRS/Univ. de Provence, Centre scientifique de* Saint-jérôme, F13397 Marseille, France

[2]Thales Electron Devices, 2 rue Latécoère, F78141 Vélizy-Villacoublay, France

(Received: 9 September 2008 / Accepted: 17 October 2008)

Beside its extensive use as a powerful wideband amplifier, the traveling-wave tube is a simple and efficient tool to study one dimensional wave-particle interaction also central in plasma physics. In latter utilization, the traveling-wave tube plays the part of a plasma without its unwanted instabilities and noise. In both situations, numerical simulation plays an important part in directing experiments and developing industrial tubes. In this paper, we propose a new model for the traveling-wave tube interaction. Whereas all previous models based on Pierce's work use a frequency-domain approach, we propose to treat the interaction in time-domain. By this approach, one becomes able to observe phenomena that happen at frequencies that are unknown *a priori*. This approach also allows taking into account reflected waves without additional effort. This model is based on a reduced model of the propagating structure coupled with a vlasovian model of a one dimensional electron beam. This paper shows two time-domain methods to modelize the propagating structure and its interaction with the electron beam.

Keywords: wave-particle interaction, traveling-wave tube, time-domain simulation, equivalent circuit, model order reduction.

## 1. Introduction

The traveling-wave tube (TWT) is a device wherein an electron beam interacts with a longitudinal propagating electric wave.

Kompfner invented the TWT in 1943 as a microwave amplifier, and it is still used in spatial telecommunication, counter-measure and radar domains where robustness, large bandwidth and high output power are required [1].

Beside its industrial use, the TWT is used as a simple and efficient tool to study the one dimensional wave-particle interaction in plasma physics. In this utilization, the wave-guide plays the part of a plasma without its instabilities and noise. Non-linear instabilities and chaos control have been studied in such a device [2,3].

In both contexts, numerical simulation plays an essential part in directing experiments, and in industrial development. Indeed, it allows understanding a particular situation before its actual realization.

Since the invention of TWT, several models have been developed and used [4-6]. A common feature of these models is their frequency approach, *i.e.* they assume that only a finite number of frequencies are involved in the wave-particle interaction. A problem with this approach is that it assumes one knows *a priori* the frequencies that will be involved. But several phenomena like instabilities may happen at frequencies that are not simply connected with the pilot frequency, and consequently require a preliminary work to determine involved frequencies. A remedy to this problem is a time domain approach.

After a brief description of the TWT interaction, we shall show two time-domain methods to simulate the interaction that lies in a TWT. To modelize the propagating structure the first method is based on the use of an equivalent circuit, and the second one is based on Model Order Reduction. Both Methods are coupled with a one-dimensional Vlasov model of the beam. Then some results are shown.

## 2. The TWT interaction

In order to achieve strong interaction between electric wave and electrons, both must propagate at roughly the same velocity. One uses a slow-wave structure (SWS) made of a cylindrical metallic tube that contains a metallic helix as sketched in Fig.1. As the electric wave travels along the helix, its longitudinal velocity nears the electrons velocity.

Simulating this interaction by resolving

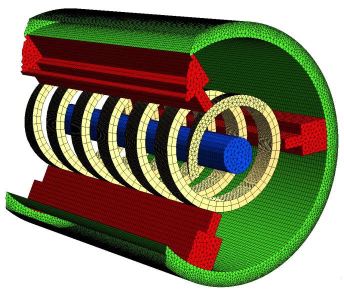

Fig.1 A cut of the helix slow wave structure. The metallic helix is maintained in the center of the waveguide by three ceramics rods.

author's e-mail: anass.aissi@thalesgroup.com, frederic.andre@thalesgroup.com, fabrice.doveil@univ-provence.fr

electrodynamics equations require solving huge equations at each time step. Indeed, once discretised, such a system has typically several millions of degrees of freedom. Therefore, to simulate the TWT interaction within a reasonable amount of time one have to develop lighter models for the propagating structure.

## 3. The equivalent circuit approach

As seen in Fig.2 each point on the helix is coupled capacitively with a point on the metallic sheath and two points that lie on the helix one turn forward and one turn backward. It is also connected with two neighboring points on the helix. The lowest level of discretization is to take two points per helix turn. It gives the equivalent circuit seen in Fig.3.

As seen in Fig. 4, this simple circuit exhibits a rather correct dispersion features over a wide frequency band. It is able to propagate direct and backward waves, and even to show a band gap.

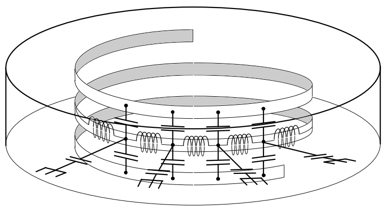

Fig.2 Construction of the equivalent circuit.

