J. Plasma Fusion Res. SERIES, Vol. 8 (2009)

Electrical Breakdown in Gases via a New Mechanism of Avalanche and Streamer Multiplications

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(Received: 4 August 2008 / Accepted: 2 November 2008)

The present streamer theory fails in explaining severe systematic deviations of electron avalanches from the Furry population statistics that is believed to be a governing statistical law. The present analysis tends towards formulating a modified theoretical concept supplementing the streamer theory by a new statistical scheme unifying population statistics of all avalanches regardless of their type, i.e. Townsend's, pre-streamer and streamer avalanches. The concept is based on the generalized photoionization mechanism leading to fractal side branching of avalanches. The fractal branching was developed into a mathematical form which resulted in a general statistical pattern capable to provide a unifying description of population statistics of electrical pre-breakdown and breakdown states of gases. The theory is supplemented by electrical and optical experiments performed within the last twenty years as well as by uv-experiments that have been accomplished only recently. The latter experiments represent an original contribution to the discussion of population statistics of electron avalanches and provide further evidence of soundness of the theoretical concept proposed.

Keywords: Electron avalanches, streamers, population statistics, Furry and Pareto statistics, fractal branching of avalanches, fractal dimension, generalized statistical pattern.

1. Introduction

Almost 70 years have passed since the time when Raether [1], Loeb [2] and Meek [3] formulated their new theory of streamer and spark breakdowns in gases. Since that time the breakdown mechanism has been explored by many researchers. Despite the research effort there are still many open questions and unclear points waiting for their solutions.

In the fifties and sixties of the last century Raether and his collaborators were studying extensively properties of electron avalanches as initiators of streamer discharges. Among the properties, which were analyzed, it was the population statistics that considerably attracted their attention. The reason is evident. The extent of electron population in avalanches is one of the main factors governing the cross-over of avalanches into streamers. Raether [1] found the average electron population $\bar{n} \approx 10^8$ as a favorable value for streamer development in many gases. This phenomenon is, however, highly stochastic and the population statistics is a convenient tool for a description of this stochasticity. Therefore, the density of probability $w(n)$ was measured with various gases and at various physical conditions. A nice summary of papers concerning this topic is given in Raether's book "Electron Avalanches and Breakdown in Gases" [1].

As far as the behavior of $w(n)$ is concerned, Raether's research group revealed an interesting phenomenon, which can be described as follows. Low populated avalanches ($\bar{n} < 10^5$) showed a perfect exponential behavior

$$w(n) = 1/\bar{n} \cdot \exp(-n/\bar{n}) , \quad (1)$$

$$\bar{n} = \exp[\int_0^x \alpha(x')\, dx'] \quad (2)$$

which is in fact an asymptotic form ($\bar{n} \to \infty$) of the Furry distribution [4,5]

$$w(n) = \frac{1}{\bar{n}}\left[1 - \frac{1}{\bar{n}}\right]^{n-1} \quad (3)$$

Here the Townsend first ionization coefficient α is used to express the average value $\bar{n}$ in Eq. (2). However, as soon as the population increases ($\bar{n} \geq 10^5$) the probability density $w(n)$ deviates more and more from the exponential behavior and shows a long pronounce bending tail as was shown by Frommhold [6]. When the population reaches typical streamer values ($\bar{n} \geq 10^8$) the bending is so large (see e.g. statistics measured by Richter [7]) that the function $w(n)$ does not resemble the exponential one at all. Nevertheless, at that time the Furry distribution (3) in its exponential form (1) was accepted as a fundamental distribution law of avalanche populations. The faith in its correctness was so strong that when experimental data of high populated avalanches ($\bar{n} > 10^5$) showed clear devi-

