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A One-dimensional Kinetic Model of a Bounded Plasma System Containing Hot and Emitted Electrons with Drifting Velocity Distributions

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A one dimensional fully kinetic model of a bounded plasma system that contains two electron populations with drifting Maxwellian distributions is presented. These two groups of electrons are the hot electrons and the electrons emitted from the collector. Both drifts even if they are relatively small have a strong effect on the floating potential of the collector and other parameters.

Keywords: plasma diode, sheath, Bohm criterion, two-electron temperature plasma, electron emission

1. Introduction

Bounded plasma systems are important because of their relevance in various areas of plasma technology as well as in fusion research. For an extensive review on the kinetic and particle simulation of the bounded plasma systems the reader is directed to the review paper by Kuhn [1]. A fully kinetic one-dimensional model developed for the analysis of monotonic potential profiles in bounded plasma systems for an arbitrary ion to electron temperature ratio was presented by Schwager and Birdsall [2]. This model turned out as very appropriate for various purposes and has been used and modified by several authors [3] - [9]. Also we have extended this model and included two additional electron populations into it - the hot and the emitted electrons [7] - [9]. It has been observed experimentally [10, 11] that electron emitting electrodes may have triple floating potentials in the presence of an additional energetic electron population. Our model qualitatively reproduces these experimental results. This work presents further extension of this model. A non-zero drift velocities are included into the distribution functions of the hot and of the emitted electrons. In addition a non-zero electric field at the source is also included into the model. In the next section mathematical details of the model are presented. In section 3 some preliminary results related to the finite drifts are shown and in section 4 conclusions are given.

2. Model

An infinitely large planar electrode (collector) has its surface perpendicular to the x axis and is located at $x = 0$. This electrode absorbs all the particles that hit it. On the other hand it may also emit electrons. This electron emission can be thermal or secondary triggered by the impact of incoming electrons and/or ions. The details of the emission mechanism are not essential for the model.

An infinitely large planar plasma source has also its surface perpendicular to the x axis (figure 1). This source is located at a certain distance $x = L$ from the collector. The distance L is not crucial for the results of the model. This source injects 3 groups of charged particles into the system: singly charged positive ions (index i), the cool electrons (index 1) and the hot electrons (index 2). The emitted electrons from the collector have index 3. The particles i, 1 and 2 are injected from the source with half-maxwellian velocity distribution functions with temperatures T_i, T_1 and T_2. Also the emitted electrons have a half-

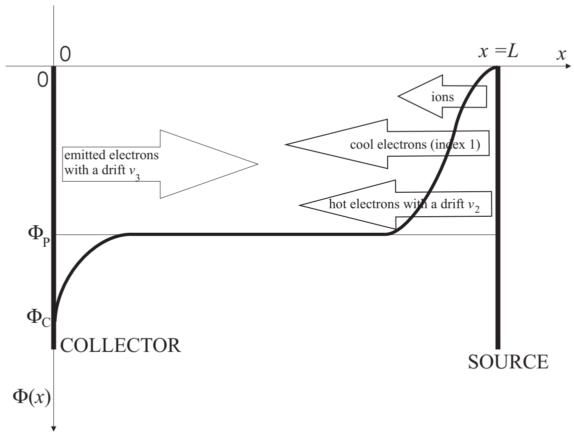

Fig. 1 Schematic of the model.

maxwellian velocity distribution function at the collector with the temperature T_3. In principle all four temperatures may be arbitrary. The electrons emitted from e.g. emissive probes or any other electron emitting electrodes are very often treated as monoenergetic [3]. So in this work we shall always assume that $T_3 \ll T_1$. The electrons 2 are called the hot electrons. This implies $T_2 \gg T_1$. A non-zero drift velocity v_2 is included into the velocity distribution of the hot electrons. The emitted electrons have a finite drift velocity v_3 in the direction towards the source. In an experiment the emitted electrons could get a small non-zero

author's e-mail: tomaz.gyergyek@fe.uni-lj.si initial velocity if predominant mechanism of the electron emission is secondary emission and the emitted electrons are kicked out of the collector surface by the impacting particles with a finite initial velocity.

The potential profile $\Phi(x)$ is determined by the Poisson equation:

$$\frac{d^2\Phi}{dx^2} = -\frac{e_0}{\varepsilon_0}\left(n_i - n_1 - n_2 - n_3\right). \tag{1}$$

where e_0 is the elementary charge and ε_0 is the permittivity of free space.