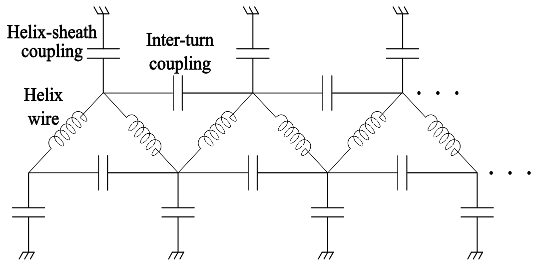


Fig.3 The simplest equivalent circuit is described by three parameters. One could add resistances to describe resistive effects.

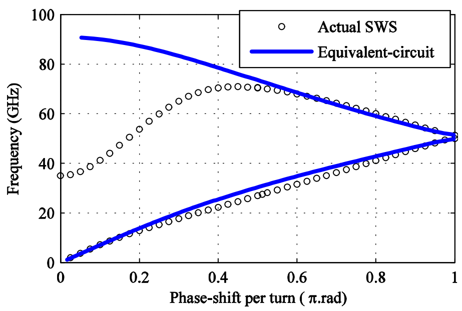


Fig.4 Comparison of the dispersion curves of both actual slow wave structure and equivalent circuit.

Time evolution of the equivalent circuit is simply obtained by solving Kirchoff's equations. If the vector $U$ contains the capacitance voltages and the inductance currents of the circuit, then its evolution is given by a differential equation in the form:

$$\dot{U} = M.U \quad (1)$$

when the matrix $M$ contains the coefficients given by Kirchoff's equations applied to the equivalent circuit. The solution of this equation is:

$$U(t) = e^{M.t}U(0) \quad (2)$$

where one uses the exponential of a matrix.

Next, this equivalent circuit is coupled with an electron beam. Since the TWT is essentially longitudinal, we assume that the electron beam is one-dimensional. The beam is described by its distribution function $f(x,v)$, and its evolution is given by the Vlasov's equation:

$$\frac{df}{dt} + v\frac{\partial f}{\partial x} + \frac{q}{m}(E_{SWS} + E_{SC})\frac{\partial f}{\partial v} = 0 \quad (3)$$

where $q$ is the electron charge, $m$ is the electron mass, $E_{SWS}(x)$ is the electrical field due to the slow wave structure and $E_{SC}(x)$ is the space-charge field. The solution of Vlasov equation is calculated by time-splitting the Vlasov equation (3) into two advective equations [7], and then solving successively both equations with the "Piecewise Parabolic Method" [8]. This method is fast and conserves positivity and monotonicity of $f(x,v)$ [9]. The electric field $E_{SWS}(x)$ is calculated at each time step as a function of the voltages of the equivalent circuit; and $E_{SC}(x)$ is obtained by solving Poisson's equation. We use an analytical solution for a one-dimensional discretized beam in a metallic cylinder.

While the electron beam is submitted to electric fields, it induces current in the SWS. This current is induced in the inter-turn capacitances, which are the only parts of the equivalent circuit that act on the electrons. Shockley-Ramo's theorem is used to calculate at each time step the variation of the voltage of inter-turn capacitances [10].

The behavior of a TWT involves several phenomena, and time evolution of each phenomenon is described by an equation. Hence, the whole time evolution is obtained by successive resolution of these equations. This approach is called time splitting or fractional step method [7]. Simply resolving successively the equations is a first order time splitting scheme.

We present now two simulations. In both simulations the electrons are emitted at the left-hand side of the SWS. In the first case, we inject a continuous beam along with a harmonic signal, and observe in Fig. 5 the bunching of the beam. In the second simulation, shown in Fig. 6, we inject a modulated beam and no signal. We then observe the growth and saturation of a signal. Computation time is about 30s on a standard office computer (2GHz, 1GB).

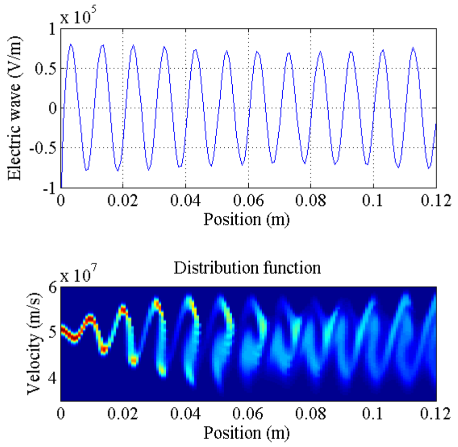


Fig.5 The wave modulates and traps the beam. The higher graphics represents the electric field in inter-turn capacitances. In the lower graphics the distribution function $f(x,v)$ is red when it is maximum and dark blue when it is null.