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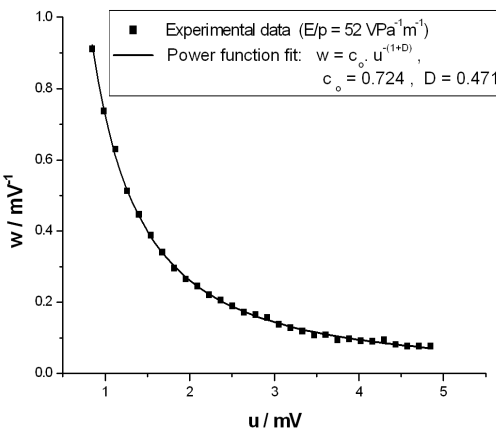

Fig. 1 Avalanche statistics in air at normal laboratory conditions [21]. Reproduced by the courtesy of IOP Publishing, UK.

ation from it [6–9], the scientific community – at its head with Professor Raether – tried to advocate the Furry distribution using the argument of "experimental artifact" (see Raether's book [1] on page 59). Raether believed that experimental devices used for acquisition of electron avalanches were not fast enough to recognize avalanches coming within very short time intervals and as a consequence such avalanches might have been integrated into a one big avalanche. He supposed that the integration resulted in an artificial increase of fraction of high populated avalanches and a fraction decrease of low populated avalanches which might cause a pronounce bending of the probability density function $w(n)$. Unfortunately, this speculation has not been verified until recently and for many years it has remained as an unsolved puzzle. Even today many researchers believe in general validity of the Furry distribution (3), and its exponential approximation (1) is still used in technical literature with high populated avalanches.

2. Analysis of Previous Experimental Data

For the last two decades our laboratory has been interested, among others, also in properties of electron avalanches [10-22] and paid a great attention especially to deviations from the Furry distribution [12], [14-19]. In order to prove thoroughly that the deviations have nothing to do with experimental artifacts, it was necessary to built up *an ultra-fast digitizer* for acquisition of avalanche electrical pulses that would be capable to distinguish and register all the pulses without the loss of any of them or without their integration into a single larger avalanche which was the main argument in favour of the experimental artifact. Indeed, such a digitizer was built up in our laboratory [14], [15] and thanks to its *new principle of digital acquisition* the ultra-fast capturing was accomplished. All the measurements performed with this unique device have shown [14-18] *clear systematic deviations from the Furry distribution* (3)/(1).

Fig. 1 shows one of our typical avalanche statistics measured with the ultra-fast digitizer. Avalanches were detected across a resistance* as short voltage pulses with random heights u. Since we did not calibrate the voltage pulses u against the number of electrons n, our resulted distribution curves w are dependent on u instead of n. From Fig. 1 it can be seen that a power function $w(u) = c_o \cdot u^{-(1+D)}$ represents an excellent fit of the measured data. Assuming linear proportionality $u \approx c \cdot n$, our curve $w(u)$ will preserve the same shape as $w(n) = c \cdot n^{-(1+D)}$, i.e. they both will possess the same value of the exponent $(1+D)$.

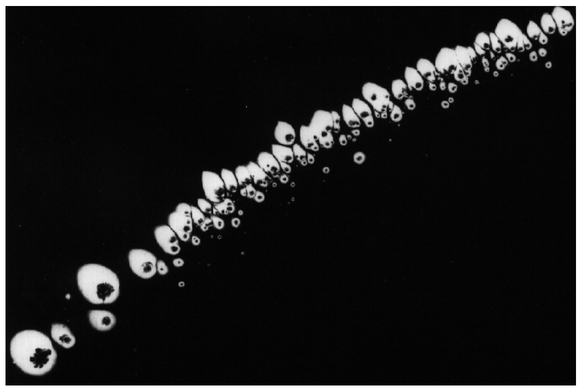

Fig. 2 Microdischarge spots on the dielectric barrier [21]. Reproduced by the courtesy of IOP Publishing, UK.