The potential at the source is set to zero. We are only interested in the solutions where the potential $\Phi(x)$ increases monotonically from the value Φ_C at the collector to the zero value at the source. Because of that the electric field everywhere in the system must be negative or zero. The absolute value of the electric field at the source is labeled by E_L. So the boundary conditions for the equation (1) are:

$$\Phi(x = L) = 0, \ \frac{d\Phi}{dx}(x = L) = -E_L. \tag{2}$$

The ion velocity distribution function at some position x between the source and the collector is given by:

$$\begin{aligned} f_i = n_i\sqrt{\tfrac{m_i}{2\pi kT_i}}\exp\left(-\tfrac{e_0\Phi(x)}{kT_i}\right)\times \\ \times\exp\left(-\tfrac{m_i v^2}{2kT_i}\right). \end{aligned} \tag{3}$$

Here m_i is the ion mass and n_i is the density that the ions would have at the source if their velocity distribution there was fully maxwellian and k is the Boltzmann constant. The distribution function for the cool electrons is given by:

$$\begin{aligned} f_1 = n_1\sqrt{\tfrac{m_e}{2\pi kT_1}}\exp\left(\tfrac{e_0\Phi(x)}{kT_1}\right)\times \\ \times\exp\left(-\tfrac{m_e v^2}{2kT_1}\right). \end{aligned} \tag{4}$$

Here n_1 is the density that the cool electrons would have at the source, if their velocity distribution there was fully maxwellian. For the hot electrons the distribution function is written in a similar way. But one must take into account that the hot electrons have a finite drift velocity v_2 towards the collector:

$$\begin{aligned} f_2 = n_2\sqrt{\tfrac{m_e}{2\pi kT_2}}\exp\left(\tfrac{e_0\Phi(x)}{kT_2}\right)\times \\ \times\exp\left(-\tfrac{m_e (v+v_2)^2}{2kT_2}\right). \end{aligned} \tag{5}$$

Here n_2 is is the density that the hot electrons would have at the source, if their velocity distribution there was fully maxwellian. The emitted electrons have a finite drift velocity v_3 towards the source, so their distribution function is written as:

$$\begin{aligned} f_3 = n_3\sqrt{\tfrac{m_e}{2\pi kT_3}}\exp\left(\tfrac{e_0(\Phi(x)-\Phi_C)}{kT_3}\right)\times \\ \times\exp\left(-\tfrac{m_e (v-v_3)^2}{2kT_3}\right). \end{aligned} \tag{6}$$

Here n_3 is is the density that the emitted electrons would have at the collector, if their velocity distribution there was fully maxwellian.

The potential of the collector is Φ_C. We assume that the plasma is collisionless and the energy and the flux of the particles are conserved. An ion that is born at the source with zero velocity has at the distance x from the collector the velocity:

$$v_{mi} = -\sqrt{-\frac{2e_0\Phi(x)}{m_i}}, \tag{7}$$

in the direction towards the collector. Because of this a negative sign is in front of the square root. So the ion velocity distribution function (3) actually has a cutoff at v_{mi} given by (7). Something similar is valid for the electrons. An electron that leaves the collector with a negligibly small initial velocity will have at the position x the velocity

$$v_{me} = \sqrt{\frac{2e_0\left(\Phi(x) - \Phi_C\right)}{m_e}}, \tag{8}$$

in the direction towards the source. So the electron velocity distribution functions (4)-(6) have a cutoff at v_{me} given by the (8).

The following variables are introduced:

$$\begin{aligned} &\mu = \tfrac{m_e}{m_i},\ \tau = \tfrac{T_i}{T_1},\ \Theta = \tfrac{T_2}{T_1}, \\ &\sigma = \tfrac{T_3}{T_1},\ \Psi = \tfrac{e_0\Phi(x)}{kT_1}, \\ &\Psi_C = \tfrac{e_0\Phi(x=0)}{kT_1} = \tfrac{e_0\Phi_C}{kT_1}, \\ &\alpha = \tfrac{n_i}{n_1},\ \beta = \tfrac{n_2}{n_1},\ \varepsilon = \tfrac{n_3}{n_1}, \\ &v_0 = \sqrt{\tfrac{2kT_1}{m_e}},\ \ u = \tfrac{v}{v_0},\ u_2 = \tfrac{v_2}{v_0},\ u_3 = \tfrac{v_3}{v_0}, \\ &u_{me} = \tfrac{v_{me}}{v_0} = \sqrt{\Psi - \Psi_C}, \\ &u_{mi} = \tfrac{v_{mi}}{v_0} = -\sqrt{-\mu\Psi}. \end{aligned} \tag{9}$$

With these variables the distribution functions (3) - (6) are written in the following way:

$$F_i\left(u,\Psi\right) = \frac{\alpha}{\sqrt{\pi\tau\mu}}\ \exp\left(-\frac{\Psi}{\tau}\right)\exp\left(-\frac{u^2}{\mu\tau}\right), \tag{10}$$

$$F_1\left(u,\Psi\right) = \frac{1}{\sqrt{\pi}}\ \exp\left(\Psi\right)\exp\left(-u^2\right), \tag{11}$$

$$\begin{aligned} F_2\left(u,\Psi\right) = \tfrac{\beta}{\sqrt{\pi\Theta}}\ \exp\left(\tfrac{\Psi}{\Theta}\right)\times \\ \times\exp\left(-\tfrac{(u+u_2)^2}{\Theta}\right), \end{aligned} \tag{12}$$

$$\begin{aligned} F_3\left(u,\Psi\right) = \tfrac{\varepsilon}{\sqrt{\pi\sigma}}\ \exp\left(\tfrac{\Psi-\Psi_C}{\sigma}\right)\times \\ \times\exp\left(-\tfrac{(u-u_3)^2}{\sigma}\right). \end{aligned} \tag{13}$$

