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Repeated Interactions of Thermal Ions with an Oblique Shock Wave

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The repeated acceleration of thermal ions by an oblique shock wave is studied by means of fully kinetic, electromagnetic particle codes. When thermal ions encounter a shock wave, some of them are reflected at the shock front. It is theoretically shown that if $\theta \simeq 45°$, where θ is the angle between the wave normal and external magnetic field, the reflected ions can be repeatedly accelerated by the shock wave because they can stay near the shock front for long periods of time. This is confirmed by a one-dimensional simulation for a planer shock wave with $\theta = 45°$. Furthermore, a cylindrical shock wave, in which the value of θ depends on the position on the front, is simulated with a two-dimensional particle code. It is demonstrated that the ions that are reflected at the front with $\theta \simeq 45°$ can be repeatedly energized.

Keywords: planar shock wave, cylindrical shock wave, particle reflection, repeated acceleration, particle simulation

1. Introduction

It has been shown with theory and particle simulations [1, 2, 3] that nonthermal, energetic ions barely entering a shock wave can be further accelerated to higher energies by the transverse electric field in the shock wave because, while they are in the shock region, their gyromotions are nearly parallel to the transverse electric field. If these ions move with the shock wave for long periods of time, the acceleration processes can be repeated several times; their energies increase once in a gyroperiod. The condition for the repeated acceleration is given by [2]

$$v_{\parallel} \cos\theta \simeq v_{\rm sh} \tag{1}$$

where $v_{\rm sh}$ is the shock propagation speed, $v_{\parallel}$ is the particle velocity parallel to the magnetic field, and θ is the shock propagation angle, i.e., the angle between the wave normal and external magnetic field. If the relation $v_{\rm sh} \sim c\cos\theta$ is satisfied, where c is the light speed, energetic particles could be indefinitely accelerated owing to the relativistic effects [3].

The above studies were concerned with the acceleration of nonthermal ions; that is, it was assumed that energetic ions were present from the beginning. In this paper, we study repeated interactions of thermal ions with an oblique shock wave; we do not assume the presence of the energetic ions at $t = 0$. In Sec. 2, we consider the acceleration of thermal ions by a one-dimensional, planar shock wave. When the thermal ions encounter the shock wave for the first time, some of them are energized by the reflection from the shock front [4, 5]. We theoretically estimate their parallel velocities in the upstream region immediately after the reflection to be $v_{\parallel} \simeq 2v_{\rm sh}\cos\theta$. We then show that if $\theta = 45°$, the reflected ions can be further accelerated by the shock wave with the mechanism discussed in Refs. [1, 2, 3] because the condition (1) is satisfied. This is verified with one-dimensional (one space coordinate and three velocity components), fully kinetic, electromagnetic particle simulations. It is observed that in a planar shock wave with $\theta = 45°$, some of the thermal ions are accelerated three times; the first energy gain is from the longitudinal electric field, and the second and third ones are from the transverse electric field. The parallel velocities of these ions also increase several times, which is due to the magnetic field [2], and thus these ions eventually move away to the upstream region. In Sec. 3, by means of a two-dimensional code, we study the acceleration by a cylindrical shock wave. In such a wave, the value of θ depends on the position on the front. It is shown that the repeated acceleration can occur near the front with $\theta \simeq 45°$. In Sec. 4, we summarize our work.

2. Repeated acceleration by a planar shock wave

We consider the acceleration of thermal ions by a planar shock wave propagating in the x direction ($\partial/\partial y = \partial/\partial z = 0$) in an external magnetic field in the (x, z) plane,

$$\boldsymbol{B}_0 = B_0(\cos\theta, 0, \sin\theta). \tag{2}$$

Under these circumstances, the magnetic field B_z is enhanced in the shock region while B_x is constant [6], and thus the direction of $\boldsymbol{B}$ changes.

The upstream region ions are supposed to be in thermal equilibrium with the thermal speed much smaller than $v_{\rm sh}$. When the ions encounter the shock wave, some of them are reflected at the shock front and are accelerated to the speed $v \simeq 2v_{\rm sh}$ in the direction

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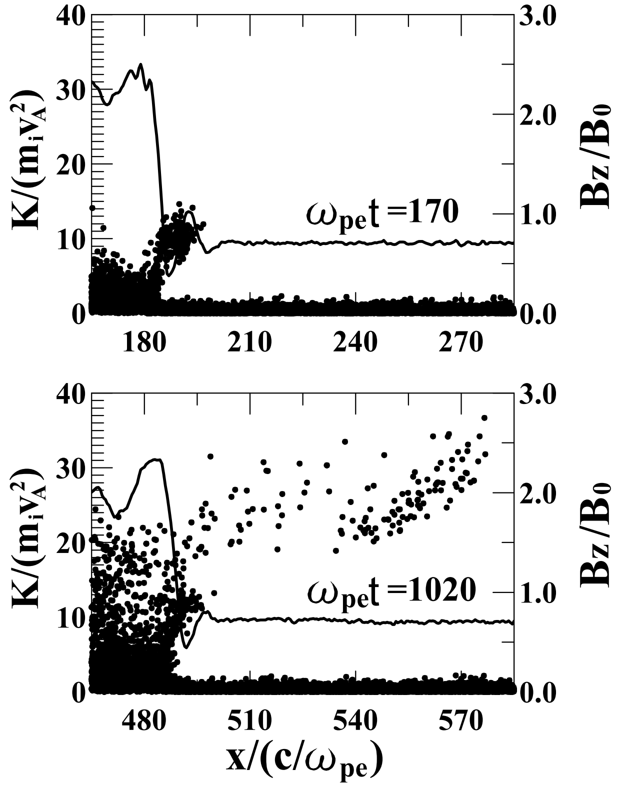

Fig. 1 Ion phase space plots (x, K) and profiles of B_z for a one-dimensional shock wave with $\theta = 45°$.

