J. Plasma Fusion Res. SERIES, Vol. 8 (2009)

Acceleration of Relativistic Electron Beam Trapped in Extraordinary Beat Wave

Reiji Sugaya and Tsunehiro Maehara

Department of Physics, Faculty of Science, Ehime University, 2-5 Bunkyo-cho, Matsuyama 790-8577, Japan

(Received: 2 September 2008 / Accepted: 23 December 2008)

Relativistic electron beam acceleration due to the extraordinary beat wave induced by nonlinear electron Landau and cyclotron damping of electromagnetic waves is investigated theoretically on the basis of the relativistic equations of motion for beam electrons trapped in the beat wave. The equations of motion in the moving frame of reference with the velocity of the electron beam were analyzed analytically and numerically, where the relationship between the moving and laboratory frames is given by the Lorentz transformation. The spatial and temporal evolutions of the energy and momentum of the trapped beam electrons in the moving and laboratory frames were studied numerically.

Keywords: relativistic electron beam acceleration, extraordinary beat wave, nonlinear Landau and cyclotron damping, equations of motion, trapped beam electrons, moving and laboratory frames of reference, Lorentz transformation

1. Introduction

Relativistic electron beam acceleration due to the extraordinary beat wave induced by nonlinear electron Landau and cyclotron damping of the electromagnetic waves in a magnetized plasma is investigated theoretically and numerically on the basis of the relativistic equations of motion for beam electrons trapped in the beat wave [1-6]. In order to investigate the highly relativistic electron beam, the equations of motion in the moving frame with the velocity of the electron beam $\mathbf{v}_b = (0,0,v_b)$ were analyzed numerically, where the relationship between the moving and laboratory frames is given by the Lorentz transformation [4,6,7]. The beat waves are excited via nonlinear electron Landau and cyclotron damping of the two electromagnetic waves and trap the beam electrons, satisfying the resonance condition in the moving frame, $\tilde{\omega}_{\tilde{\mathbf{k}}''} - \tilde{k}''_{\parallel} v'_{\parallel} = m\omega'_{ce}$, where $\omega'_{ce} = eB_0/\gamma' m_e c$ is the relativistic electron cyclotron frequency for beam electrons in the moving frame, γ' is the Lorentz factor in the moving frame, and $\tilde{\omega}_{\tilde{\mathbf{k}}''} = \tilde{\omega}_{\tilde{\mathbf{k}}} - \tilde{\omega}_{\tilde{\mathbf{k}}'}$ and $\tilde{\mathbf{k}}'' = \tilde{\mathbf{k}} - \tilde{\mathbf{k}}' = (\tilde{k}''_{\perp}, 0, \tilde{k}''_{\parallel})$ are the wave frequency and wave vector of the beat wave in the moving frame, respectively. It is proved that the acceleration rate in the laboratory frame increases approximately in proportion to β, where $\beta = (1 - v_b^2/c^2)^{-1/2}$ and c is the light speed.

The detailed acceleration mechanism for the cases of $m = 0$ and 1 was clarified by the numerical analysis of the spatial and temporal evolutions of the energy and momentum of the trapped beam electrons in the moving and laboratory frames.

2. Basic Equations

2.1 Lorentz transformation

The Lorentz transformation of the laboratory frame of reference (x, y, z, t) to the moving frame of reference (x', y', z', t') is expressed by

$x' = x,\ y' = y,\ z' = \beta(z - v_b t),\quad t' = \beta(t - v_b z/c^2)$,

$\tilde{\omega}_{\tilde{\mathbf{k}}} = \beta(\omega_{\mathbf{k}} - k_{\parallel} v_b),\ \tilde{k}_{\perp} = k_{\perp}$, and $\tilde{k}_{\parallel} = \beta(k_{\parallel} - \omega_{\mathbf{k}} v_b/c^2)$,