The distribution functions (10)-(13) are normalized to the density that the cool electrons would have at the source, if their velocity distribution there was fully maxwellian:

$$n_1\sqrt{\frac{m_e}{2\pi kT_1}}\int_{-\infty}^{\infty}\exp\left(-\frac{m_e v^2}{2kT_1}\right)dv = \tag{14}$$

$$= \frac{1}{\sqrt{\pi}}\frac{n_1}{v_0}\int_{-\infty}^{\infty}\exp\left(-\frac{v^2}{v_0^2}\right)dv = n_1 \equiv 1.$$

The zero moments of the distribution functions give the particle densities, while the first moments give the particle fluxes. Integration over the velocity u goes only to the respective cutoff velocities u_{mi} and u_{me}.

$$N_i(\Psi) = \int_{-\infty}^{-\sqrt{-\mu\Psi}} F_i(u,\Psi)\,du, \tag{15}$$

$$J_i(\Psi) = \int_{-\infty}^{-\sqrt{-\mu\Psi}} uF_i(u,\Psi)\,du, \tag{16}$$

$$N_1(\Psi) = \int_{-\infty}^{\sqrt{\Psi-\Psi_C}} F_1(u,\Psi)\,du, \tag{17}$$

$$J_1(\Psi) = \int_{-\infty}^{-\sqrt{\Psi-\Psi_C}} uF_1(u,\Psi)\,du, \tag{18}$$

$$N_2(\Psi) = \int_{-\infty}^{\sqrt{\Psi-\Psi_C}} F_2(u,\Psi)\,du, \tag{19}$$

$$J_2(\Psi) = \int_{-\infty}^{-\sqrt{\Psi-\Psi_C}} uF_2(u,\Psi)\,du, \tag{20}$$

$$N_3(\Psi) = \int_{\sqrt{\Psi-\Psi_C}}^{\infty} F_3(u,\Psi)\,du, \tag{21}$$

$$J_3(\Psi) = \int_{\sqrt{\Psi-\Psi_C}}^{\infty} uF_3(u,\Psi)\,du, \tag{22}$$

Note that in (18) and (20) we have integrated only over the velocity of those electrons that actually reach the collector and are not repelled back into the system.

The densities (15), (17), (19) and (21) are inserted into the Poisson equation (1). The following equation is obtained:

$$\begin{aligned}\frac{d^2\Psi}{dz^2} &= -\frac{\alpha}{2}\exp\left(-\frac{\Psi(z)}{\tau}\right)\operatorname{erfc}\left(\sqrt{-\frac{\Psi(z)}{\tau}}\right)+\\ &+\frac{1}{2}\exp(\Psi(z))\left[1+\operatorname{erf}\left(\sqrt{\Psi(z)-\Psi_C}\right)\right]+\\ &+\frac{\beta}{2}\exp\left(\frac{\Psi(z)}{\Theta}\right)\left[1+\operatorname{erf}\left(\frac{u_2+\sqrt{\Psi(z)-\Psi_C}}{\sqrt{\Theta}}\right)\right]+\\ &+\frac{\varepsilon}{2}\exp\left(\frac{\Psi-\Psi_C}{\sigma}\right)\operatorname{erfc}\left(\frac{\sqrt{\Psi(z)-\Psi_C}-u_3}{\sqrt{\sigma}}\right),\end{aligned} \tag{23}$$

where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp\left(-t^2\right)dt,$$

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp\left(-t^2\right)dt.$$

The space coordinate x has been normalized to the Debye length λ_D in the following way:

$$z = \frac{x}{\lambda_D},\ \lambda_D = \sqrt{\frac{\varepsilon_0 kT_1}{n_1 e_0^2}}. \tag{24}$$

The boundary conditions (2) become:

$$\Psi\left(z=\frac{L}{\lambda_D}\right) = 0,\ \frac{d\Psi}{dz}\left(z=\frac{L}{\lambda_D}\right) = -\eta, \tag{25}$$

where

$$\eta = \frac{E_L}{\frac{kT_1}{e_0\lambda_D}} = E_L\sqrt{\frac{\varepsilon_0}{kT_1 n_1}}. \tag{26}$$

At some point ($z = z_0$) between the collector and the source, the plasma is quasi-neutral and the potential at that point is $\Psi(z = z_0) = \Psi_P$. From the Poisson equation the neutrality condition can therefore be derived in the following form:

$$\begin{aligned}\left(\frac{d^2\Psi}{dz^2}\right)_{z=z_0} &= -\alpha\exp\left(-\frac{\Psi_P}{\tau}\right)\operatorname{erfc}\left(\sqrt{-\frac{\Psi_P}{\tau}}\right)+\\ &+\exp(\Psi_P)\left[1+\operatorname{erf}\left(\sqrt{\Psi_P-\Psi_C}\right)\right]+\\ &+\beta\exp\left(\frac{\Psi_P}{\Theta}\right)\left[1+\operatorname{erf}\left(\frac{u_2+\sqrt{\Psi_P-\Psi_C}}{\sqrt{\Theta}}\right)\right]+\\ &+\varepsilon\exp\left(\frac{\Psi_P-\Psi_C}{\sigma}\right)\operatorname{erfc}\left(\frac{\sqrt{\Psi_P-\Psi_C}-u_3}{\sqrt{\sigma}}\right)=0.\end{aligned} \tag{27}$$