To verify more thoroughly our doubts about correctness of the Furry distribution of high populated avalanches, *another experimental procedure was tested* [19] which consists in the following. High populated avalanches, being in their streamer states, are leaving some light spots in places of their impact on the dielectric/insulation barrier (see Fig. 2). The area of each spot is proportional to the charge (population) transferred by the avalanche. The discharge spots can be *visualized* [20] by photographic technique, their areas measured and the corresponding *statistics created*. Naturally, *these statistics are equivalent to those of avalanche populations*. When the task was accomplished [19], the results again *clearly confirmed the systematic deviations from the Furry distribution (Fig. 3)*. The tests

* The resistance (R=100 kΩ) was connected in series to the discharge gap (C) so that both the components formed a classical RC-circuit – for more details see ref. [16].

of this kind were also of great relevance because they

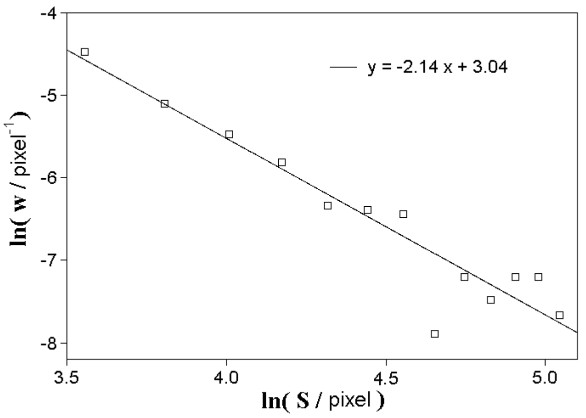

Fig. 3 Statistics of streamer spots in log-log co-ordinates [21]. Reproduced by the courtesy of IOP Publishing, UK.

they were established on a purely optical basis so that *no artifacts (drawbacks) consisting in imperfect electrical capturing may have been posed*.

3. Analysis of Recent Experimental Data

Another kind of experiments have been accomplished in our laboratory only recently. They have been based on photomultiplier registration of ultraviolet radiation emitted by electron avalanches. The reasons why we realized such measurements were the following.

Historically, there were three main experimental methods for studying avalanches:

(i) The electrical method relying on electrical circuits which detect and processed the pulsating signal caused by avalanches. The shape of a pulse bears the information on either the ion and electron components. This method in diverse arrangements is used very frequently also in recent time and has been used in our laboratory as well (Fig. 1)

(ii) The optical method using the Wilson chamber for visualization of ion tail of avalanches. There are many photographs of ion tails developed in Wilson chamber, which were published by Raether [1]. Nowadays this method is used only scarcely in this context. Nevertheless, we also used a certain kind of photographic visualization to study avalanches, namely microdischarge spots manifested on dielectric barriers (Figs. 2, 3).

(iii) The method employing photomultipliers for registration of ultraviolet radiation accompanying collisional ionization, i.e. only electron component is explored. This method has been used so far in many laboratories for various purposes but, unfortunately, it has not been available in experimental arsenal of our laboratory until recently. Since this method offers direct information on electron components, it has been important to verify behavior of population statistics also using this unique method. Therefore, the corresponding results can be reported in the present paper for the first time.

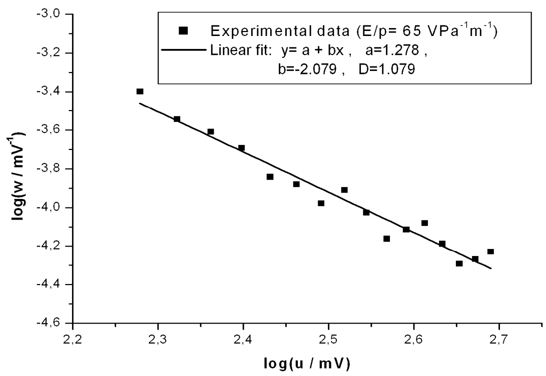

Fig. 4 Streamer statistics as a result of the photomultiplier registration.

The results of the analysis of ultraviolet pulses have unmistakably showed again the power function to be a convenient approximation for population statistics of high populated avalanches (see Fig. 4). So it has been *necessary to consider seriously the deviations from the Furry distribution as a real, natural fact and not as an artifact*. In addition, it has been desirable to find an analytical interpretation of the "deviated" distribution curves and in this way to determine a *new distribution law* applicable to high populated avalanches ($\bar{n} > 10^5$). All the measured statistics – both "electrical" [13 - 21] and "optical" [19,20] – obeyed very well the Pareto distribution

$$w(n) = const \cdot n^{-(D+1)} \tag{4}$$

where D is the so-called fractal dimension. Pareto statistics are inherently fractal statistics. All fractal objects are governed by those statistics. To understand better meaning and consequences of the Pareto statistics, it was inevitable *to study fractal geometry and try to be active e als o in its field* [22] - [27]. The problem proved to be quite interdisciplinary.