where $\omega_{\mathbf{k}}$ and $\mathbf{k} = (k_{\perp}, 0, k_{\parallel})$ are the wave frequency and wave vector in the laboratory frame, respectively, and $\tilde{\omega}_{\tilde{\mathbf{k}}}$ and $\tilde{\mathbf{k}} = (\tilde{k}_{\perp}, 0, \tilde{k}_{\parallel})$ are the wave frequency and wave vector in the moving frame, respectively. The electric and magnetic fields of the beat wave in the moving frame are also provided by means of the Lorentz transformation and are represented as follows [4,6,7]:

$$\tilde{\mathbf{E}}^{(2)}_{\tilde{\mathbf{k}}''} = \mathbf{E}^{(2)}_{\parallel\mathbf{k}''} + \beta\left(1 - \frac{k''_{\parallel} v_b}{\omega_{\mathbf{k}''}}\right)\mathbf{E}^{(2)}_{\perp\mathbf{k}''} + \frac{\beta v_b}{\omega_{\mathbf{k}''}}\mathbf{k}''_{\perp} E^{(2)}_{\parallel\mathbf{k}''},$$

$$\tilde{\mathbf{B}}^{(2)}_{\tilde{\mathbf{k}}''} = \mathbf{B}^{(2)}_{\parallel\mathbf{k}''} + \beta\mathbf{B}^{(2)}_{\perp\mathbf{k}''} - \frac{\beta}{c}\mathbf{v}_b \times \mathbf{E}^{(2)}_{\mathbf{k}''}. \qquad (1)$$

Here, $\mathbf{E}^{(2)}_{\mathbf{k}''}$ and $\mathbf{B}^{(2)}_{\mathbf{k}''}$ refer to the laboratory frame, and $\tilde{\mathbf{E}}^{(2)}_{\tilde{\mathbf{k}}''}$ and $\tilde{\mathbf{B}}^{(2)}_{\tilde{\mathbf{k}}''}$ refer to the moving frame. It is found from the above equations that the magnitude of the perpendicular components of the electric and magnetic fields in the moving frame increases about β times that in the laboratory frame.

2.2 Equations of motion for the beam electrons

The beam electrons trapped in the beat wave are governed by the following equation of motion in the moving frame [4,6,7]:

$$\frac{d\mathbf{p}'}{dt'} = -e\tilde{\mathbf{E}}^{(2)}_{\tilde{\mathbf{k}}''} - \frac{e}{\gamma' m_e c}\mathbf{p}' \times \left(\mathbf{B}_0 + \tilde{\mathbf{B}}^{(2)}_{\tilde{\mathbf{k}}''}\right), \qquad (2)$$

author's e-mail: nt5r-sgy@asahi-net.or.jp

where $d\mathbf{p}'_0/dt' = -\left(e/\gamma' m_e c\right)\mathbf{p}'_0 \times \mathbf{B}_0$, $\mathbf{p}' = \mathbf{p}'_0 + \mathbf{p}'^{(2)}$, $\gamma' = \left(1 + p'^2/m_e^2c^2\right)^{1/2} = \left(1 - v'^2/c^2\right)^{-1/2}$, and $\mathbf{B}_0 = \left(0, 0, B_0\right)$. Thus, Eq. (2) becomes

$$\frac{d\mathbf{p}'^{(2)}}{dt'} = -e\tilde{\mathbf{E}}^{(2)}_{\tilde{\mathbf{k}}''} - \frac{e}{\gamma' m_e c}\mathbf{p}'_0 \times \tilde{\mathbf{B}}^{(2)}_{\tilde{\mathbf{k}}''} - \frac{e}{\gamma' m_e c}\mathbf{p}'^{(2)} \times \mathbf{B}_0 , \qquad (3)$$

where the higher order term $-\left(e/\gamma' m_e c\right)\mathbf{p}'^{(2)} \times \tilde{\mathbf{B}}^{(2)}_{\tilde{\mathbf{k}}''}$ is neglected. As was proved previously, this equation shows that the trapped beam electrons are accelerated and decelerated by the parallel electric field of the beat wave for the case of $m = 0$ (nonlinear electron Landau damping due to the first term of Eq. (3)) and by the Lorentz force arising from the perpendicular magnetic field of the beat wave for the case of $m = 1$ (nonlinear electron cyclotron damping due to the second term of Eq. (3)) [4,6].