It is known that

$$\frac{1}{2}\frac{d}{dz}\left(\frac{d\Psi}{dz}\right)^2 = \frac{d\Psi}{dz}\frac{d^2\Psi}{dz^2}.$$

So the Poisson equation (23) is multiplied by $d\Psi/dz$. The differentials dz cancel out and the equation can be integrated once over the potential from $\Psi = 0$ (at $z = L/\lambda_D$) to $\Psi = \Psi_P$ (at $z = z_0$):

$$\begin{aligned}&\left(\frac{d\Psi}{dz}\right)^2_{\Psi=\Psi_P} - (-\eta)^2 =\\ &= -\frac{\alpha}{2}\int_0^{\Psi_P}\exp\left(-\frac{\Psi}{\tau}\right)\operatorname{erfc}\left(\sqrt{-\frac{\Psi}{\tau}}\right)d\Psi+\\ &+\frac{1}{2}\int_0^{\Psi_P}\exp(\Psi)\left[1+\operatorname{erf}\left(\sqrt{\Psi-\Psi_C}\right)\right]d\Psi+\\ &+\frac{\beta}{2}\int_0^{\Psi_P}\exp\left(\frac{\Psi}{\Theta}\right)\left[1+\operatorname{erf}\left(\frac{u_2+\sqrt{\Psi-\Psi_C}}{\sqrt{\Theta}}\right)\right]d\Psi+\\ &+\frac{\varepsilon}{2}\int_0^{\Psi_P}\exp\left(\frac{\Psi-\Psi_C}{\sigma}\right)\operatorname{erfc}\left(\frac{\sqrt{\Psi-\Psi_C}-u_3}{\sqrt{\sigma}}\right)d\Psi=\\ &= -\eta^2.\end{aligned} \tag{28}$$

Since the second derivative of the potential at $\Psi(z = z_0) = \Psi_P$ is zero, the first derivative, which is proportional to the electric field, is a constant. Since $L \gg \lambda_D$, the electric field at $\Psi(z = z_0) = \Psi_P$ may be set to zero.

If the emission of the electrons from the collector increases, eventually the density of the emitted electrons in front of the collector becomes so high, that the electric field at the collector becomes zero. In this case the emission is space charge limited, or critical. The value of ε, at which the electric field at the collector becomes zero, is called the critical emission coefficient. This fact can be used to derive another expression relating Ψ_P and Ψ_C. This relation is called zero field condition at the collector and is derived in very similar way as the zero field condition at the inflection point (28). In fact only the boundaries of the integration are changed. The integration goes from $\Psi = \Psi_P$ (at $z = z_0$) to $\Psi = \Psi_C$ (at $z = 0$). This gives:

$$\begin{aligned}&\left(\frac{d\Psi}{dz}\right)^2_{\Psi=\Psi_P} - \left(\frac{d\Psi}{dz}\right)^2_{\Psi=\Psi_C} =\\ &= -\frac{\alpha}{2}\int_{\Psi_P}^{\Psi_C}\exp\left(-\frac{\Psi}{\tau}\right)\operatorname{erfc}\left(\sqrt{-\frac{\Psi}{\tau}}\right)d\Psi+\\ &+\frac{1}{2}\int_{\Psi_P}^{\Psi_C}\exp(\Psi)\left[1+\operatorname{erf}\left(\sqrt{\Psi-\Psi_C}\right)\right]d\Psi+\\ &+\frac{\beta}{2}\int_{\Psi_P}^{\Psi_C}\exp\left(\frac{\Psi}{\Theta}\right)\left[1+\operatorname{erf}\left(\frac{u_2+\sqrt{\Psi-\Psi_C}}{\sqrt{\Theta}}\right)\right]d\Psi+\\ &+\frac{\varepsilon}{2}\int_{\Psi_P}^{\Psi_C}\exp\left(\frac{\Psi-\Psi_C}{\sigma}\right)\operatorname{erfc}\left(\frac{\sqrt{\Psi-\Psi_C}-u_3}{\sqrt{\sigma}}\right)d\Psi=0.\end{aligned} \tag{29}$$

