Collector Floating Potentials in a Discharge Plasma with Two Species of Positive Ions

Milan ČERČEK[1,2), Tomaž GYERGYEK[1,3)] and Gregor FILIPIČ[1)]

1) *Jožef Stefan Institute, Jamova 39, 1000 Ljubljana, Slovenia*
2) *Faculty of civil engineering, University of Maribor, Smetanova 17, 2000 Maribor, Slovenia*
3) *Faculty of electrical engineering, University of Ljubljana, Tržaška 25, 1000 Ljubljana, Slovenia*

(Received: 31 August 2008 / Accepted: 26 December 2008)

In this contribution we present the study on potential formation near a floating collector in discharge plasma with two ion populations and with a bi-Maxwellian electron distribution. It is performed analytically and complemented by computer simulations. In the analytical study we use a fully kinetic plasma sheath model, originally developed by Schwager and Birdsall [L. A. Schwager, C. K. Birdsall, Phys. Fluids B, **2**, (1990) 1057] and later extended in order to include additional particle species like hot electrons [M. Čerček, T. Gyergyek, J. Phys. D: Appl. Phys. **34**, (2001) 330] and/or negative ions. The collisionless one-dimensional pre-sheath/sheath plasma region is bounded by a planar plasma source on one side and by a floating collector on the other side. Two positive ion populations are injected from the Maxwellian plasma source into the system with an accelerated half-Maxwellian distribution, and the electron population is modeled with a truncated bi-Maxwellian distribution. The collector floating potential and the pre-sheath potential are calculated as functions of positive ion density fraction. For the simulations we use XPDP1 particle-in-cell simulation code for a bounded plasma system developed at Berkeley University. The potential and particle density profiles are examined and resulting particle velocity distribution functions along the system are calculated and displayed. Good agreement with analytical results is obtained.

Keywords: two-electron temperature plasma, double-ion plasma, double layer, plasma simulations, bounded plasma

1. Introduction

In this research we continue the series of bounded plasma research. In [5] effects of addition of energetic electrons to one electron-ion plasma were investigated. It was shown that presheath boundary potential and collector potential decrease with increasing hot to cold electron density ratio for small values of this ratio (≤ 0.4). For larger values of the ratio the values of presheath potential decreases less rapidly, while collector potential settles to more or less constant value. An important phenomenon is the formation of a double layer at the value of the hot to cold electron density ratio around 0.25. This time we have added another species of ions and we are interested in potential dependence on hot to cold electron density ratio at three ion density ratios, and in double layer formation. We approach the problem from two angles: first we modify the kinetic model of Schwager and Birdsall, [3] and obtain the boundary conditions for numerical solution of the Poisson equation which gives a theoretical prediction for the presheath and the collector potentials. Second we simulate our four particle plasma with the code XPDP1. Results from both methods are in good agreement.

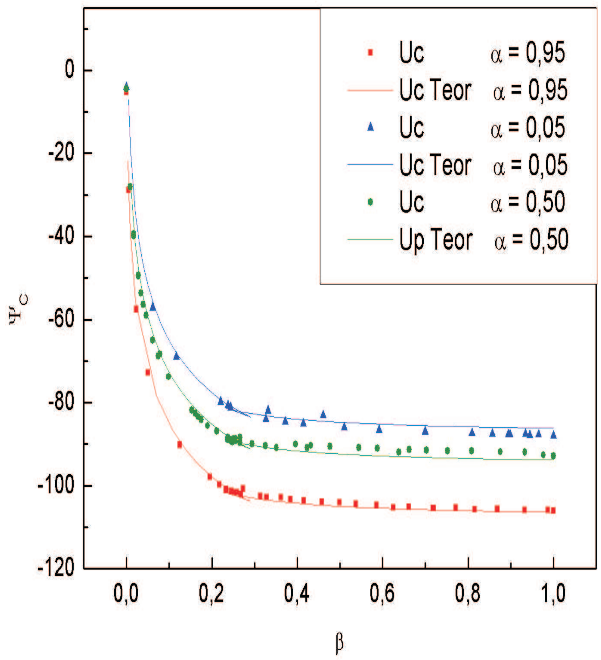

Fig. 1 Calculated values for collector potential for three different α ratios as function of β