3. Numerical Analysis

We performed the numerical analysis of Eq. (3) for the cases of $m = 0$ and 1, and the spatial and temporal evolutions of the energy and momentum of the trapped beam electrons in the moving and laboratory frames were studied. The spatial and temporal evolutions for $m = 0$ are calculated by retaining the first and third terms in the right hand of Eq. (3) and shown in Figs. 1-4. Those for $m = 1$ are calculated by retaining the second and third terms in the right hand of Eq. (3) and shown in Figs. 5-8. The spatial evolutions along the magnetic field ($z'-$axis) are obtained from the relation of $d\mathbf{p}'^{(2)}/dz'_2 = \left(\gamma' m_e / p'^{(2)}_{z'}\right) d\mathbf{p}'^{(2)}/dt'$. Here, $\beta = 1000$, $v'_\parallel / c = 0.1$, and $\left|\tilde{k}''_\parallel c\right| / \omega_{ce0} = 0.1$. For the six curves in each figure, the value of $\left|e\tilde{E}_{\tilde{\mathbf{k}}''} / \tilde{k}''_\parallel m_e c^2\right|$ is given as a parameter. For simplicity, it is assumed that $\tilde{E}_{\tilde{\mathbf{k}}''} = \tilde{E}^{(2)i}_{\tilde{\mathbf{k}}''} = \tilde{B}^{(2)i}_{\tilde{\mathbf{k}}''}$, $i = x', y', z'$. The absolute values of the energy and momentum of the trapped beam electrons increase with $\left|e\tilde{E}_{\tilde{\mathbf{k}}''} / \tilde{k}''_\parallel m_e c^2\right|$ in each figure. The trapping frequency for the beam electrons in the moving frame is defined such that $\omega'_B = \left|e\tilde{E}_{\tilde{\mathbf{k}}''}\tilde{k}''_\parallel / m_e\right|^{1/2}$.

Figures 1 and 2 show the spatial and temporal evolutions in the moving frame, where $m = 0$, $g^* = \gamma'$, $\mathrm{P}^*_x = p'^{(2)}_{x'} / m_e c$, $\mathrm{P}^*_y = p'^{(2)}_{y'} / m_e c$, $\mathrm{P}^*_z = p'^{(2)}_{z'} / m_e c$, $z'_2 = z' - v'_\parallel t'$, $z^* / \mathrm{a}^* = z'_2 / \mathrm{a}^*$, $\left|\tilde{k}''_\parallel\right| \mathrm{a}^* = 0.05$, $\mathrm{T}^* = \left|\tilde{k}''_\parallel\right| ct' / \tau'_{\max}$, $\left|e\tilde{E}_{\tilde{\mathbf{k}}''} / \tilde{k}''_\parallel m_e c^2\right|$ =400, 800, 1200, 1600, 2000, 2400, $\tau'_{\max} = \left|\tilde{k}''_\parallel\right| ct'_{\max} = 0.213$, 0.188, 0.178, 0.173, 0.17, 0.168, and $\omega'_B / \omega_{ce0} = 2$, 2.83, 3.46, 4, 4.47, 4.9 ($\omega_{ce0} = eB_0 / m_e c$). Here, $\tau'_{\max}$ decreases and ω'_B / ω_{ce0} increases with $\left|e\tilde{E}_{\tilde{\mathbf{k}}''} / \tilde{k}''_\parallel m_e c^2\right|$. The initial values at $t' = 0$ are $\left|\tilde{k}''_\parallel\right| z'_2 = -\left|\tilde{k}''_\parallel\right| \mathrm{a}^*$, $\mathrm{P}^*_x = \mathrm{P}^*_y = 0$ and $\mathrm{P}^*_z = 0.1$. The effective trapping frequency in the moving frame should be given by $\omega'_T = \pi / t'_{\max} = \pi \left|\tilde{k}''_\parallel\right| c / \tau'_{\max}$, and

