Simulation of Electron Beam Acceleration by Electromagnetic Field in Static Inhomogeneous Magnetic Field

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An autoresonance relativistic motion of 0.1 mA electron beam in a combined steady-state inhomogeneous magnetic field and a stationary microwave field is studied. The space profile of the axially symmetric magnetic field is fitted to maintain the interaction of electrons with the microwave field of cyclotron resonance type on all their trajectories. The Newton-Lorentz equation is simulated simultaneously with the Poisson equation by using the particle-in-cell method with the Boris leap-frog procedure and the Fast Fourier transform respectively. It is shown that the beam electrons can be accelerated up to 0.23 MeV by 2.45 GHz microwave field of TE_{112} mode and 0.85 MeV by the 100 MHz TE_{111} standing mode. In either case the amplitude of the microwave electric component is taken equal to 6 kV/cm.

Keywords: spatial autoresonance, cyclotron resonance, electron beam, particle-in-cell method, fast Fourier transform, self-consistent field.

1. Introduction

It is well known that the autoresonance regime which consists in a constant maintenance of equality between the electromagnetic field frequency and the cyclotron frequency can be satisfied in various ways [1]. The autoresonance realized through an increase in time of a homogeneous magnetic field level with a rate to compensate the relativistic electron mass rise is one of the most extensively studied [2-9]. A cyclotron resonance acceleration of electrons by a traveling electromagnetic wave with adiabatically varying parameters in a magnetostatic guiding field is identified as a spatial autoresonance cyclotron acceleration (or SACA) [10,11] . In the numerical experiments on plasma dynamics in the minimum-B trap, the effect of high energy gained by electrons was attributed to a continuous self-sustenance of the cyclotron resonance conditions [12]. A possibility of the cyclotron resonance acceleration of electrons with pulsed microwaves was also demonstrated in the experiments on plasma heating in an adiabatic mirror magnetic trap [13]. Recently, we have analyzed the cyclotron resonance speeding of electrons in combined inhomogeneous steady-state magnetic and stationary microwave fields in the one particle approximation through numerical experiments [14]. This type of acceleration can be named spatial autoresonance acceleration, abbreviated to SARA. In this paper, a relativistic motion of 0.1 mA electron beam in the SARA conditions is subjected to a numerical study using the particle-in-cell method [15] and the Boris leap-frog procedure [16]. The self-consistent electrostatic fields for the electron beam are found through the numerical solution of the Poisson equation in a finite-difference form via fast Fourier transform [17].

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2. Simulation model

The geometry of the numerical experiment is shown in Fig.1. An electron beam is injected into the cylindrical cavity along its axis which is taken as a z-axis. The magnetic field is generated by three axisymmetric bobbins with a cavity. The acceleration of the beam electrons is studied for two microwave frequencies, i.e. 2.45 GHz and 100 MHz. In both cases, the microwave field amplitude was 6 kV/cm. The magnetic field and the cavity parameters are controlled by the microwave frequency values. For the case of 2.45 GHz, the cavity radius and the length are 4.54 cm and 20 cm while those for 100 MHz are 2.0 m and 1.33 m respectively. These radii are longer than the respective maximum electron Larmor radius.

The magnetic field in the point of the beam entrance into the cavity (see Fig.1) was taken equal to the classical cyclotron resonance value of 0.0875 T for the 2.45 GHz microwave frequency and 3.57×10^{-3} T for the 100 MHz field. The magnetic field dependence on the (r,z) coordinates is fitted in such a way as not to let the electrons leave the acceleration phase band on all their trajectories. A specific function $B(r,z)$ for 2.45 GHz case was deduced using the bobbins geometries and for the

100 MHz case was deduced from the equation $\nabla \cdot \vec{B} = 0$ in an approximation of a low transversal inhomogeneity.

