Electron-Inertia Effects On The Transverse Gravitational Instability

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This paper studies the non-ideal electron inertial effects on the gravitational instability of a medium permeated by a magnetic field both in the presence and absence of rotation. It is seen that the inertial effects become important when considering the transverse perturbations as, in this case, the compressional wave, modified by the electron-inertial terms (inertial compressional wave - ICW), can modify the Jeans gravitational criterion. A band of wavenumber is introduced around the Jeans wavenumber for which the ICW is found to have a destabilizing effect. The width of the band depends on the parameter β, the ratio of sound to Alfvén speed, and the electron inertial length δ. In the case of rotation it is seen that when the perturbations are transverse to the direction of the magnetic field and the rotational axis the system is stable for all wave numbers for rotational frequencies greater than a critical value. Otherwise, the unstable range of wavenumbers introduced by the inertial effects is also modified by the Coriolis force.

Keywords: Self-gravitating systems, gravitational instability, electron inertia, compressional Alfvén wave, Hall effect, Jeans instability criterion, Coriolis force.

1. Introduction

The gravitational instability of a plasma media is important for understanding various astrophysical problems and since the first stability analysis given by Jeans [1] there has been great interest in studying the onset of gravitational instabilities in rotating, magnetized or turbulent media [2]. Recently there has been interest in understanding the transverse gravitational instabilities of magnetized plasmas, especially to understand the theory of bi-modal star formation. An important point to be noted in this case is that the magnetic field opposes gravitational instability and the critical Jeans wavelength for the instability is increased by a factor of $\left(\frac{\beta}{1+\beta}\right)^{-1/2}$, where the constant β is the ratio of thermal over magnetic pressure [3]. In this paper we argue that for the case where the perturbations are perpendicular to the direction of the magnetic field the situation can arise when the perpendicular wavelengths become short in comparison with the finite Larmor radius or the electron-inertial length and it becomes important to include the Hall effects.

The Hall current dynamics of self-gravitating systems have been studied by many authors [4], but almost all of them have neglected electron inertial effects. The aim of the present paper is to study the transverse gravitational instability taking account of electron inertial terms in the generalized Ohm's Law [5].

The inclusion of the finite Larmor radius effect, as any other dissipative effect, is seen to oppose the stabilizing effect of the magnetic field [4, 6]. When electron inertia is taken into account however we find that the Alfvén waves are modified and, in the case of transverse perturbations, we get a wave which, following the classification [7] for the Alfven Waves in the presence of non-ideal effects, we call an inertial compressional Alfvén wave. In this paper we essentially discuss the compressional inertial acoustic-gravitational mode interaction and show the changes brought in the Jeans instability criterion both in the absence and presence of rotation.

2. Basic Equations

Consider a self-gravitating system immersed in a uniform magnetic field $\vec{H}$ rotating with uniform angular velocity $\vec{\Omega}$. The equilibrium density and the pressure are represented, respectively, by ρ_0 and p_0. Let $\rho_1, \vec{v}_1, p_1, U$ and $\vec{h}$ be, in order, the perturbed density, velocity, pressure, the gravitational potential and magnetic field. With these definitions the following linearized basic equations can be set down as governing the system:

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \text{ div } \vec{v}_1 = 0, \tag{1}$$

$$\rho_0 \left[\frac{\partial \vec{v}_1}{\partial t} + 2\vec{\Omega} \times \vec{v}_1\right] = \text{grad } p + \rho_0 \nabla(U) + \frac{1}{4\pi}[(\nabla \times \vec{h}) + \vec{H}_0], \tag{2}$$

$$p_1 = S^2 \rho_1, \tag{3}$$

where S is the sound speed corresponding to the unperturbed medium;

$$\nabla^2 U = -4\pi G \rho_1, \tag{4}$$

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$$\mathrm{div}\vec{h} = 0, \tag{5}$$

and

$$\frac{\partial \vec{h}}{\partial t} = \mathrm{curl}(\vec{v}_1 \times \vec{H}_0) - \frac{c}{4\pi n_e} \times (\nabla \times \vec{h} \times \vec{H}_0) + \frac{c^2}{\omega_{pe}^2}\frac{\partial(\nabla^2 \vec{h})}{\partial t}, \tag{6}$$

where

$$\omega_{pe} = \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2}$$

is the electron plasma frequency, with e, m_e, respectively representing the charge and mass of the electron and n_e the particle density of the electron gas.

Setting the magnetic field and the rotational frequency as $\vec{H}_0 = (H_x, 0, 0)$ and $\vec{\Omega} = (\Omega_x, \Omega_y, 0)$ we look for transverse perturbations taking the time and space dependencies as $\exp i(\omega t + kz)$.

3. Dispersion Equations

The dispersion equation for transverse perturbations in the absence of rotation becomes

$$\omega^2 = \frac{k^2 V_x^2}{1 + \frac{c^2 k^2}{\omega_{pe}^2}} + S^2 k^2 - 4\pi G \rho_0, \tag{7}$$

where

$$V_x = \left(\frac{H_x^2}{4\pi \rho_0}\right)^{1/2}$$

is the Alfvén velocity.