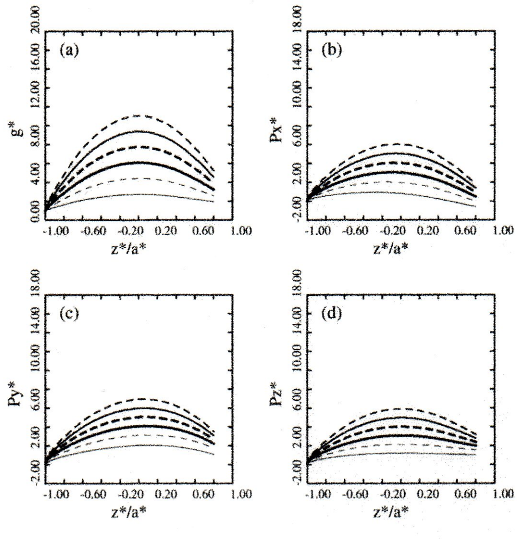

Fig.1 Spatial evolutions of the energy and momentum of the trapped beam electrons in the moving frame for $m = 0$ are shown.

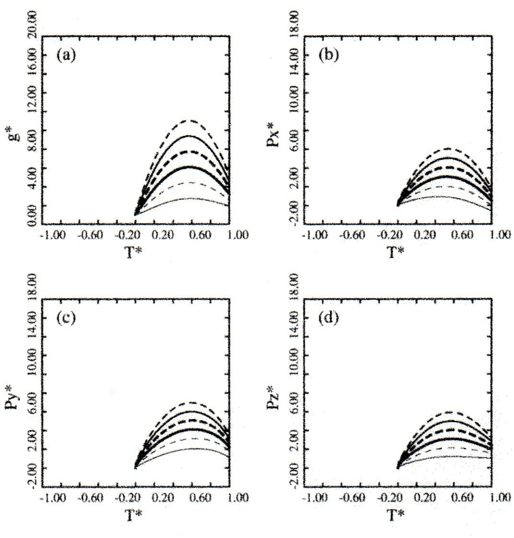

Fig.2 Temporal evolutions of the energy and momentum of the trapped beam electrons in the moving frame for $m = 0$ are shown.

$\omega'_T / \omega_{ce0} = 1.47$~1.87. This value is rather smaller than ω'_B / ω_{ce0}. Figures 3 and 4 show the spatial and temporal evolutions in the laboratory frame, where $g / b = \gamma / \beta$, $\mathrm{P}_x = p^{(2)}_x / m_e c$, $\mathrm{P}_y = p^{(2)}_y / m_e c$, $\mathrm{P}_z / b = p^{(2)}_z / \beta m_e c$, $\gamma = \left(1 + p^2 / m_e^2 c^2\right)^{1/2} = \left(1 - v^2/c^2\right)^{-1/2}$. The quantities in the laboratory frame are expressed as $\gamma = \beta\left(\gamma' + v_b p'_{z'} / m_e c^2\right) \simeq \beta\gamma'$, $p^{(2)}_x = p'^{(2)}_{x'}$, $p^{(2)}_y = p'^{(2)}_{y'}$, $p^{(2)}_z = \beta p'^{(2)}_{z'}$, $z_2 = z - v_b t = \left(z'_2 + t' p'_{z'} / \gamma' m_e\right) / \beta \simeq z'_2 / \beta$, $t = \beta\left[\left(1 + p'_{z'}\left(1 - \beta^{-2}\right)^{1/2} / \gamma' m_e c\right) t' + \left(1 - \beta^{-2}\right)^{1/2} z'_2 / c\right]$ $\simeq \beta t'$, $\omega_B \simeq \omega'_B / \beta$, $\omega_T \simeq \omega'_T / \beta$, $m = 0$, $z / \mathrm{a} = z_2 / \mathrm{a}$,