2. Analytical model

System consists of two planar electrodes and empty space between. At the first electrode (source electrode) at $x = 0$ particles are injected into the system. Sum of injected ion particle currents is equal to sum of injected

author's e-mail: milan.cercek@ijs.si

electron particle currents. Second electrode (collector) at $x = L$, where L is length of the system, is at floating potential. Lighter and therefore faster electrons reach the collector first and charge it negative. Potential barrier repels nearly all electrons, which return towards the source. There they are refluxed back into the system. Electrostatic potential is assumed to monotonically decrease with distance from the source. Ions are attracted towards the collector, they are accelerated and cooled. At the collector they are all absorbed. Both ion species are thus described with half-Maxwellian velocity distribution and both electron species with truncated full-Maxwellian distribution:

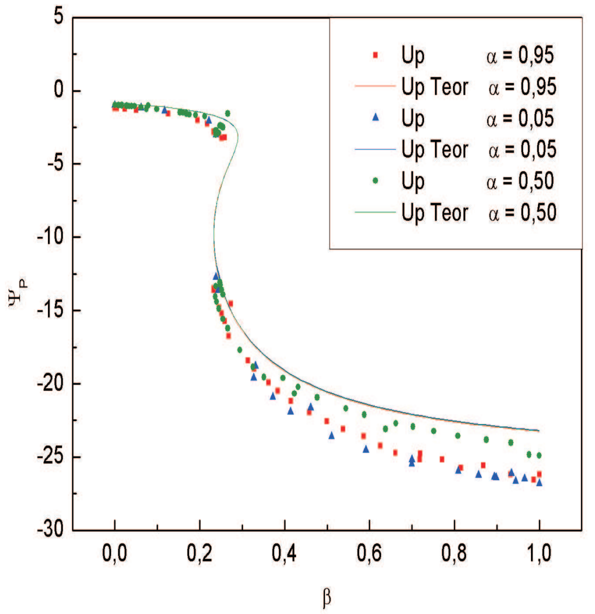

Fig. 2 Calculated values for plasma potential for three different α ratios as function of β

$$F_c = \frac{(1-\beta)}{\sqrt{\pi}}\, e^{-u^2+\Psi}\, H\left(\sqrt{\Psi-\Psi_c}-u\right) \tag{1}$$

$$F_h = \frac{\beta}{\sqrt{\pi t}}\, e^{-\frac{u^2}{t}+\frac{\Psi}{t}}\, H\left(\sqrt{\Psi-\Psi_c}-u\right) \tag{2}$$

$$F_A = \alpha\, \gamma\, \sqrt{\frac{\nu}{\tau_A\, \pi}}\, e^{-u^2\frac{\nu}{\tau_A}-\frac{\Psi}{\tau_A}} \times \times H\left(-\sqrt{-\frac{\Psi}{\nu}}-u\right) \tag{3}$$

$$F_H = (1-\alpha)\, \gamma\, \sqrt{\frac{\mu}{\tau_H\, \pi}}\, e^{-u^2\frac{\mu}{\tau_H}-\frac{\Psi}{\tau_H}} \times \times H\left(-\sqrt{-\frac{\Psi}{\mu}}-u\right) \tag{4}$$

$H(x)$ is a step function (it truncates the electron velocity distributions and limits the ion velocity distributions to just half-Maxwellian). The distribution functions (1)-(4) are normalized to the cold electron density at the source. The following variables were introduced:

$$\begin{array}{lll} t = \frac{T_h}{T_c} & \tau_A = \frac{T_A}{T_c} & \tau_H = \frac{T_H}{T_c} \\ \mu = \frac{m_H}{m_e} & \nu = \frac{m_A}{m_e} & \\ \Psi = \frac{e\phi}{kT_e} & u = \frac{v}{v_o} & v_o = \sqrt{\frac{2kT_c}{m_e}} \\ \alpha = \frac{n_{oA}}{n_{oH}+n_{oA}} & \beta = \frac{n_{oh}}{n_{oc}+n_{oh}} & \gamma = \frac{n_{oA}}{n_{oc}} \end{array} \tag{5}$$

where T denotes temperature, m mass, ϕ electrostatic potential, v velocity and n_o are densities of full Maxwellian source. Index $_H$ denotes helium ions, $_A$ argon ions, $_e$ electrons, $_c$ cold electrons and $_h$ hot electrons.