4. Fractal Model

On the basis of experimental observations and deductions [21] it is probable that the multiplication mechanism of high populated avalanches, whose populations follow Pareto's distribution, may be governed by a fractal scenario with the following properties (see Fig. 5).

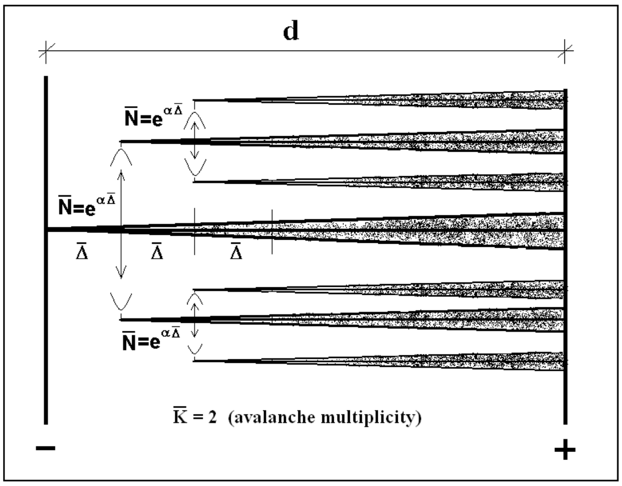

Fig. 5 Scheme of fractal avalanche multiplication [21]. Reproduced by the courtesy of IOP Publishing, UK.

(i) Besides a parent avalanche a series of additional smaller avalanches arise as a consequence of UV radiation inside the discharge gap. These smaller avalanches are generated in a hierarchical manner with different mean populations $\bar{n}_d$

$$\left\{\bar{n}_d\right\}_{j=0}^{J} = \left\{e^{\alpha(d-j\bar{\Delta})}\right\}_{j=0}^{J}, J \le \frac{d}{\Delta} - 1 \quad . \qquad (5)$$

In this way the number of less populated avalanches increases and, as a consequence, deviations from the exponential distribution may occur.

(ii) Multiplication of high populated avalanches with mean populations $\left\{\bar{n}_d\right\}_{j=0}^{J}$ must be generated according to a fractal scenario based on branching or partitioning like most fractals when going to smaller scales. Therefore, some type of fractal avalanche branching should be explored. The branching should originate with a parent avalanche possessing a mean population $n_{d,0} = e^{\alpha d}$ (fractal initiator). After having passed a certain distance $\bar{\Delta} > 0$ and gathered a certain number of energetic electrons $\bar{N} = e^{\alpha d} \ge 1$, which are capable of creating a group of UV photons, a photoionization process may start and a swarm of $\bar{K} \ge 1$ smaller avalanches with mean populations $\bar{n}_{d,1} = e^{\alpha(d-\bar{\Delta})}$ may appear beside the parent avalanche. Let us call them the "side avalanches of the first generation". The side avalanches of the first generation actually represent the so-called fractal generator given by the multiplicity $\bar{K}$. The side avalanches, once created, become parent avalanches for the next generation of new side avalanches. So, the side avalanches of the first generation become parent avalanches for the side avalanches of the second fractal generation with the mean population $\bar{n}_{d,2} = e^{\alpha(d-2\bar{\Delta})}$. This process of avalanche multiplication may or may not continue up to the last possible generation $J = d/\bar{\Delta} - 1$ with the mean population $\bar{n}_{d,J} = e^{\alpha(d-J\cdot\bar{\Delta})}$. Provided the multiplicative process reaches the *j*th generation, the mean (average) total number of side avalanches is just $\bar{K}^j$. The multiplicative process described yields a hierarchy of avalanches and when extended to infinity ($J \to \infty$), it yields an infinite set of avalanches that is similar to the well-known fractal object called the Cantor fractal set. By using the mentioned similarity a relation between avalanche characteristics and properties of the fractal set can be easily found