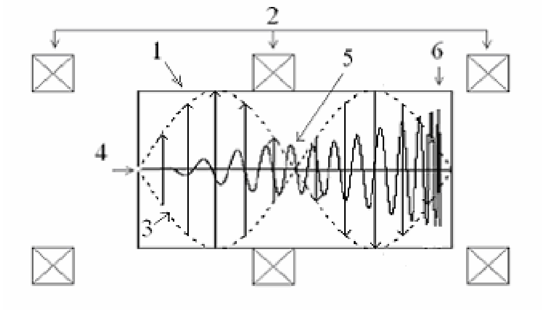

Fig.1. Schematic presentation of the SARA type accelerator: 1 – cylindrical cavity, 2 – d.c. current bobbins, 3 – microwave standing wave profile, 4 – entrance for electron beam, 5 – electron beam, 6-stop plane.

The components $B_r(r,z)$ and $B_z(r,z)$ were calculated on a 65x128 rectangular mesh. The azimuth angle θ was introduced indirectly through the (x,y) coordinates. The magnetic field in the particle position points was found through the bilinear interpolation method. The right hand polarized electric component of the microwave field can be expressed as:

$$\vec{E} = E_0(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})\sin(2\pi z/d), \quad (1)$$

which determines the local field in the particle points for each time step of the simulation procedure. The space charge field distribution was calculated beginning with the finite difference Poisson equation:

$$\frac{\partial^2\Phi(\vec{r}\,')}{\partial x'^2} + \frac{\partial^2\Phi(\vec{r}\,')}{\partial y'^2} + \frac{\partial^2\Phi(\vec{r}\,')}{\partial z'^2} = -\rho(\vec{r}\,')/\varepsilon_0 \quad , \quad (2)$$

where: $0 \le x' \le L_x$, $0 \le y' \le L_y$, $0 \le z' \le L_c$, $x' = x + R_c$, $y' = y + R_c$, $z' = z$ and $L_x = L_y = 2R_c$ (R_c and L_c are the radius and length of the cavity respectively); $\Phi^* = \Phi/(B_0 c r_{l0})$, $\rho^* = \rho r_{l0}/(\varepsilon_0 B_0 c)$ are the normalized electrostatic beam potential and charge density, respectively; $x'^* = x'/r_{l0}$, $y'^* = y'/r_{l0}$, $z'^* = z'/r_{l0}$ and $r_{l0} = c/\omega$ are coordinates and relativistic Larmor radius in the adimensional forms. The charge density $\rho(i\Delta x', j\Delta y', k\Delta z')$ in the mesh points was determined in accordance with the space particle distribution. To solve the equation (2), the normalized potential is given by the double Fourier series:

$$\Phi^*(x'^*, y'^*, z'^*) = \sum_{K_1,K_2=0}^{\infty} U_{K_1,K_2}(z'^*)\sin(K_1\pi x'^*/L_x^*) \quad (3)$$
$$\times \sin(K_2\pi y'^*/L_y^*)$$

and the function $\rho^*(x'^*, y'^*, z'^*)$ as:

$$\rho^*(x'^*, y'^*, z'^*) = \sum_{K_1,K_2=0}^{\infty} V_{K_1,K_2}(z'^*)\sin(K_1\pi x'^*/L_x^*) \quad (4)$$
$$\times \sin(K_2\pi y'^*/L_y^*),$$

where

$$V_{K_1,K_2}(z'^*) = \frac{4}{L_x^* L_y^*}\int_0^{L_x^*}\int_0^{L_y^*}\rho^* \sin(K_1\pi x'^*/L_x^*)$$
$$\times \sin(K_2\pi y'^*/L_y^*)dx'^* dy'^*, \quad (5)$$

where $L_x^* = L_x/r_L$ and $L_y^* = L_y/r_L$.