The particle fluxes are given by the equations (16), (18), (20) and (22). J_i, J_1 and J_2 have negative sign because of their direction towards the collector, while J_3 has positive direction towards the source. If the negative charge of the electrons and the positive charge of the ions is taken into account, the total electric current density J_t to the collector is given by:

$$\begin{aligned} J_t = &-\frac{\alpha}{2}\sqrt{\frac{\mu\tau}{\pi}} - \frac{\varepsilon}{2\sqrt{\pi}}\exp\left(\frac{\Psi-\Psi_C}{\sigma}\right)\times \\ &\times\left[\begin{array}{c}\sqrt{\sigma}\exp\left(-\frac{\left(\sqrt{\Psi-\Psi_C}-u_3\right)^2}{\sigma}\right)+ \\ +u_3\sqrt{\pi}\,\mathrm{erfc}\left(\frac{\sqrt{\Psi-\Psi_C}-u_3}{\sqrt{\sigma}}\right)\end{array}\right]+ \\ &+\frac{1}{2\sqrt{\pi}}\exp\left(\Psi_C\right)+\frac{\beta}{2\sqrt{\pi}}\exp\left(\frac{\Psi}{\Theta}\right)\times \\ &\times\left[\begin{array}{c}\sqrt{\Theta}\exp\left(-\frac{\left(\sqrt{\Psi-\Psi_C}-u_2\right)^2}{\Theta}\right)+ \\ +u_2\sqrt{\pi}\,\mathrm{erfc}\left(\frac{\sqrt{\Psi-\Psi_C}-u_2}{\sqrt{\Theta}}\right)\end{array}\right]. \end{aligned} \tag{30}$$

In this way the direction of the electric current is defined in technical sense.

The neutrality condition (27), the zero electric field condition at the inflection point (28), the zero electric field condition at the collector (29) and the expression for the current density to the collector (30) form the basis of the model. The parameters that in an experiment are determined by the selection of the gas, the method of the plasma production and the material properties of the collector are selected as independent parameters of the model. These parameters are the ion mass μ the density and temperature ratios β, Θ, τ and σ, the drift velocities u_2 and u_3 and the electric field at the source η. If the emission is below the space charge limit, ε is also a given, independent parameter. In such a case the equations (27), (28) and (30) form a system of 3 equations for 3 unknown quantities. If the collector is floating, these quantities are the floating potential of the collector Ψ_C, the potential at the inflection point Ψ_P and the neutralization parameter α. If the collector is floating, J_t in (30) is set to zero. If a given potential Ψ_C is applied to the collector, then Ψ_P and α are found from (27) and (28) and then J_t is found from (30). If the emission is space charge limited also the equation (29) must be added into the system of equations and the critical emission coefficient ε is found as a solution of the system of equations together with Ψ_C, Ψ_P and α.

3. Results

In this section we show some results of our model. We first illustrate some general properties of our model. The following parameters are selected: μ = 1/1836, Θ = 100, σ = 0.01, τ = 0.1, u_2 = 0.1, u_3 = 0.004 and η = 0. The hot to cool electron density ratio β is gradually increased and the system of equations (27), (28), (29) and (30) is solved for Ψ_P, Ψ_C, α and ε. In this way one gets the dependence of Ψ_P, Ψ_C, α and ε on β. The plot is shown in figure 2. For very small values of β the system of equations (27), (28), (29) and (30) only has 1 solution. We call it the low solution because the absolute value of the corresponding Ψ_P is the smallest. When β reaches the value around 0.094 suddenly 2 branches of a new solution appear. We call it the middle solution because the absolute value of the corresponding Ψ_P is the intermediate. One branch of the middle solution merges with the low solution when the value of β exceeds 0.331 and the other branch of the middle solution merges with the high solution when β exceeds 0.165. The high solution is the third solution of the system (27), (28), (29) and (30) with the largest absolute value of the corresponding Ψ_P. This solution appears when β exceeds 0.134. This solution also has 2 branches. One of them joins with the middle solution when β exceeds 0.165 and the other branch extends to the values of β that are even larger than 0.331 where the low and the middle solution join. In addition the low solution splits into 3 branches when β is between 0.162 and 0.227. So for β between 0.162 and 0.165 the system of equations (27), (28), (29) and (30) has 7(!) solutions all together.

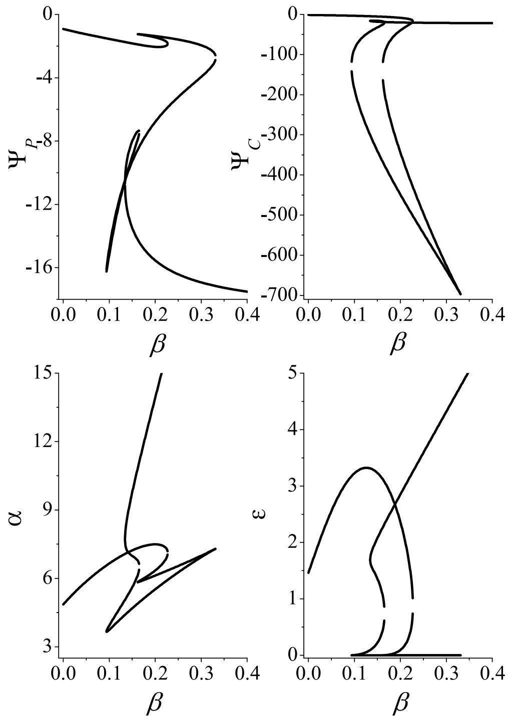

Fig. 2 Solutions Ψ_P, Ψ_C, α and ε of the system of equations (27), (28), (29) and (30) versus β for μ = 1/1836, Θ = 100, σ = 0.01, τ = 0.1, u_2 = 0.1, u_3 = 0.004 and η = 0.