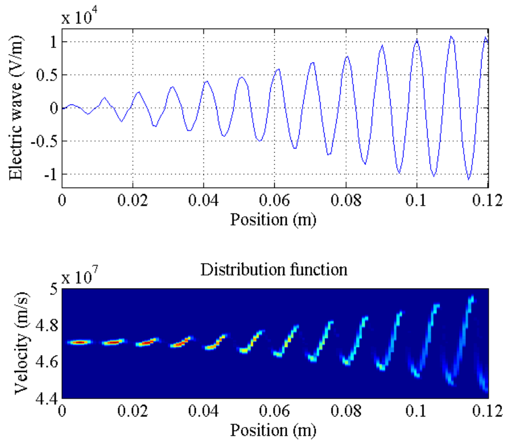


Fig.6 The modulated beam generate and amplifies an electric wave. The higher graphics represents the electric field in inter-turn capacitances. In the lower graphics the distribution function $f(x,v)$ is red when it is maximum and dark blue when it is null.

## 4. The Model Order Reduction approach

To solve numerically the Maxwell's equations, one usually discretizes them over a mesh that describes a particular electromagnetic structure and obtains a differential equation in the form:

$$M.\frac{\partial^2 U(t)}{\partial t^2}+C.\frac{\partial U(t)}{\partial t}+K.U(t)=R(t) \quad (4)$$

where:

- $U(t)$ is a vector that describes the electromagnetic field in the structure,
- $M$ is the mass matrix,
- $C$ is the damping matrix,
- $K$ is the stiffness matrix,
- and $R(t)$ is a vector that contains excitation terms due to charges and currents.

This equation is general and do not depend on the method used to descretize the Maxwell's equations (Finite difference, finite elements, etc…).

Next, one could resolve the equation (4) by using, for example, the Newmark method [11], but the size of the descetized equation is actually very large and it is not reasonable to solve it in time-domain.

Thanks to model order reduction methods [12], it could be possible to integrate huge systems of equations such as equation (4). These methods aim to replace a large differential system by a smaller one that preserves the behavior of the original one.

Applied to a dynamical system in the form:

$$\begin{cases} M.\dfrac{\partial^2 U(t)}{\partial t^2}+C.\dfrac{\partial U(t)}{\partial t}+K.U(t)=B.x(t) \\ y(t)=C.U(t) \end{cases} \quad (5)$$

where $x(t)$ is the input vector, and $y(t)$ is the output vector, the model order reduction methods construct a projection basis $V$ that change the system (5) to a smaller system:

$$\begin{cases} m.\dfrac{\partial^2 u(t)}{\partial t^2}+c.\dfrac{\partial u(t)}{\partial t}+k.u(t)=b.x(t) \\ y(t)=c.u(t) \end{cases} \quad (6)$$

where :

- $u(t)=V^T U(t)$,
- $m=V^T MV$,
- $c= V^T CV$,
- $k= V^T KV$,
- $b=V^T B$,
- $c=CV^T$.

Notice that both systems (5) and (6) have the same input and output vectors $x(t)$ and $y(t)$. Indeed, while reducing the system (5), one demand that its transfer function:

$$G(s)=\frac{y(s)}{x(s)} \quad (7)$$

is preserved, *i.e.* the transfer function of the reduced system (6):

$$g(s)=\frac{y(s)}{x(s)}\approx G(s) \quad (8)$$

around a certain frequency $s_0$.

Once (5) has been reduced into (6), time integration could be much faster.

We can apply this formalism to our electromagnetic structure because we are interested by the time evolution of only a little part of the electromagnetic field. In our case, the input vector $x(t)$ is related to the structure ends and the region where the electron beam evolves, because the electromagnetic structure is excited only by its ends and by the beam. The output vector $y(t)$ is related to the same region, because one just need to know the electromagnetic field at the ends of the structure, and the field that acts on electrons. Hence, an electromagnetic structure can be viewed as an input/output dynamical system.

We are implementing a reduction method found in [13] to reduce a propagating structure of a TWT wherin we have discretized Maxwell's equations by the finite elements method. The reduction method is based on Krylov subspaces to construct the projection basis $V$. As Krylov subspaces construction is essentially based on matrix-vector multiplication, it enables to obtain a fast and parallelized reduction method [14].