Normalizing the equation (2) and substituting the equations (3) and (4), we obtain:

$$\frac{d^2}{dz'^{*2}}U_{K_1,K_2}(z'^*) - \left(\frac{K_1^2}{L_x^{*2}} + \frac{K_2^2}{L_y^{*2}}\right)\pi^2 U_{K_1,K_2}(z'^*) \quad (6)$$
$$= V_{K_1,K_2}(z'^*),$$

where $K_1 = 0,1,2,\ldots,\infty$ and $K_2 = 0,1,2,\ldots,\infty$.

Now, we can discretize the problem in the x' and y'- directions by truncating our Fourier expansion by solving the Eq.(6) for $K_1 = 0,1,2,\cdots I$ and $K_2 = 0,1,2,\cdots J$. This truncating is equivalent to discretization in the x' and y'-directions on the equally-spaced mesh points $(x_i'^*, y_j'^*) = (iL_x^*/I, jL_y^*/J)$. The discretization in the z'-direction is fulfilled by dividing $L_z^* = L_c/r_L$ into equal segment lengths of $\delta z'^* = L_z^*/(N+1)$ and representing the operator d^2/dz'^{*2} in the second-order centred difference form. Thus, since $L_x^* = L_y^* = L^*$ for $k = 1,2,\ldots,N$, $K_1 = 0,1,2,\cdots I$ and $K_2 = 0,1,2,\cdots J$ we obtain:

$$U_{K_1,K_2,k-1} - \left[2 + (K_1^2 + K_2^2)G^2\right]U_{K_1,K_2,k} \quad (7)$$
$$+ U_{K_1,K_2,k+1} = V_{K_1,K_2,k}(\delta z'^*)^2.$$

Here $U_{K_1,K_2,k} \equiv U_{K_1,K_2}(z_k'^*)$, $V_{K_1,K_2,k} \equiv V_{K_1,K_2}(z_k'^*)$ and $G = \pi\delta z'^*/L^*$. The boundary conditions in z'-direction are reduced to:

$$U_{K_1,K_2,0} = U_{K_1,K_2,N+1} = 0 \quad (8)$$

for $K_1 = 0,1,2,\ldots,I$ and $K_2 = 0,1,2,\ldots,J$.

The equations (7) and (8) constitute a set of $(I+1)(J+1)$ uncoupled tri-diagonal matrix equations. They can be inverted to obtain the values of $U_{K_1,K_2,k}$. Finally, the $\Phi^*(x_i'^*, y_j'^*, z_k'^*)$ values can be reconstructed

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via inverse fast Fourier transform (see equation (3)).

The self-consistent electric field in the mesh points (i, j, k) is determined as

$$\left(g_{i,j,k}^{sn}\right)_x = \frac{\Phi^{*n}(i-1,j,k)-\Phi^{*n}(i+1,j,k)}{2\Delta x^*} \qquad (9)$$

The electron movement is described by the relativistic Newton Lorentz equation which in the finite difference form can be written as:

$$\frac{\vec{u}^{n+1/2}-\vec{u}^{n-1/2}}{\Delta\tau} = \vec{g}^n + \frac{\vec{u}^{n+1/2}+\vec{u}^{n-1/2}}{2\gamma^n}\times\vec{b}^n, \qquad (10)$$

where $\vec{u}=\gamma\vec{v}$, $\vec{g}^n=\vec{E}^n/B_0c$, $\vec{b}^n=\vec{B}^n/B_0$, $B_0=m_e\omega/e$ and $\gamma=(1+u^2)^{1/2}$ is the relativistic factor, $\tau=\omega t$, n is the index of the time step number, $\vec{g}^n$ is the vector of the total electric field consisting of the microwave $\vec{g}^{wn}$ and electrostatic $\vec{g}^{sn}$ fields, $\vec{g}^n=\vec{g}^{wn}+\vec{g}^{sn}$, $\vec{B}(r,z)$ is the steady-state magnetic field. The simulation time step $\Delta\tau$ was chosen equal to 1/250 of the microwave field period. Such a relatively short time step is specified because of the rapid changes in the position of the electrons. The equation for the electron motion is assumed as relativistic not only because of the fact that the electrons can achieve high energies under the spatial autoresonance conditions but also because of the exceptional sensitivity of the cyclotron resonance to the phase shift between the TE electric field and the particle velocity. For the (n+1) simulation step, the electron x-coordinate is calculated accordingly to the relation $x^{*n+1}=x^{*n}+u^{n+1/2}\Delta\tau/\gamma^{n+1/2}$, where $x^*=x/r_{l0}$. The other two components are found in a similar way.