The zero moments of the distribution functions give the particle densities, while the first moments give the particle fluxes:

$$N_j = \int_{-\infty}^{\infty} F_j\, du \tag{6}$$

$$J_j = \int_{-\infty}^{\infty} u\, F_j\, du \tag{7}$$

For theoretical calculation of the presheath potential (Ψ_p) and of the collector potential (Ψ_c) we integrate the Poisson equation. It is of second order, so we need two boundary conditions. By reflux any charge accumulation is prevented and therefore zero electric field is enforced at the source plane. Somewhere in the middle of the system an inflection point is assumed where plasma is quasi-neutral and the potential is constant ($\Psi = \Psi_P$). Therefore electric field is zero. Poisson equation is rearranged in terms of the differentials of the potential:

$$d\left(\frac{d\Psi}{dz}\right)^2 = -2(N_i - N_e)\, d\Psi \tag{8}$$

The space coordinate x has been normalized to the Debye length λ_D in the following way:

$$z = \frac{x}{\lambda_D},\ \lambda_D = \sqrt{\frac{\varepsilon_0 k T_c}{n_{oc} e_0^2}}. \tag{9}$$

We integrate equation (8) from potential at the source to the one at the inflection point. Left hand side of the integrated equation is zero at both boundaries, so the integral yields:

$$\int_{\Psi_p}^{\Psi=0} (N_i - N_e)\, d\Psi = 0 \tag{10}$$

Besides Ψ_p and Ψ_c, α, β and γ are also unknown. α and β are manually appointed, but for γ and also Ψ_P and Ψ_C we need two more equations. At the inflection point we have neutrality condition, where density of positive ions must be equal to density of electrons:

$$aN_i\Big|_{\Psi=\Psi_p} = N_e\Big|_{\Psi=\Psi_p} \tag{11}$$

The last condition and equation we get from the floating condition of the collector. Because it is not connected to outer world, no net current is possible. Thus the current of ions must be equal to the current of electrons:

$$aJ_i\Big|_{\Psi=\Psi_c} = J_e\Big|_{\Psi=\Psi_c} \tag{12}$$

Equations (10), (11) and (12) form a system of equations, from which we numerically calculate Ψ_P, Ψ_C and γ belonging to appropriate values of α and β. Result of calculations for $\alpha = 0.05, 0.50, 0.95$ are plotted in Fig. 1 and Fig. 2.

3. Simulations

The purpose of the simulations was to verify the theoretical calculations of potentials. Three series were simulated: $\alpha = 0.95$, $\alpha = 0.50$ and $\alpha = 0.05$. In each series