In figures 3 and 4 the dependence of Ψ_P, Ψ_C, α and ε on drift velocities u_3 and u_2 is plotted. For the figure 3 the following parameters are selected: μ = 1/1836, Θ = 100, σ = 0.01, τ = 0.1, u_2 = 0.1, β = 0.163 and η = 0. The drift velocity of the emitted electrons u_3 is varied between 10^{-4} and 10^{-2}. The low, the middle and the high solution can be found in the entire interval of u_3 shown in figure 3. For the values of u_3 between 0.0019 and 0.0066 the low solution is

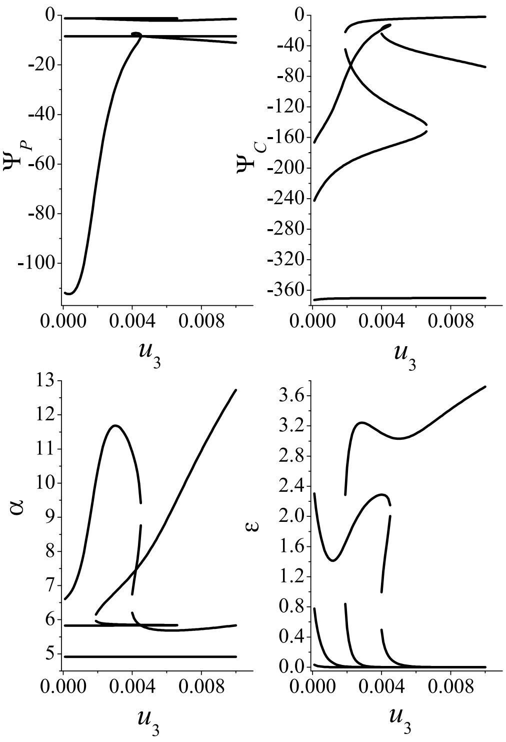

Fig. 3 Solutions Ψ_P, Ψ_C, α and ε of the system of equations (27), (28), (29) and (30) versus u_3 for μ = 1/1836, Θ = 100, σ = 0.01, τ = 0.1, u_2 = 0.1, β = 0.163 and η = 0.

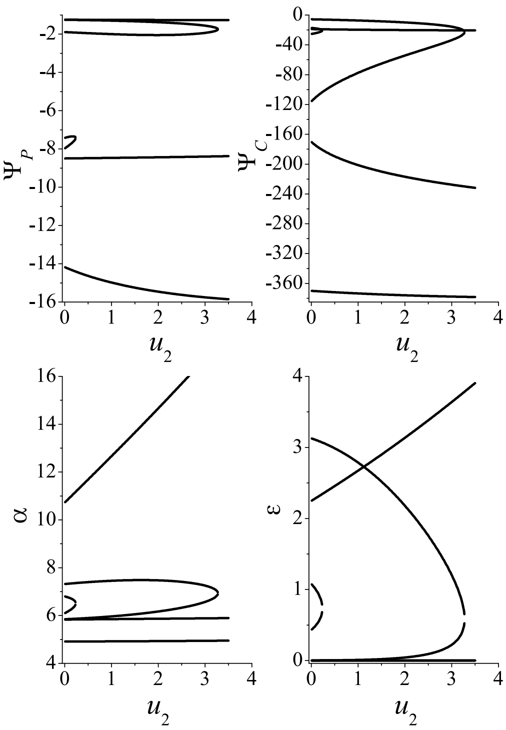

Fig. 4 Solutions Ψ_P, Ψ_C, α and ε of the system of equations (27), (28), (29) and (30) versus u_2 for μ = 1/1836, Θ = 100, σ = 0.01, τ = 0.1, u_3 = 0.004, β = 0.163 and η = 0.

divided into 3 branches. The high solution has 2 parts. The first branch exist for the values of u_3 below 0.0045. The second branch of the high solution starts when u_3 exceeds 0.0040 and extends towards the values of u_3 above 0.0045. Also the middle solution has 2 branches. One of them has almost constant values of Ψ_P, Ψ_C, α and ε in the entire interval of u_3 shown in the figure. The second branch of the middle solution can be found for the values of u_3 between 0.0040 and 0.0045. In this interval of u_3 the system of equations (27), (28), (29) and (30) has 7 solutions.