The simultaneous numerical solution of the Newton-Lorentz and Poisson equations permits to find the beam trajectory, particle energy, longitudinal velocity and a phase difference between the vector of the particle velocity and the electric component of the microwave field. The simulations are stopped when the first injected electrons reach the stop plane.

3. Results and Discussions

A 0.1 mA electron beam of initial energy of 10 keV is injected through an orifice of 0.3 cm in diameter into the cavity along its axis. The injection point on the cavity axis is taken as the origin of the coordinates (z-axis coincides with the cavity axis). In the coordinate origin the magnetostatic field is equal to the classical cyclotron resonance value for the used 2.45 GHz microwaves of amplitude of 6 kV/cm. The space dependence of the magnetic field is fitted in such a way that it does not let the electrons leave the acceleration phase band [14].

The evolution of the average beam electron energy is shown in Fig. 2 (the solid line). One can see that in the range of 8÷12 cm the energy is near-constant because in this range the microwave field is insignificant. The maximum average energy of 230 keV is achieved in the plane z = 18.2 cm where the electron longitudinal motion stops due to the diamagnetic force action [14]. All the electrons injected into the cavity under the experimental conditions reach the stop plane. To maintain the microwave amplitude on the 6 kV/cm level, the microwave power is to be equal to 17 W. The difference in the beam electron average energy (Fig.2, the solid line) and that of the single electron (Fig.2, the circles) is due to two factors: the radial magnetic field inhomogeneity and the self consistent electric field of the beam. The outer beam electrons are in the magnetic field which is slightly different from the magnetic field inside the beam and are under the greater influence of the self-consistence electrostatic field.

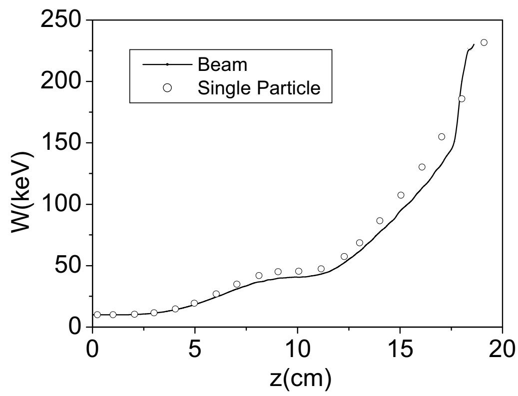

Fig.2. Average energy as a function of z – coordinate for the 2.45 GHz case.

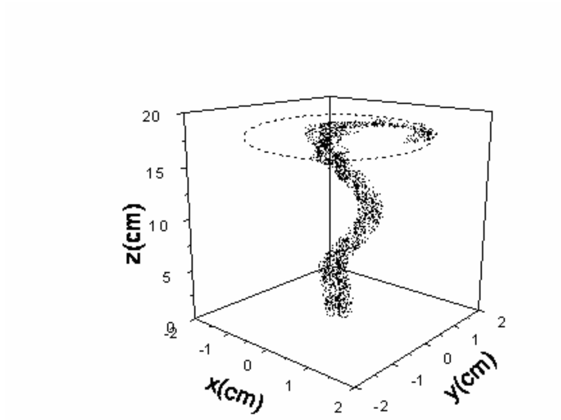

Fig.3. Electron beam instant picture after 11.5 microwave cycles and the electron trajectory in the stop plane (the dash circle).