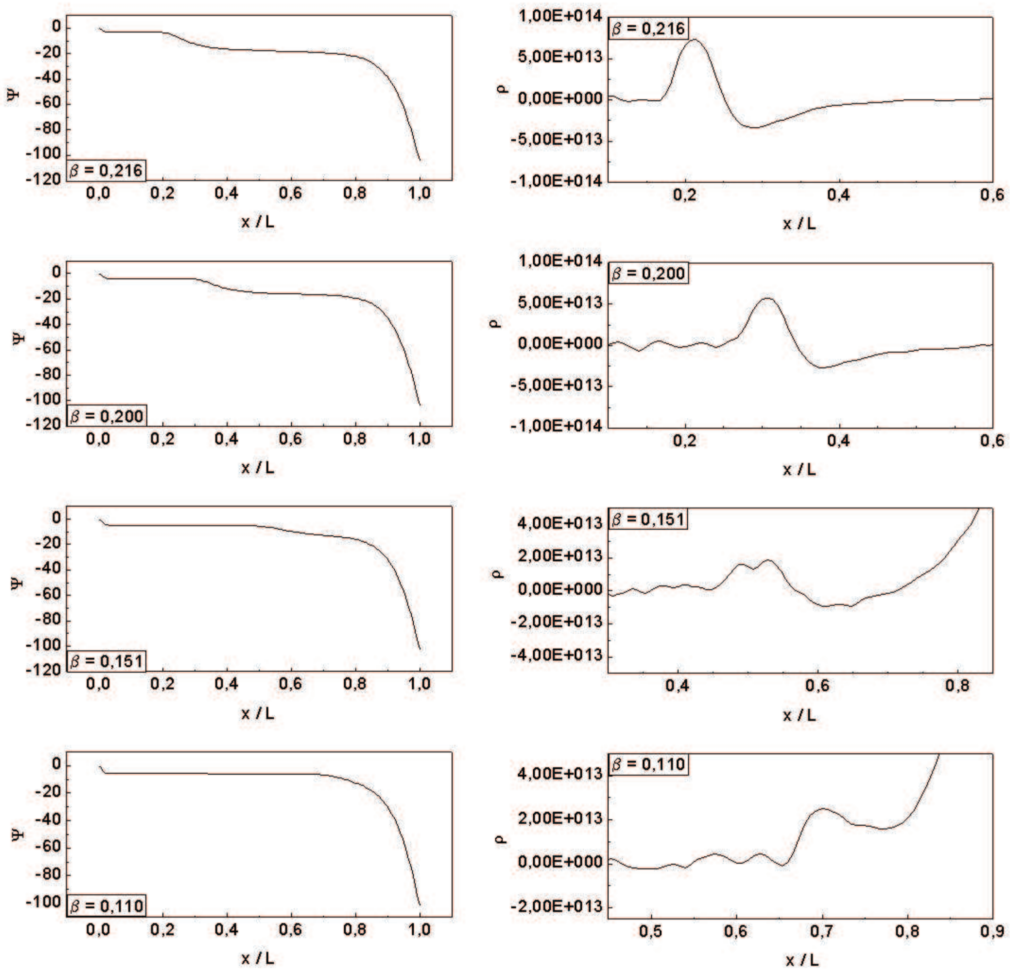

Fig. 3 Potential and charge distributions of simulations for $\alpha = 0.95$ for different β, at which double layer is formed.

simulated potentials were obtained for β in range form 0 to 1. α ratio was set in the input file with injected ion current ratio, while β ratio was established by pre-simulation. First the system was completely empty, then we ran a simulation with injection of all 4 particle species. When the number of cold electrons in the system reached predetermined value the simulation was saved to a dump file. The input file was edited so that the injection of the cold electrons was set to zero and the influx of the hot electrons was set equal to the sum of both ion influxes. Then simulation continued from the dump file, but with the new input file. Simulation was over when number of particles reached a constant value. Length of the system was changed to get a stable simulation. For α 0.95 and 0.05 length was from 0.01 m for small values of β to 0.08 m for larger β. For $\alpha = 0.50$ length interval was 0.05-0.4 m. In our analysis we used electrons with 2 eV and 40 eV of temperature and both ions had temperature 0.1 eV. Mass of ions was equal to helium and argon ions.

As addition to previous simulations we made some changes in parameters to simulate plasma similar to plasma in our linear plasma machine. Discharge potential for accelerating electrons into our plasma system is of the order of 50 V, so we set lower and higher limit to electron velocity with cutoff velocity equal to 50 eV of kinetic energy. Again we were interested in presheath and collector potential. In the end we changed temperature of hot electrons to 12 eV and made simulations with and without cutoff velocity.

4. Results

Potentials Ψ_P and Ψ_C have similar behavior due to β as in [5]. Both decrease, Ψ_P slowly at first, than faster and towards large β (≥ 0.4) the decrease is more gentle again. Ψ_C falls rapidly for small β and at around $\beta = 0.4$ settles at almost constant value. For presheath potential Ψ_p (Fig. 2) there is no difference in calculated values among different α ratios, but in simulations there is. The calculated values of the Collector potential (Fig. 1) agree with simulations. Differences are seen between α-series. Larger values of α bring lower values of the collector potential which is expected because of slower positively charged particles.

Double layers are formed for all three α in intervals of β around 0.23 as seen in Fig. 2, where at same β we had two values of Ψ_P.

In Fig. 3 are shown potential profiles along the system (left column) and inserts of charge density profiles (right column). In first double layer is seen as sheath between two locally constant potentials and in later we see the rise of positive charge followed by an excess of negative charge. It is seen how double layer travels down the system with decreasing β and eventually gets assimilated into collector plasma sheath.