$$\bar{K}^j = \left(\frac{L_o}{l_j}\right)^D = \left(\frac{\bar{n}_{d,o}}{\bar{n}_{d,j}}\right)^D \Rightarrow$$
$$\Rightarrow \bar{K}^j = \left(\bar{N}^j\right)^D \Rightarrow D = \frac{\ln\bar{K}}{\ln\bar{N}} \quad . \qquad (6)$$

Since all fractal objects obey the Pareto statistics with the probability density in the form of the power law (4), the studied avalanche set, being of fractal nature, will also follow this statistical law

$$w(n) = c_o \cdot n^{-(1+D)} = c_o \cdot n^{-(1+\frac{\ln\bar{K}}{\ln\bar{N}})} . \qquad (7)$$

Such a strictly deterministic mechanism, which has already been described, might hardly be expected in a real situation. Instead, a strongly stochastic mechanism is more probable with certain distributions of detectable quantities Δ, K and N. However, using their *average* values $\overline{\Delta}$, $\overline{K}$ and $\overline{N}$ makes the treatment *more realistic* and advocates the deterministic view of the problem.

(iii) The described fractal mechanism of multiplication of high populated avalanches anticipates that the most probable place where a parent avalanche initiates side avalanches is in some of the first Δ-intervals because due to diffusion at a larger distance the parent avalanche is broadened enough to integrate the side avalanches.

Taking into account all the foregoing considerations and summarising the exponential probability density functions for all avalanche generations, one can obtain [21] the following general expression that contains both the exponential and Pareto distributions as special asymptotic cases

$$w(n) \approx \frac{G}{\overline{n}_d} \sum_{j=0}^{J} (\overline{K} \cdot \overline{N})^j \cdot \exp\left(- \frac{n \cdot \overline{N}^{\,j}}{\overline{n}_d} \right), \qquad (8)$$

where G is a normalization constant.

The statistical pattern (8) has been tested on a large set of experimental data in the course of the last two years and has proved to be a reliable fitting pattern that faithfully follows all the experimental results obtained in our laboratory. To illustrate this fact, two graphs (Fig. 6 and 7) have been chosen as typical representatives of the fitting procedures performed with high populated (pre-streamer) avalanches (Fig. 6) and streamers (Fig. 7). As is seen from these figures, the agreement between experiments and the model function (8) is very good.

Fig. 7 is a typical representative of ultraviolet experiments which have been performed in our laboratory only recently. They have not been published so far and represent an original contribution to the discussion of population statistics of electron avalanches since the statistics published by other authors have been based solely on electrical method (see point (i) in section 3). To our knowledge, this is a first presentation of population statistics of electron avalanches based on ultraviolet method (Figs. 4 and 7).

5. Localized and Delocalized Breakdown

Considering gas discharge breakdowns in uniform electric fields, usually two basic mechanisms are mentioned in discharge physics, namely, the Townsend delocalized breakdown and localized channel breakdown.

The Townsend delocalized breakdown is based on multiplication of electron avalanches that start from the cathode as a consequence of ion impact (secondary

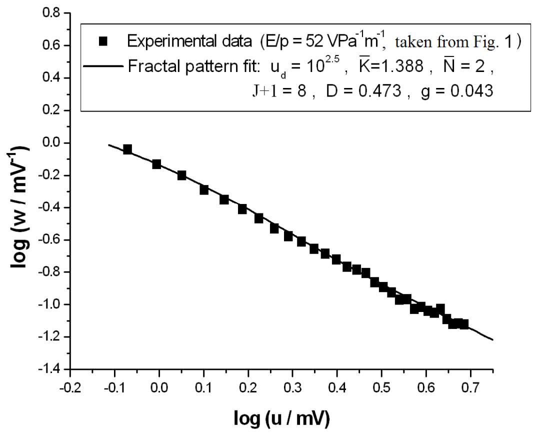

Fig. 6 Fractal pattern fit of pre-streamer statistics [21]. Reproduced by the courtesy of IOP Publishing, UK.