Fig.3 shows the electron beam instant picture after 11.5 microwave cycles and the electrons trajectories in the stop-plane where the electrons form a ring of 1.6 cm in radius in the plane z = 18.2 cm (Fig.3, the dashed line)

Fig.4 presents an instant energy distribution of the beam electrons when the first beam electrons achieve the stop-plane (the case of Fig.3). The peak at 40 keV owes to the fact that the electron energy is held approximately constant in the range of 8÷12 cm. It must be emphasized that almost all electrons are accelerated up to 220 keV.

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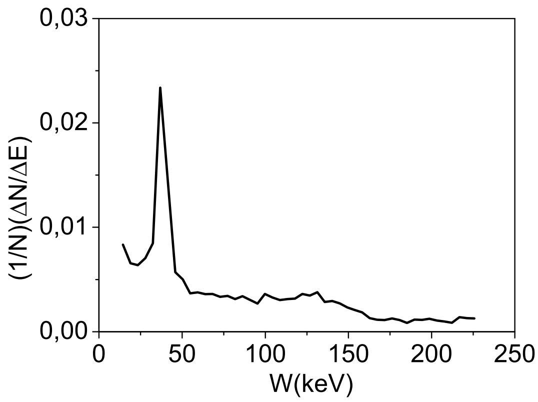

Fig.4. Energy distribution in the beam for the moment corresponding to Fig.3.

Fig.5 (the solid line) shows the dependence of the average energy of the beam electrons which moves forward in the 100 MHz TE_{111} cavity which is immersed into the magnetic field

$$\vec{B}(r,z) \cong -(1/2)\left[r dB_z(z)/dz\right]\hat{r} + B_z(z)\hat{z} \qquad (11)$$

where $B_z(z) = B_0[\gamma_0 + \alpha z]$, $\alpha = \gamma_0(R_m - 1)/L_c$, $R_m = 1.3$ and γ_0 is the relativistic factor corresponding to the initial energy. The initial beam energy is chosen 20 keV and the microwave amplitude is taken equal to that of the 2.45 GHz, which resulted in the maximum average energy of 0.85 MeV in the z = 66 cm stop plane. Such high value in comparison with the 2.45 GHz case can be accounted for the fact that for the electrons in their motion up to the stop plane are exposed to the autoresonance acceleration for a longer time. The difference with the single electron energy evolution which is observed in the range z > 55 cm can be attributed to the self-consistent electric field influence. The 100 MHz microwave power absorbed by the beam is 86 W.

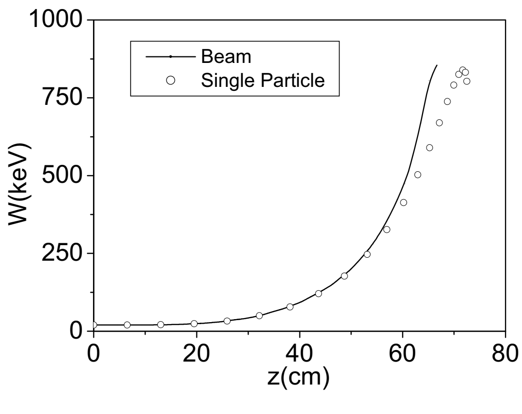

Fig.5. Average beam energy as a function of z – coordinate for the 100 MHz case.

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CONCLUSIONS

We have shown our preliminary results on the autoresonance acceleration of an electron beam by a combined field comprised of a space inhomogeneous magnetostatic field, a standing microwave field and a beam self-consistent electric field at the cyclotron resonance conditions. It is demonstrated that the electrons of a 0.1 mA beam can be accelerated in the SARA mode of 2.45 GHz up to 0.22 MeV and up to 0.85 MeV by the 100 MHz microwaves at a distance smaller than 1 m. For further study of the SARA acceleration mechanism, we will continue the simulations of higher density beams to look for the ways to achieve energies of order of the tens Mega-electron-Volts range.

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