On the Formation of Plasma Structures in our Magnetized Dusty Universe

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Our universe is magnetized and its constituents are the electrons, anti-electrons (positrons), ions and dust particles. Our objective here is to discuss the formation of large scale plasma structures based on a model associated with the Jeans instability in a self-gravitating multi-component fully ionized magnetoplasma. Assuming that the plasma particles are magnetized and charged dust grains form the stationary background, we derive a new dispersion relation for the low-frequency electrostatic waves in a self-gravitating pairs-ion-dust plasma. The dispersion relation is analyzed to obtain the growth rates and thresholds of the Jeans instabilities. It is found that the ambient magnetic field plays a stabilizing role. The implication of the present work to the formation of large scale structures in our dusty universe is highlighted.

Keywords: plasma structures, universe, dusty plasma, magnetic field

1. Introduction

The power spectrum of the large-scale structure, together with the cosmic microwave background (CMB) fluctuations, provide invaluable informations on the composition and the dynamical evolution of our Universe. Contrary to CBM fluctuations, which have been frozen since the recombination time, the evolution of the matter power spectrum reveals that the Universe has essentially been in a fully ionized plasma state. The latter is composed of the electrons, positrons, ions, and dust particles, which support numerous collective plasma effects to exhibit influence on the formation of different scale structures in our dusty universe.

It is well known that the Jeans instability [1, 2] in a self-gravitating system sets in provided that the gravitational force exceeds the pressure gradient. The resulting collapse of self-gravitating bodies is then responsible for the formation of large scale structures in our Universe.

Recently, Chen and Lai [3] discussed the role of the plasma on the formation of large scale structures caused by the Jeans instability in an unmagnetized Universe, by assuming that the latter is composed of the electrons and baryons/ions only. Such an assumption is invalid since a fully-ionized Universe contains the electrons, positrons, ions, and charged dust particles [4, 5, 6, 7]. The role of the positrons and dust grains has been ignored in the analysis of Ref. [3].

In this paper, we present an investigation of the Jeans instability in a fully-ionized self-gravitating [8] dusty magnetoplasma. It is found that the presence of the positrons, charged dust grains, and magnetic field alters the Jeans instability criteria.

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2. Dispersion Relations

We consider a self-gravitating fully-ionized dusty magnetoplasma composed of the electrons, positrons, ions, and charged dust grains. Thus, at equilibrium, the quasi-neutrality condition reads [9, 10] $Z_i n_{i0} + n_{p0} = n_{e0} + \epsilon Z_d n_{d0}$, where Z_i is the ion charge state, Z_d is the number of charges on the dust, $\epsilon = +1(-1)$ for the negative (positive) dust, and n_{j0} is the unperturbed number density of the particle species j (j equals e for the electrons, i for the ions, p for the positrons, and d for the dust grains). We assume that the dust mass density $n_d m_d$ is much smaller than the ion mass density $n_i m_i$, and therefore consider the dust grains to form the neutralizing background. Here, n_i is the ion number density, and m_d and m_i are the masses of the dust grains and ions, respectively. The external magnetic field is $B_0\hat{\mathbf{z}}$, where B_0 is the strength of the external magnetic field and $\hat{\mathbf{z}}$ is the unit vector along the z axis in a Cartesian coordinate system.

In our self-gravitating dusty magnetoplasma, the gravitational potential ϕ_g [8] is determined from the gravitational Poisson equation

$$k^2\phi_g = -4\pi G m_i n_{i1}, \tag{1}$$

where k is the wave number of the electrostatic perturbation, G is the gravitational constant, and $n_{i1}(\ll n_{i0})$ is the ion number density perturbation.