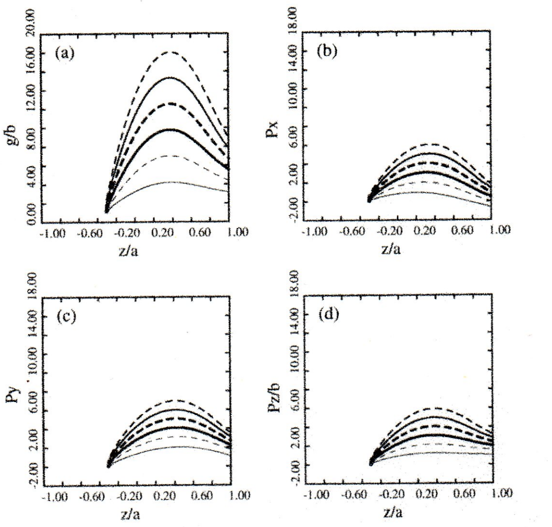

Fig.3 Spatial evolutions of the energy and momentum of the trapped beam electrons in the laboratory frame for $m = 0$ are shown.

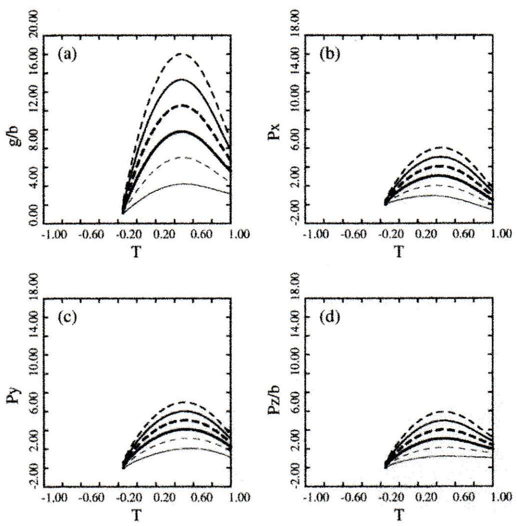

Fig.4 Temporal evolutions of the energy and momentum of the trapped beam electrons in the laboratory frame for $m = 0$ are shown.

$\left|\tilde{k}''_{\parallel}\right| \mathrm{a} = \xi_{\max}$, $\mathrm{T} = \left|\tilde{k}''_{\parallel}\right| ct / \tau_{\max}$, $\xi_{\max} = 1.732\times10^{-4}$, 1.703×10^{-4} , 1.665×10^{-4} , 1.634×10^{-4} , 1.61×10^{-4} , 1.59×10^{-4} , and $\tau_{\max} = \left|\tilde{k}''_{\parallel}\right| ct_{\max} = 0.386\times10^{3}$, 0.358×10^{3} 0.345×10^{3} , 0.336×10^{3} , 0.331×10^{3} , 0.327×10^{3} . Here, $\xi_{\max}$ and $\tau_{\max}$ correspond to the acceleration length and the acceleration time in the laboratory frame, respectively, and decrease with $\left|e\tilde{E}_{\tilde{\mathbf{k}}'} / \tilde{k}''_{\parallel} m_e c^2\right|$. It is noted that the obtained results depend hardly on the value of β except for the values of $\xi_{\max}$ $(\propto 1/\beta)$ and $\tau_{\max}$ $(\propto \beta)$. For $m = 0$, it is found that the energy and momentum of the trapped beam

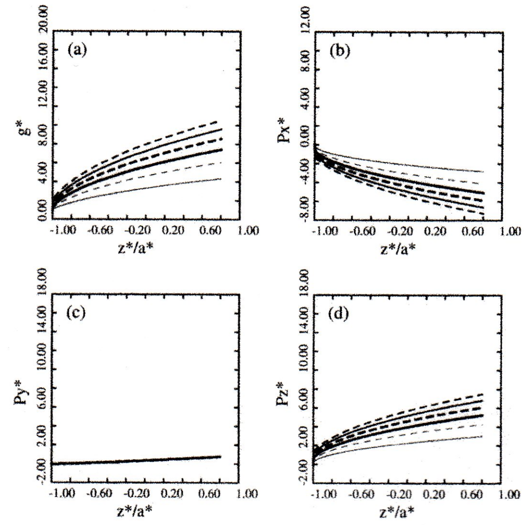

Fig.5 Spatial evolutions of the energy and momentum of the trapped beam electrons in the moving frame for $m = 1$ are shown.