The first term on the r.h.s. of eqn. (7) gives the compressional Alfvén inertial wave. The group velocity in the x-direction for this wave is given by

$$v_{gx} = \left(\frac{\partial \omega}{\partial k}\right) = \left(\frac{\omega}{k}\right)\frac{1}{(1 + \delta^2 k^2)}, \tag{8}$$

where δ is the electron inertial length c/ω_{pe}. From eqn. (8) we may note that the energy is transported in the transverse direction. For $\delta k > 1$ the group velocity is slower than the phase velocity in the transverse direction.

It is seen from eqn. (7) that the compressional inertial wave couples with acoustic and gravity modes.

When $k^2 > \left(\frac{4\pi G \rho_0}{S^2}\right) \equiv k_J^2$, the system is stable and Jeans stability holds well. When $k^2 < k_J^2$, the system can be either stable or unstable for $k < k_c$, where k_c, the critical wavenumber, is given by the equation:

$$\delta^2 k^4 + k^2 \left[\frac{\beta + 1}{\beta} - k_J^2 \delta^2\right] - k_J^2 = 0, \tag{9}$$

with $\beta = \left(\frac{S^2}{V_x^2}\right)$.

We note that eqn. (9) has one positive and one negative root. The positive root k_c^2 can be easily checked to be less than k_J^2 but is greater than k_{JM}^2, that is $k_{JM} < k_c < k_J$, where k_{JM} is the wavenumber in the presence of a magnetic field:

$$k_{JM} = k_J \left(\frac{\beta + 1}{\beta}\right)^{1/2}.$$

Hence electron inertial terms oppose the stability introduced by the magnetic field, just as in the case of other dissipative effects, but not completely, for all wavenumbers $k_{JM} < k < k_c$ the system which was stable becomes unstable.

An interesting case arises when

$$k_J = \frac{1}{\delta}\left(\frac{\beta + 1}{\beta}\right)^{1/2}.$$

Then

$$k_c = \left(\frac{k_J}{\delta}\right)^{1/2} \tag{10}$$

is greater than k_J since $k_J > 1/\delta$. Hence the electron inertia has a destabilizing effect and the system which is stable for wavenumbers $k > k_J$ becomes unstable for the range $k_J < k < \left(\frac{k_J}{\delta}\right)^{1/2}$.

4. Effect of Rotation

In the presence of Coriolis Force with the rotational axis along the direction of the magnetic field the dispersion relation becomes

$$\omega^2 = \frac{k^2 V^2}{1 + k^2 \delta^2} + S^2 k^2 - 4\pi G \rho_0 + 4\Omega^2. \tag{11}$$

We wish to point out, at first, that the coupling of the Coriolis force and the magnetic field arises only in the case where the rotational axis lies along the magnetic field direction as, otherwise, the rotation gets decoupled.

From eqn. (11), an important, well-known result [2], is apparent that when $\Omega^2 > \pi G \rho_0$ the system remains stable. This shows that when rotational frequency is large it is impossible to break the system into fragments due to gravitational instability.

We shall now discuss the case when $\Omega^2 < \pi G \rho_0$. Here again, for $k^2 < k_J^2$, the system is stable, as before, in the Jeans gravitational sense but the criterion changes to $k^2 > (k_J^2 - k_\Omega^2)$ where $k_\Omega^2 = \frac{4\Omega^2}{S^2}$. When $k^2 < k_J^2$, however, the instability recurs for $k^2 < k_c^2$, the critical wavenumber being given by the equation:

$$\delta^2 k^4 + k^2 \left[\frac{\beta + 1}{\beta} - (k_J^2 - k_\Omega^2)\delta^2\right] - (k_J^2 - k_\Omega^2) = 0. \tag{12}$$

On taking $(k_J^2 - k_\Omega^2) \equiv K^2$, eqn. (12) is found to be identical to eqn. (9). Therefore, we find that the critical wavenumber k_c lies between

$$(k_J^2 - k_\Omega^2)^{1/2}\left(\frac{\beta}{\beta + 1}\right)^{1/2} < k_c < (k_J^2 - k_\Omega^2)^{1/2}.$$

In the case when $k_J^2\delta^2 = \left(\frac{\beta+1}{\beta} + k_\Omega^2\delta^2\right)$ instability is introduced in the region $k > k_J$, with the critical wavenumber given by $k_c^2 = \frac{1}{\delta}(k_J^2 - k_\Omega^2)^{1/2}$.

5. Conclusion

It is apparent from this work that the finite electron inertial effects modify the Alfvén compressional wave, giving the Inertial Compressional Alfvén mode (ICW). The interaction of the ICW with acoustic and gravitational modes shows interesting characteristic changes in the Jeans gravitational instability both for non-rotating and rotating systems.

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