The dependence of the solutions Ψ_P, Ψ_C, α and ε of the system of equations (27), (28), (29) and (30) on u_2 is shown in figure 4. The other parameters are: μ = 1/1836, $\Theta = 100$, $\sigma = 0.01$, $\tau = 0.1$, $u_3 = 0.004$, $\beta = 0.163$ and $\eta = 0$. The drift velocity of the hot electrons u_2 is varied between 0.01 and 3.5. The low, the middle and the high solution can be found for all the values of u_2 shown in figure 4. For the values of u_2 below 3.27 three branches of the low solution can be found. The additional branches of the middle and of the high solution can be found only for u_2 below 0.23. For these values of u_2 the system of equations (27), (28), (29) and (30) has 7 solutions.

4. Discussion and conclusions

It is known that a necessary condition for the formation of a stable sheath in front of a negative electrode is that the ions reach the ion sound velocity at the sheath edge. For an extensive discussion of this subject see [12]. In order to fulfill this requirement the ions must be accelerated in the potential drop of the so called pre-sheath. In a bounded plasma system the pre-sheath potential drop is replaced by the potential drop between the plasma source (where the potential is zero) and the potential Ψ_P at the inflection point. In a bounded plasma system the Bohm condition is determined by 2 parameters: Ψ_P and α. If there are 2 species of negative particles with different temperatures present in the plasma, triple solutions for the Bohm condition can often be found. The potential drop that accelerates the ions to fulfill the Bohm condition can be determined by either of the negative particle species. Therefore 2 values for the Bohm condition may be expected and are physically reasonable. The third solution is usually an intermediate one and is just a mathematical result. The discussion which solution is correct for a given set of parameters is still not finished - see the discussions in [13] - [16]. In our recent paper [9] we have analyzed very similar bounded plasma system as in this work, only without drifts. Triple solutions of the model were found, as well as the splitting of the low solution into 3 parts. This splitting is in qualitative agreement with experimentally observed triple floating potentials of electron emitting electrodes [10, 11]. We have suggested that the correct solution is the one that results in a regular numerical solution of the Poisson equation of the system.

In this work the drifts have been introduced into the distribution functions of the hot and of the emitted electrons. Even when the drift velocities are relatively small when compared to the thermal velocities of the respective group of electrons, they have quite a strong effect on the potential and other plasma parameters. Furthermore 2 additional solutions of the model may appear, because the middle and the high solution can split in 2 branches. For some values of the plasma parameters up to 7 simultaneous solutions of the model are possible. Which solutions are physically possible and which Bohm condition is the correct one for a given set of plasma parameters remains the subject of future investigations. Another generalization of a one-dimensional kinetic model of a bounded plasma system presented in this work is the inclusion of a finite (non-zero) electric field at the source electrode. Also the analysis of the effects of this electric field remains the subject of future investigations.

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Appendix

The zero electric field conditions (28) and (29) in section 2 are written only in the symbolic form, with the integration over Ψ not actually carried out. For the convenience of the reader we give the list of the respective integrals in this Appendix:

$$\int_0^{\Psi_P} \exp\left(-\tfrac{\Psi}{\tau}\right) \operatorname{erfc}\left(\sqrt{-\tfrac{\Psi}{\tau}}\right) d\Psi = \\ = \tau\left(1 - \tfrac{2}{\sqrt{\pi}}\sqrt{-\tfrac{\Psi_P}{\tau}} - \exp\left(-\tfrac{\Psi_P}{\tau}\right)\operatorname{erfc}\sqrt{-\tfrac{\Psi_P}{\tau}}\right),$$

$$\int_{\Psi_P}^{\Psi_C} \exp\left(-\tfrac{\Psi}{\tau}\right) \operatorname{erfc}\left(\sqrt{-\tfrac{\Psi}{\tau}}\right) d\Psi = \\ = \tau\left[\begin{array}{l} \exp\left(-\tfrac{\Psi_P}{\tau}\right)\operatorname{erfc}\left(\sqrt{-\tfrac{\Psi_P}{\tau}}\right) - \\ -\exp\left(-\tfrac{\Psi_C}{\tau}\right)\operatorname{erfc}\left(\sqrt{-\tfrac{\Psi_C}{\tau}}\right) + \\ +\tfrac{2}{\sqrt{\pi}}\left(\sqrt{-\tfrac{\Psi_P}{\tau}} - \sqrt{-\tfrac{\Psi_C}{\tau}}\right) \end{array}\right],$$

$$\int_0^{\Psi_P} \exp(\Psi)\left[1 + \operatorname{erf}\left(\sqrt{\Psi - \Psi_C}\right)\right] d\Psi = \\ = \exp(\Psi_P)\left[1 + \operatorname{erf}\sqrt{\Psi_P - \Psi_C}\right] - \\ - \tfrac{2}{\sqrt{\pi}}\exp(\Psi_C)\left(\sqrt{\Psi_P - \Psi_C} - \sqrt{-\Psi_C}\right) - \\ - \left[1 + \operatorname{erf}\sqrt{-\Psi_C}\right],$$