Before applying this technique to an actual TWT system, we have tried it on a fictive two-dimensional periodic propagating structure shown in Fig.7. This system has 575 degrees of freedom, and we have constructed a reduced system that has 44 degrees of freedom. Then, we have compared the dispersion curves of the original system and the reduced one. The result is shown in Fig.8, and one can see that the reduced system, that is 13 times smaller, propagates waves as the original one over 4 modes. Computation time needed to reduce this example was about 0.3s on standard computer (2GHz, 1GB).

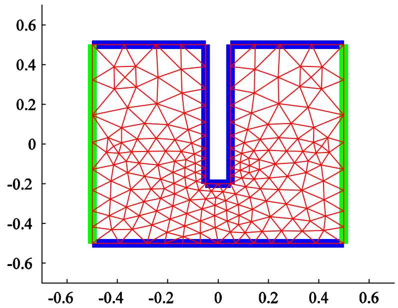


Fig.7 A step of a periodic two-dimensional propagating structure. Left and right side are the connection ports, and the remainder is a conducting boundary.

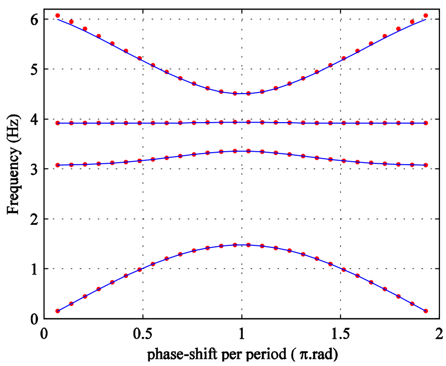


Fig.8 Comparison of dispersion features of the original structure (line) and the corresponding reduced model (points). Discrepancies are visible at the top of the fourth mode.

## 5. Conclusion and perspectives

We have proposed two methods to lighten the time-domain simulation of the traveling-wave tube.

The first method is based on the use of an equivalent circuit and showed good qualitative results. To conserve energy properly, it needs to be enhanced by using superior order time splitting scheme, and by taking into account the distribution of the electric field acting on electrons, i.e. the coupling impedance of the propagating structure.

The second method is based on the reduction of the propagating structure viewed as an input/output linear dynamical system. It showed good results in the case of a two-dimensional propagating structure, and is still in implementation for an actual traveling-wave tube propagation structure.

## 4. References

[1] A.S.Gilmour, Jr., *Principles of Traveling Wave Tubes* (Artech House, 1994).

[2] G. Dimonte and J.H. Malmberg, *Destruction of trapping oscillations*, Phy. Fluids, **21**, 1188-1206 (1978); S.I. Tsunoda, F. Doveil and J.H. Malmberg, *Experimental test of quasilinear theory,* Phys. Fluids, **B3**, 2747-2757 (1991).

[3] F. Doveil, A. Macor and A. Aïssi, *Observation of Hamiltonian chaos and its control in wave-particle interaction,* Plasma Phys. Controlled Fusion, **49**, B125-135 (2007).

[4] L. Brillouin, *The Traveling-Wave Tube*, Journal of Applied Physics **20**, 1196-1206 (1949).

[5] J.R.Pierce, *Traveling-wave tubes*, Van Nostrand Company (1950).

[6] J.E.Rowe, *A Large-Signal Analysis of the Traveling-wave Amplifier: Theory and General Results*, IRE Trans. Electron Devices, 39-56 (Jan. 1956).

[7] C.Z.Cheng, G.Knorr, *The Integration of the Vlasov Equation in Configuration Space*, Journal of computational physics **22**, 330-351 (1976).

[8] Ph.Colella, P.R.Woodward, *The piecewise Parabolic Method for Gas-Dynamical Simulations*, Journal of computational physics **54**, 174-201 (1984).

[9] T.D.Arber, R.G.L.Vann, *A Critical Comparison of Eulerian-Grid-Based Vlasov Solvers*, Journal of computational physics **180**, 339-357 (2002).

[10] M.D.Sirkis, N.Holonyak, Jr., *Currents Induced by Moving Charges*, American Journal of Physics **34**, 943-946 (1966).

[11] K.J.Bathe, E.L.Wilson, *Numerical Methods in Finite Elements Analysis* (Prentice-Hall, Inc., New Jersey, 1976)

[12] A.C.Antoulas, *An overview of approximation methods for large-scale dynamical systems*, Annual Reviews In Control **29**, 181-190 (2005).

[13] B.Salimbahrami, B.Lohmann, *Order reduction of large scale second-order systems using Krylov subspace methods*, Linear Algebra and its applications **415**, 385-405 (2006).

[14] E.J.Grimme, *Krylov projection methods for model reduction*, Thesis of the University of Illinois (1997).