In a nonuniform magnetoplasma, the ion density perturbation n_{i1} in the presence of ϕ_g and the electrostatic potential ϕ is

$$n_{i1} = -\frac{k^2}{4\pi Z_i e}\chi_i\left(\phi + \frac{m_i\phi_g}{Z_i e}\right), \tag{2}$$

where e is the magnitude of the electron charge, and the ion susceptibility is [11]

$$\chi_i = \frac{\omega_{pi}^2}{k^2 V_{Ti}^2}\Big\{1 - \sum_{n=-\infty}^{\infty} I_n(b_i)\exp(-b_i)$$
$$\times \quad \left[\omega - \omega_{i*}\left(1 - \frac{n\omega}{b_i\omega_{ci}}\right)\right]$$
$$\times \quad \int_{-\infty}^{\infty} \frac{F_{iz}dv_z}{\omega - k_z v_z - n\omega_{ci}}\Big\}. \qquad (3)$$

Here $\omega_{pi} = (4\pi n_{i0} Z_i^2 e^2/m_i)^{1/2}$ is the ion plasma frequency, $V_{Ti} = (k_B T_i/m_i)^{1/2}$ is the ion thermal speed, m_i is the ion mass, k_B is the Boltzmann constant, T_i is the ion temperature, I_n is the modified Bessel function of order n, $b_i = k_\perp^2 V_{Ti}^2/\omega_{ci}^2$, $\omega_{ci} = Z_i e B_0/m_i c$ is the ion gyrofrequency, c is the speed of light in vacuum, $\omega_{i*} = -k_y\kappa_i V_{Ti}^2/\omega_{ci}$, $\kappa_i = -n_{i0}^{-1}\partial n_{i0}/\partial x$, $F_{iz} = (2\pi V_{Ti}^2)^{-1/2}\exp(-v_z^2/2V_{Ti}^2)$, and ω is the angular frequency of the electrostatic perturbations. We note that (2) has been obtained by inserting the perturbed ion distribution function, deduced from the linearized ion Vlasov equation containing the gravitational and electrostatic forces, into the definition of the ion number density and integrating over the velocity space.

The ions are coupled with the electrons and positrons through the space charge electric field $\mathbf{E} = -i\mathbf{k}\phi$. The electron and positron number density perturbations are determined from

$$n_{e1,p1} = \pm\frac{k^2}{4\pi e}\chi_{e,p}\phi, \qquad (4)$$

where the plus (minus) sign on the right-hand side of (4) stands for the electrons (positrons), and the electron/positron susceptibility reads [11]

$$\chi_{e,p} = \frac{\omega_{pe,pp}^2}{k^2 V_{Te,Tp}^2}\Big\{1 - \sum_{n=-\infty}^{\infty} I_n(b_{e,p})\exp(-b_{e,p})$$
$$\times \quad \left[\omega - \omega_{e*}\left(1 - \frac{n\omega}{b_{e,p}\omega_c}\right)\right]$$
$$\times \quad \int_{-\infty}^{\infty} \frac{F_{ez,pz}dv_z}{\omega - k_z v_z - n\omega_c}\Big\}. \qquad (5)$$

Here $\omega_{pe,pp} = (4\pi n_{e0,p0}e^2/m_{e,p})^{1/2}$ is the electron/positron plasma frequency, $V_{Te,Tp} = (T_{e,p}/m_{e,p})^{1/2}$ is the electron/positron thermal speed, $T_{e,p}$ is the electron/positron temperature, $b_{e,p} = k_\perp^2 V_{Te,Tp}^2/\omega_c^2$, and $\omega_{e*,p*} = k_y\kappa_{e,p}V_{Te,Tp}^2/\omega_c$, $\kappa_{e,p} = -n_{e0,p0}^{-1}\partial n_{e0,p0}/\partial x$, and $F_{ez,pz} = (2\pi V_{Te,Tp}^{1/2}\exp(-v_z^2/2V_{Te,Tp}^2$. In our pair-ion-dust plasma, we typically have $T_e = T_p = T$, $m_e = m_p = m$, and $\omega_c = eB_0/mc$. The gravitational force acting on the electrons and positrons is insignificant, and therefore it is not included in (4).