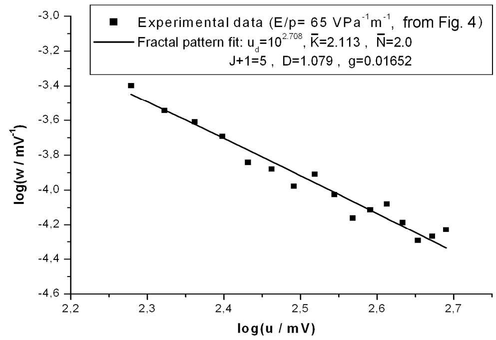

Fig. 7 Fractal pattern fit of streamer statistics.

electron emission from cathode) or due to ultraviolet radiation (photoemission from cathode). If conditions for self-sustained discharges are fulfilled, creation of electron avalanches is cyclically repeated until the entire discharge gap is filled up by plasma and a some type of discharge is established. So the essence of this delocalized mechanism is represented by the periodically increasing number of avalanches filling the discharge gap.

The second possible breakdown mechanism, the so-called streamer or spark breakdown, needs no more than one critical avalanche that is capable, under favorable conditions, to develop into a tiny plasma channel localized in a very narrow corridor of a negligible space as compared with the overall space of the discharge gap.

These two breakdown mechanisms are in fact two extreme cases – the firs one fully delocalized, the second one fully localized in space – and besides them no other types of breakdowns have been described. This seems to be rather anomalous when considering high stochasticity of both these breakdown mechanisms. The phenomenon discussed in this paper offers a certain intermediate type of breakdown that shares some features of both the localized and delocalized breakdowns. The present analysis of the fractal multiplication of avalanches and streamers has indicated that otherwise localized streamer channel breakdown may be accompanied by a swarm of side avalanches and if the multiplicity $\overline{K}$ (fractal avalanche generator) is high, this alternative of breakdown can even resemble a fully delocalized breakdown, which markedly reminds us of the final stage of the Townsend breakdown. The concept of fractal multiplication of avalanches and streamers has been derived from the statistical behavior of high populated avalanches and streamers and as such it is in agreement with experimental data.

6. Conclusion

In conclusion we would like to underline several main points:

A new concept of the fractal multiplication of high populated avalanches and streamers has been proposed. The concept is based on a generalized photoionisation mechanism leading to side branching of avalanches.

The proposed concept might be considered as a new supplement to the classical theory of avalanche and streamer multiplications.

Instead of the simple photoionisation that acts solely within the primary (parent) avalanche, the new concept of side branching allows for photoionisation going beyond the parent avalanche and creating side avalanches that accompany the parent avalanches with a certain delay (incubation time).

The branching may propagate to higher generations of side avalanches. This process is inherently stochastic and requires introducing the average multiplicity $\overline{K}$ and the average number $\overline{N}$ of initiating electrons to describe analytically the branching procedure.

The generalized probability density function (8) has been verified as a statistical distribution that faithfully describes the statistical behavior of avalanche and streamers regardless of their population stages.

The proposed theoretical concept has resulted from the previous electrical and optical data published by other authors and also by our laboratory. In the present paper this experimental data have been supplemented by new ones from missing ultraviolet experiments which bears unique information on electron avalanche components. In this way, the experimental evidence becomes complete since it contains results of all the three existing methods used for studying electron avalanches.

All the results lead us to the conclusion that the "deviated" population statistics of big electron avalanches are not artifacts, as has been suggested in the past [1], but rather a manifestation of a real physical process that is well described by the fractal side branching of avalanches.

Thanks are due to IOP Publishing Limited (UK) for the permission to reproduce Figs. 1 – 3, 5, 6 published in paper [21]. The present work was supported by the Grant Agency of the Czech Republic under the grant no. 202/07/1207.

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