Ions, which travel from the source to the collector, are accelerated in the double layer area and at the same time they are cooled down - velocity distribution is more narrow after double layer (Fig. 4). In collector sheath they get accelerated once more to a final velocity with which they hit the collector. Velocity distribution for hot electrons before and after double layer does not change, while there is no more cold electrons after double layer because the collector potential is to strong and cold electrons get repelled. Some hot electrons reach the collector and thus they have truncated velocity distribution (lower right picture in Fig. 5). All the distributions were recorded in a few, different number of simulation steps and are normalized with highest value of individual distribution.

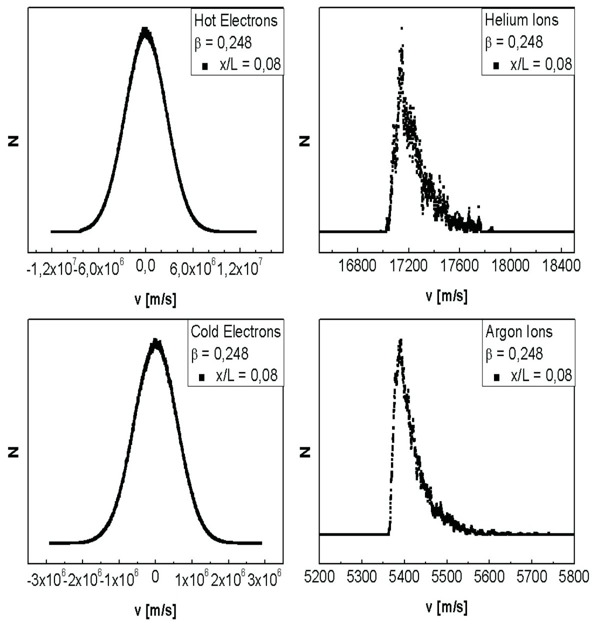

Fig. 4 Velocity distributions for all four particles. Position at which we sampled distribution is between source and double layer

Collector potential is primarily determined by fastest of hot electrons which is seen in Fig. 6. Simulations with different hot electron temperatures but same cutoff velocity have almost identical collector potential. Collector potential determines whether the double layer is formed or not. If hot electrons are not fast enough, the potential is to high and barrier for cold electrons is to small and double layer can not exist. Simulations with cutoff velocity or with lower temperature of the hot electrons ($6eV$) do not have double layer as seen in Fig. 7.

5. Conclusion

We have shown that in collisionless double-ion plasma it is also possible to obtain double plasma layer, if temperature ratio of the electrons is sufficiently large. Double layer is formed in the same β range, regardless of α ratio. On the other hand the values of α do influence the collector potential. At constant electron temperature higher α means

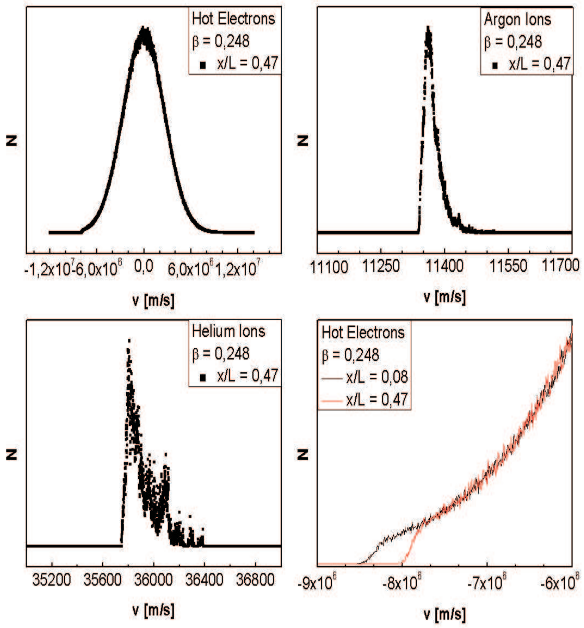

Fig. 5 Velocity distributions for all particles after the double layer, except for cold electrons. Lower left picture shows how electron distribution. It is seen, that ions accelerate and cool down when flying through double layer. Electron distributions changes only at low negative side, where electrons closer to the collector are slower.