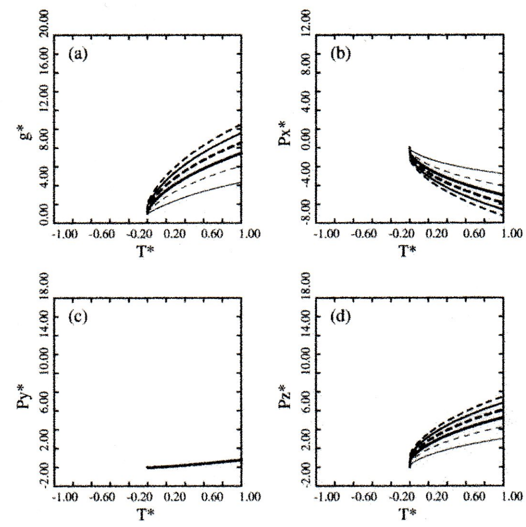

Fig.6 Temporal evolutions of the energy and momentum of the trapped beam electrons in the moving frame for $m = 1$ are shown.

electrons increase and decrease during the half period of one bounce. The increase and decrease of γ', γ , $p'^{(2)}_{z'}$ and $p^{(2)}_{z}$ mean the acceleration and deceleration of the relativistic electron beam. The increase and decrease of $p'^{(2)}_{x'}$, $p'^{(2)}_{y'}$, $p^{(2)}_{x}$ and $p^{(2)}_{y}$ mean the variation of the perpendicular energy and momentum of the relativistic electron beam.

Figures 5 and 6 show the spatial and temporal evolutions in the moving frame, where $m = 1$, $\left|\tilde{k}''_{\parallel}\right| \mathrm{a}^{*} = 0.05$, $\left|e\tilde{E}_{\tilde{\mathbf{k}}'} / \tilde{k}''_{\parallel} m_e c^2\right|$ =1000, 2000, 3000, 4000, 5000, 6000, $\tau'_{\max} = 0.14$, 0.132, 0.129, 0.128, 0.127,

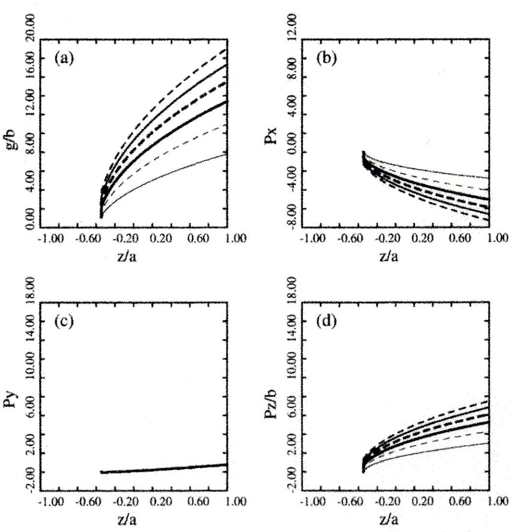

Fig.7 Spatial evolutions of the energy and momentum of the trapped beam electrons in the laboratory frame for $m=1$ are shown.

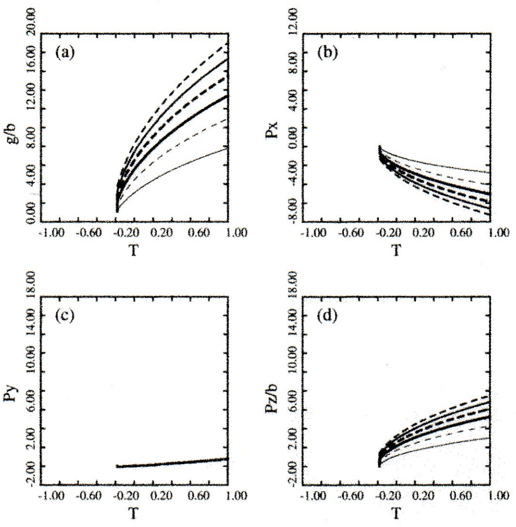

Fig.8 Temporal evolutions of the energy and momentum of the trapped beam electrons in the laboratory frame for $m=1$ are shown.