By invoking the quasi-neutrality condition, $Z_i n_{i1} + n_{p1} = n_{e1}$, which is valid for $k^2\lambda_{Di,De,Dp}^2 \ll 1$, we obtain from (2) and (4)

$$\phi = -\frac{m_i\chi_i\phi_g}{e(\chi_e + \chi_p + \chi_i)}. \qquad (6)$$

Inserting (4) into $n_{i1} = (n_{e1} - n_{p1})/Z_i$, we have

$$n_{i1} = \frac{k^2(\chi_e + \chi_p)\phi}{4\pi Z_i e}. \qquad (7)$$

Equation (7) is now combined with (1) to obtain

$$\phi_g = -\frac{Gm_i(\chi_e + \chi_p)\phi}{Z_i e}. \qquad (8)$$

Eliminating ϕ_g from (6) by using (8) we obtain the general dispersion relation

$$1 + \frac{\chi_i}{\chi_e + \chi_p} - \frac{Gm_i^2\chi_i}{Z_i^2 e^2} = 0. \qquad (9)$$

3. Analyses of the Dispersion Relation

We now analyze Eq. (9) in several limiting cases. First, we consider a uniform unmagnetized plasma and focus on the low-frequency regime $kV_{Ti} \ll \omega \ll kV_{Te,Tp}$. Hence, we write

$$\chi_{e,p} \approx \frac{1}{k^2\lambda_{De,Dp}^2}\left(1 + i\sqrt{\frac{\pi}{2}}\frac{\omega}{kV_T}\right) \qquad (10)$$

and

$$\chi_i \approx -\frac{\omega_{pi}^2}{(\omega^2 - 3k^2V_{Ti}^2)}$$
$$+i\frac{\omega}{kV_{Ti}k^2\lambda_{Di}^2}\sqrt{\frac{\pi}{2}}\exp\left(-\frac{\omega^2}{2k^2V_{Ti}^2}\right), \qquad (11)$$

where $\lambda_{De,Dp} = V_T/\omega_{pe,pp}$, $\lambda_{Di} = V_{Ti}/\omega_{pi}$, and $V_T = (T/m)^{1/2}$ is the electron/positron thermal speed. Here, Eq. (9) can be expressed as

$$\omega^2 - 3k^2V_{Ti}^2 - \frac{k^2C_s^2 kV_{Ti}}{1 + i\sqrt{\pi/2}\omega} + \omega_J^2$$
$$+i\sqrt{\frac{\pi}{2}}\frac{\omega(\omega^2 - 3k^2V_{Ti}^2)}{kV_{Ti}} = 0, \qquad (12)$$

where $C_s = \lambda_D\omega_{pi} \equiv Z_i[n_{i0}/(n_{e0}+n_{p0})]^{1/2}(T_e/m_i)^{1/2}$ is the modified dust ion-acoustic speed [10, 12], $\lambda_D = \lambda_{De}/(1 + n_{p0}/n_{e0})^{1/2}$ is the modified electron Debye radius, and $\omega_J = (4\pi Gm_i n_{i0})^{1/2}$ is the Jeans frequency involving the ion mass density (hereafter referred to as the Jeans ion mode). Equation (12), which accounts for the electron/positron and ion Landau damping rates, exhibits a linear coupling between between the dust ion-acoustic wave (DIAW) [10, 12] and the Jeans ion mode in our four component dusty universe. It generalizes the work of Ref. [3] to include the positrons and charged dust grains, as well as the electron/positron and ion Landau dampings.

In a multi-component plasma containing a fraction of charged dust grains, the DIAW is not subjected to Landau dampings, even for $T \sim T_i$. Hence, neglecting Landau dampings, we obtain from (12) by setting $\omega = i\gamma$, where $\gamma \ll kV_T$, the growth rate

$$\gamma = \left[\omega_J^2 - k^2(C_s^2 + 3V_{Ti}^2)\right]^{1/2}, \qquad (13)$$

provided that

$$k^2 < \frac{\omega_J^2}{(C_s^2 + 3V_{Ti}^2)}. \qquad (14)$$

Second, we consider the case $\omega \ll kV_{Te,Tp}, kV_{Ti}$. Here, the expressions for $\chi_{e,p}$ remains the same as for the first case, together with $\chi_i \approx (1/k^2\lambda_{Di}^2)\left(1 + i\sqrt{\pi/2}\omega/kV_{Ti}\right)$. Accordingly, Eq. (9) can be expressed as