$$\int_{\Psi_P}^{\Psi_C} \exp(\Psi)\left[1 + \operatorname{erf}\left(\sqrt{\Psi - \Psi_C}\right)\right] d\Psi = \\ = \exp(\Psi_C)\left(1 + \tfrac{2}{\sqrt{\pi}}\sqrt{\Psi_P - \Psi_C}\right) - \\ -\exp(\Psi_P)\left[1 + \operatorname{erf}\sqrt{\Psi_P - \Psi_C}\right],$$

$$\int_0^{\Psi_P} \exp\left(\tfrac{\Psi}{\Theta}\right)\left[1 + \operatorname{erf}\left(\tfrac{u_2 + \sqrt{\Psi - \Psi_C}}{\sqrt{\Theta}}\right)\right] d\Psi = \\ = \Theta\left[\begin{array}{l} \exp\left(\tfrac{\Psi_P}{\Theta}\right)\left(1 + \operatorname{erf}\left(\tfrac{u_2 + \sqrt{\Psi_P - \Psi_C}}{\sqrt{\Theta}}\right)\right) - \\ -\left(1 + \operatorname{erf}\left(\tfrac{u_2 + \sqrt{-\Psi_C}}{\sqrt{\Theta}}\right)\right) + \\ +\tfrac{\sqrt{\Theta}}{u_2\sqrt{\pi}}\exp\left(\tfrac{\Psi_C - u_2^2}{\Theta}\right) \times \\ \times \left(\begin{array}{l} \exp\left(\tfrac{-2u_2\sqrt{\Psi_P - \Psi_C}}{\Theta}\right) - \\ -\exp\left(\tfrac{-2u_2\sqrt{-\Psi_C}}{\Theta}\right) \end{array}\right) \end{array}\right],$$

$$\int_{\Psi_P}^{\Psi_C} \exp\left(\tfrac{\Psi}{\Theta}\right)\left[1 + \operatorname{erf}\left(\tfrac{u_2 + \sqrt{\Psi - \Psi_C}}{\sqrt{\Theta}}\right)\right] d\Psi = \\ = \Theta\left[\begin{array}{l} \exp\left(\tfrac{\Psi_C}{\Theta}\right)\left(1 + \operatorname{erf}\left(\tfrac{u_2}{\sqrt{\Theta}}\right)\right) + \\ +\tfrac{\sqrt{\Theta}}{u_2\sqrt{\pi}}\exp\left(-\tfrac{u_2^2}{\Theta}\right) \times \\ \times\left(1 - \exp\left(-\tfrac{2u_2\sqrt{\Psi_P - \Psi_C}}{\Theta}\right)\right) - \\ -\exp\left(\tfrac{\Psi_P}{\Theta}\right)\left(1 + \operatorname{erf}\left(\tfrac{u_2 + \sqrt{\Psi_P - \Psi_C}}{\sqrt{\Theta}}\right)\right) \end{array}\right],$$

$$\int_0^{\Psi_P} \exp\left(\tfrac{\Psi - \Psi_C}{\sigma}\right)\operatorname{erfc}\left(\tfrac{\sqrt{\Psi - \Psi_C} - u_3}{\sqrt{\sigma}}\right) d\Psi = \\ = \sigma\left[\begin{array}{l} \tfrac{\sqrt{\sigma}}{u_3\sqrt{\pi}}\exp\left(-\tfrac{u_3^2}{\sigma}\right) \times \\ \times\left(\begin{array}{l} \exp\left(\tfrac{2u_3\sqrt{\Psi_P - \Psi_C}}{\sigma}\right) - \\ -\exp\left(\tfrac{2u_3\sqrt{-\Psi_C}}{\sigma}\right) \end{array}\right) + \\ +\exp\left(\tfrac{\Psi_P - \Psi_C}{\sigma}\right)\operatorname{erfc}\left(\tfrac{\sqrt{\Psi_P - \Psi_C} - u_3}{\sqrt{\sigma}}\right) - \\ -\exp\left(-\tfrac{\Psi_C}{\sigma}\right)\operatorname{erfc}\left(\tfrac{\sqrt{-\Psi_C} - u_3}{\sqrt{\sigma}}\right) \end{array}\right],$$

$$\int_{\Psi_P}^{\Psi_C} \exp\left(\tfrac{\Psi - \Psi_C}{\sigma}\right)\operatorname{erfc}\left(\tfrac{\sqrt{\Psi - \Psi_C} - u_3}{\sqrt{\sigma}}\right) d\Psi = \\ = \sigma\left[\begin{array}{l} \tfrac{\sqrt{\sigma}}{u_3\sqrt{\pi}}\exp\left(-\tfrac{u_3^2}{\sigma}\right) \times \\ \times\left(1 - \exp\left(\tfrac{2u_3\sqrt{\Psi_P - \Psi_C}}{\sigma}\right)\right) + \\ +\operatorname{erfc}\left(-\tfrac{u_3}{\sqrt{\sigma}}\right) - \\ -\exp\left(\tfrac{\Psi_P - \Psi_C}{\sigma}\right)\left(\operatorname{erfc}\left(\tfrac{\sqrt{\Psi_P - \Psi_C} - u_3}{\sqrt{\sigma}}\right)\right) \end{array}\right].$$