0.127, and $\omega_B'/\omega_{ce0}=3.16$, 4.47, 5.48, 6.32, 7.07, 7.75. Similarly, $\tau'_{\max}$ decreases with $\left|e\tilde{E}_{\tilde{\mathbf{k}}''}/\tilde{k}''_{\parallel}m_e c^2\right|$. The initial values at $t'=0$ are $\left|\tilde{k}''_{\parallel}\right|z'_2=-\left|\tilde{k}''_{\parallel}\right|\mathrm{a}^*$ and $\mathrm{P}_x^*=\mathrm{P}_y^*=\mathrm{P}_z^*=0.1$. The effective trapping frequency in the moving frame becomes $\omega_T'/\omega_{ce0}=2.24$~2.47, which is also smaller than ω_B'/ω_{ce0}. Figures 7 and 8 show the spatial and temporal evolutions in the laboratory frame, where $m=1$, $\xi_{\max}=1.509\times10^{-4}$, 1.461×10^{-4}, 1.444×10^{-4}, 1.435×10^{-4}, 1.43×10^{-4}, 1.427×10^{-4} and $\tau_{\max}=0.291\times10^3$, 0.278×10^3, 0.274×10^3, 0.272×10^3, 0.27×10^3, 0.27×10^3. Here, $\xi_{\max}$ and $\tau_{\max}$ decrease also with $\left|e\tilde{E}_{\tilde{\mathbf{k}}''}/\tilde{k}''_{\parallel}m_e c^2\right|$. It is also noted that the obtained results depend hardly on β except for the values of $\xi_{\max}$ $(\propto 1/\beta)$ and $\tau_{\max}$ $(\propto \beta)$. For $m=1$, it is found that the energy and momentum of the trapped beam electrons increase monotonically during the half period of one bounce. The monotonic increase of γ, γ', $p'^{(2)}_{z'}$ and $p^{(2)}_z$ means the acceleration of the relativistic electron beam. The increase of the absolute values of $p'^{(2)}_{x'}$, $p'^{(2)}_{y'}$, $p^{(2)}_x$ and $p^{(2)}_y$ means the increase of the perpendicular energy and momentum of the relativistic electron beam.