$$\omega = -i\sqrt{\frac{2}{\pi}} kV_{Ti}\left(1 - \frac{\omega_J^2}{\omega_{pi}^2 k^2\lambda_D^2}\right)\left(1 + \frac{\lambda_{Di}^2}{\lambda_D^2}\right), \qquad (15)$$

which admits a purely growing instability ($\omega = i\omega_i$) if

$$k^2 < \frac{\omega_J^2}{\omega_{pi}^2\lambda_D^2}. \qquad (16)$$

The growth rate above threshold is

$$\omega_i \approx \frac{V_{Ti}\omega_J^2(\lambda_D^2 + \lambda_{Di}^2)}{k\omega_{pi}^2\lambda_D^4}. \qquad (17)$$

We note that the quasi-mode instability above is caused by the combined action of the ion Landau damping and gravitational attraction.

Next, we examine the effect of the external magnetic field on the Jeans instability. For $\omega \ll k_zV_T$ and $b_e \ll 1$, we have from (5)

$$\chi_{e,p} \approx \frac{1}{k^2\lambda_{De,Dp}^2}\left[1 + i\sqrt{\frac{\pi}{2}}\frac{(\omega - \omega_{e*,p*})}{k_zV_T}\right]. \qquad (18)$$

Furthermore, for $k_zV_{Ti} \ll \omega \ll \omega_{ci}$, the ion susceptibility reads [13]

$$\chi_i \approx \frac{1}{k^2\lambda_{Di}^2}\left\{1 - \frac{(\omega - \omega_{i*})\Gamma_0(b_i)}{\omega}\right.$$
$$\times \left.\left[1 + \frac{k_z^2V_{Ti}^2}{\omega^2} - i\left(\frac{\pi}{2}\right)^{1/2}\xi_i\exp(-\xi^2/2)\right]\right\}, \qquad (19)$$

where $\Gamma_0 = I_0(b_i)\exp(-b_i)$ and $\xi = \omega/k_zV_{Ti}$.

In a uniform magnetoplasma, the appropriate susceptibilities (18) and (19) are

$$\chi_{e,p} \approx \frac{1}{k^2\lambda_{De,Dp}^2}\left(1 + i\sqrt{\frac{\pi}{2}}\frac{\omega}{k_zV_T}\right). \qquad (20)$$

and

$$\chi_i \approx \frac{1}{k^2\lambda_{Di}^2}\left\{1 - \Gamma_0(b_i)\right.$$
$$\times \left.\left[1 + \frac{k_z^2V_{Ti}^2}{\omega^2} - i\left(\frac{\pi}{2}\right)^{1/2}\xi_i\exp(-\xi^2/2)\right]\right\}. \qquad (21)$$

By using (20) and (21) in (9) we observe that the external magnetic field reduces the growth rate of the Jeans instability.

4. Summary

In this paper, we have derived a new dispersion relation (9) in a self-gravitating magnetized dusty Universe whose constituents are the electrons, positrons, ions, and charged dust grains. Our dispersion relation for an unmagnetized plasma admits two classes of the Jeans instability in two different frequency regimes. In both cases, the growth rates and thresholds of the Jeans instabilities are affected by the presence of the charged dust grains and positrons, since $n_{i0} \neq n_{e0}$. When dust grains are negatively (positively) charged, we have $n_{i0} > (<) n_{e0}$. Furthermore, the magnetic field and equilibrium density inhomogeneities are found to affect the electron/positron and ion susceptibilities. Accordingly, the threshold criteria for the Jeans instabilities are modified. In conclusion, we stress that the present Jeans instabilities may be responsible for the formation of large scale structures in dusty cosmological environments, such as the milky way and interstellar media.

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