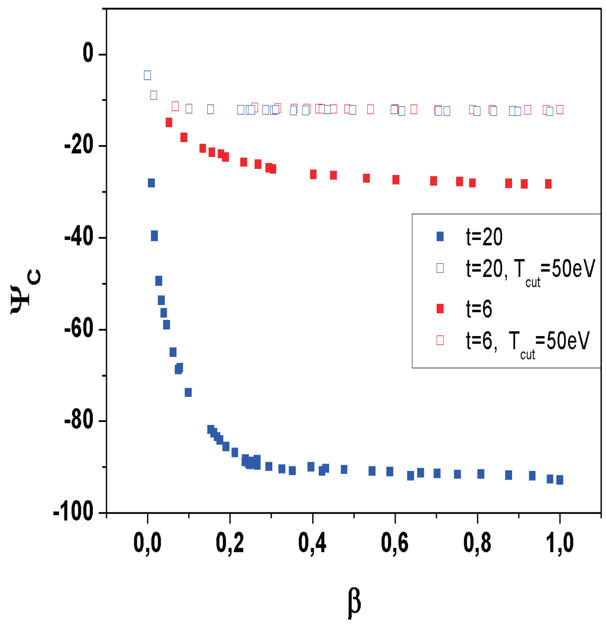

Fig. 6 Collector potential for simulations with hot electrons temperature 40 eV and 6 eV, with and without cutoff velocity at 50 eV

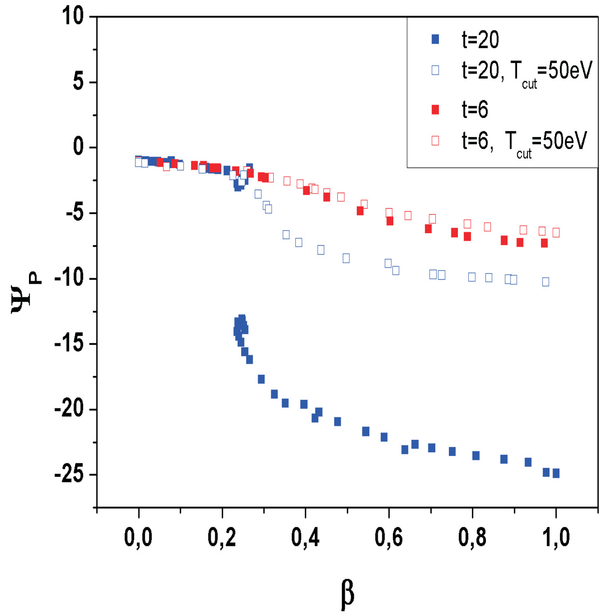

Fig. 7 Presheath potential for simulations with hot electrons temperature 40 eV and 6 eV, with and without cutoff velocity at 50 eV

lower Ψ_C. At constant α ratio, Ψ_C rises with decrease in electron temperature. Thus heavy ion to hot electron particle current determines the Ψ_C.

Cutoff velocity has similar effect as lowering the temperature of hot electrons. It can prevent double layer to form. Because of that we do not expect double layer to form in plasma in an experiment in a real machine.

Acknowledgement

This work, supported by the European Communities under the contract of Associations between EURATOM and MHEST, was carried out within the framework of the European Fusion Development Agreement. The content of the publication is the sole responsibility of its authors and it does not necessarily represent the views of the Commission or its services. This work was also supported by projects L2-0858 and J2-9160 of the Slovenian Research Agency.

[1] R.N. Franklin, J. Phys. D: Appl. Phys. **34** (2001) 1959.

[2] D. Lee, N. Hershkowitz, G.D. Severn, Appl. Phys. Lett. **91** (2007) 041505.

[3] L. A. Schwager, C. K. Birdsall, Phys. Fluids B, **2**, (1990) 1057.

[4] M. Čerček, T. Gyergyek, J. Phys. D: Appl. Phys. **34** (2001) 330.

[5] M. Čerček, T. Gyergyek, M. Stojanević, Contrib. Plasma Phys. **39**, 541 (1991).

[6] T. Gyergyek, B. Jurčič-Zlobec, M. Čerček, Phys. Plasmas, **15**, 063501 (2008).