We consider the actual system where the relativistic electron beam is injected axially into the magnetized plasma and the intense laser beam is launched. The acceleration quantities in the laboratory frame are deduced on the basis of the numerical results for the condition of $B_0=10\,\mathrm{T}$, $h\equiv\left|e\tilde{E}_{\tilde{\mathbf{k}}''}/\tilde{k}''_{\parallel}m_e c^2\right|=400$, $\beta=1000$ and $\beta m_e c^2=510\,\mathrm{MeV}$. It is assumed that the change of the initial phase $\left|\tilde{k}''_{\parallel}\right|z'_2=-\left|\tilde{k}''_{\parallel}\right|\mathrm{a}^*=-0.05\rightarrow-0.05\alpha$ leads to the changes of $h\rightarrow h/\alpha$, $\tau_{\max}\rightarrow\alpha\tau_{\max}$ and $\xi_{\max}\rightarrow\alpha\xi_{\max}$. For $m=0$, $\tau_{\max}\simeq400$ and $\xi_{\max}\simeq1.7\times10^{-4}$ are obtained. Under the condition of $\alpha=20$ and $\left|\tilde{k}''_{\parallel}c\right|/\omega_{ce0}=10^{-3}$, the acceleration time $t_a=\alpha\tau_{\max}/\left|\tilde{k}''_{\parallel}\right|c$ ($\mathrm{T}=\left|\tilde{k}''_{\parallel}\right|ct_a/\tau_{\max}$ =1), acceleration length $z_a=\alpha\xi_{\max}/\left|\tilde{k}''_{\parallel}\right|$ ($z/\mathrm{a}=z_2/\mathrm{a}=$ $\left|\tilde{k}''_{\parallel}\right|z_a/\xi_{\max}$ =1), the energy gain $W_a\equiv\Delta\left(\beta m_e c^2\right)$, the acceleration gradient $g_a\equiv\Delta\left(\beta m_e c^2\right)/z_a$, and the electric field of the laser $\left|E_{\mathbf{k}}\right|$ are estimated: $t_a=4.5\,\mu\mathrm{s}$, $z_a=5.7\times10^{-4}\,\mathrm{m}$, $W_a\simeq3\beta m_e c^2=1.5\,\mathrm{GeV}$, $g_a\simeq2.7\,\mathrm{TeV/m}$, and $\left|E_{\mathbf{k}}\right|=6.1\times10^9\,\mathrm{V/m}$. Here, the magnitude of the electric field of the beat wave is assumed to be $\left|\tilde{E}_{\tilde{\mathbf{k}}''}\right|=h\left|\tilde{k}''_{\parallel}m_e c^2/e\alpha\right|=10^{-2}\left|E_{\mathbf{k}}\right|$. For $m=1$, $\tau_{\max}\simeq320$ and $\xi_{\max}\simeq1.6\times10^{-4}$ are obtained. Under the condition of $\alpha=20$ and $\left|\tilde{k}''_{\parallel}c\right|/\omega_{ce0}=1$, the estimated values of $t_a=3.6\,\mathrm{ns}$, $z_a=5.4\times10^{-7}\,\mathrm{m}$, $W_a\simeq3\beta m_e c^2=1.5\,\mathrm{GeV}$, $g_a\simeq2.8\,\mathrm{PeV/m}$, and $\left|E_{\mathbf{k}}\right|=6\times10^9\,\mathrm{V/m}$ are obtained. Here, the magnitude of the electric field of the beat wave is assumed to be $\left|\tilde{E}_{\tilde{\mathbf{k}}''}\right|=h\left|\tilde{k}''_{\parallel}m_e c^2/e\alpha\right|=10^{-2}\left|E_{\mathbf{k}}\right|\beta$, ($\left|\tilde{E}_{\tilde{\mathbf{k}}''}\right|\simeq\beta\left|E_{\mathbf{k}''}\right|$, $\left|E_{\mathbf{k}''}\right|\simeq10^{-2}\left|E_{\mathbf{k}}\right|$). It is found that the acceleration rate for $m=1$ is considerably large compared with that for $m=0$. This is owing to the acceleration mechanism in which the beat wave acceleration for $m=1$ results from the Lorentz force due to the perpendicular components of the magnetic field of the beat wave whose magnitude in the moving frame increases about β times that in the laboratory frame. This is consistent with the previously obtained results [4,6].

4. Conclusion

It is verified theoretically and numerically from the relativistic equation of motion for the trapped beam electrons that the highly relativistic electron beam can be accelerated by the extraordinary beat wave induced via the nonlinear electron Landau and cyclotron damping of the intense electromagnetic waves. The beat wave acceleration induced by the nonlinear scattering by the extremely high-power laser injected into the magnetized plasma may be available usefully for the highly relativistic electron beam accelerator [8].

The authors wish to thank Professor Y. Kitagawa for useful discussions. Numerical calculations were made at the Computation Center of Nagoya University.

[1] R. Sugaya, Phys. Plasmas **10**, 3939 (2003).
[2] R. Sugaya, Phys. Plasmas **11**, 5634 (2004).
[3] R. Sugaya, Phys. Plasmas **13**, 053102 (2006).
[4] R. Sugaya, Phys. Plasmas **15**, 013104 (2008).
[5] R. Sugaya and T. Maehara, *Proceedings of International Conference on Inertial Fusion Sciences and Applications,* 2007, Kobe, Japan, ThPo76 (2007).
[6] R. Sugaya and T. Maehara, *Proceedings of International Conference on Inertial Fusion Sciences and Applications,* 2007, Kobe, Japan, ThPo77 (2007).
[7] R. Sugaya, J. Plasma Phys. **64**, 109 (2000).
[8] Y. Kitagawa, T. Matsumoto, T. Minamihara *et al.*, Phys. Rev. Lett. **68**, 48 (1992).