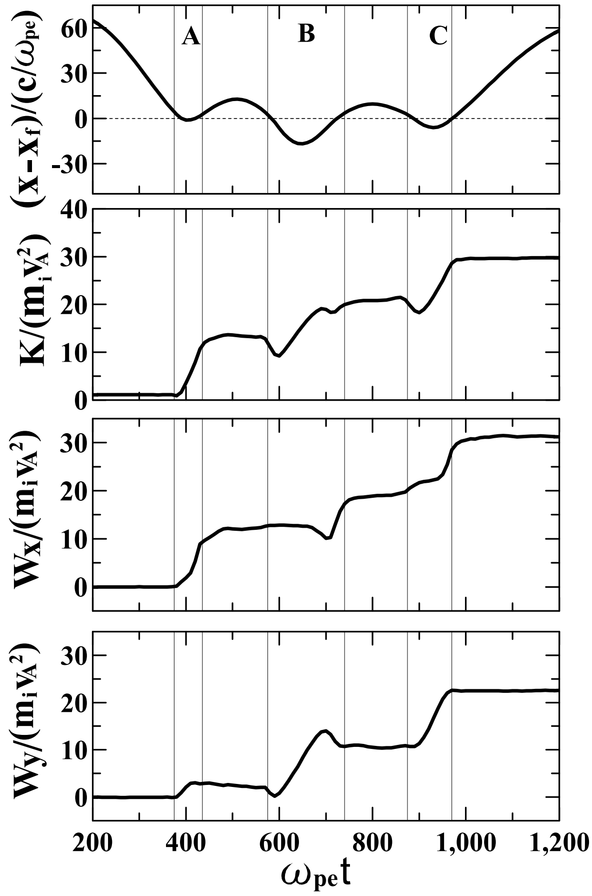

Fig. 2 Time variations of the x position, kinetic energy K, and works W_x and W_y done by the electric field E_x and E_y, respectively, of a repeatedly accelerated ion. The periods for which the ion is in the shock region are denoted by A, B, and C.

nearly perpendicular to $\boldsymbol{B}$; the acceleration is caused by the electric field, whose parallel component is usually small in magnitude in the magnetosonic shock wave. Since $\boldsymbol{B}$ is approximately in the z direction in the shock wave, the directions of the velocities of reflected ions are nearly parallel to the x axis. When they go out to the upstream region, therefore, their parallel velocities are

$$v_{\parallel} \simeq 2v_{\rm sh}\cos\theta. \tag{3}$$

If the x component of the parallel velocity, which is given by $v_{\parallel}\cos\theta$, is close to $v_{\rm sh}$, this particle can move with the shock wave for a long time and can interact with it repeatedly. The condition $v_{\parallel}\cos\theta \simeq v_{\rm sh}$ is satisfied when $\theta \simeq 45°$.

We now study the acceleration by means of a one-dimensional (one space coordinate and three velocity components), electromagnetic particle code with full ion and electron dynamics. The simulation size is $L_x = 8192\Delta_g$, where Δ_g is the grid spacing. The total number of simulation particles is $N \sim 1.0\times10^6$. The ion-to-electron mass ratio is $m_i/m_e = 100$. The thermal speeds are $v_{\rm Te}/(\omega_{\rm pe}\Delta_g) = 1.0$ and $v_{\rm Ti}/(\omega_{\rm pe}\Delta_g) = 0.1$. The electron skin depth is $c/\omega_{\rm pe} = 4$. The ratio of the electron gyrofrequency to the electron plasma frequency is $|\Omega_{\rm e}|/\omega_{\rm pe} = 1.0$ in the upstream region; the Alfvén speed is then $v_{\rm A}/(\omega_{\rm pe}\Delta_g) = 0.4$. The propagation angle is $\theta = 45°$, for which the repeated acceleration of thermal ions is expected.

Figure 1 shows ion phase space plots (x, K), where K is the kinetic energy, and the profiles of B_z for a shock wave with $v_{\rm sh} = 2.6v_{\rm A}$. At $\omega_{\rm pe}t = 170$, some ions are energized via the reflection at the shock front at $x/(c/\omega_{\rm pe}) \simeq 180$. At $\omega_{\rm pe}t = 1020$, ions with much higher energies are present. These ions have been produced by the repeated interaction with the shock wave.

The top panel of Fig. 2 shows time variation of the x position of a repeatedly accelerated ion, where x_f is the position of the shock front (the location where the field quantities begin to sharply rise); the upstream region is $x - x_f > 0$. This ion interacts with the shock wave three times; the periods for which it is in the shock region are denoted by A, B, and C. After the period C ($\omega_{\rm pe}t > 960$), this ion is in the upstream region moving faster than the shock wave. From the second panel, we see that the kinetic energy K jumps three times, and the jumps occur when the ion is in the

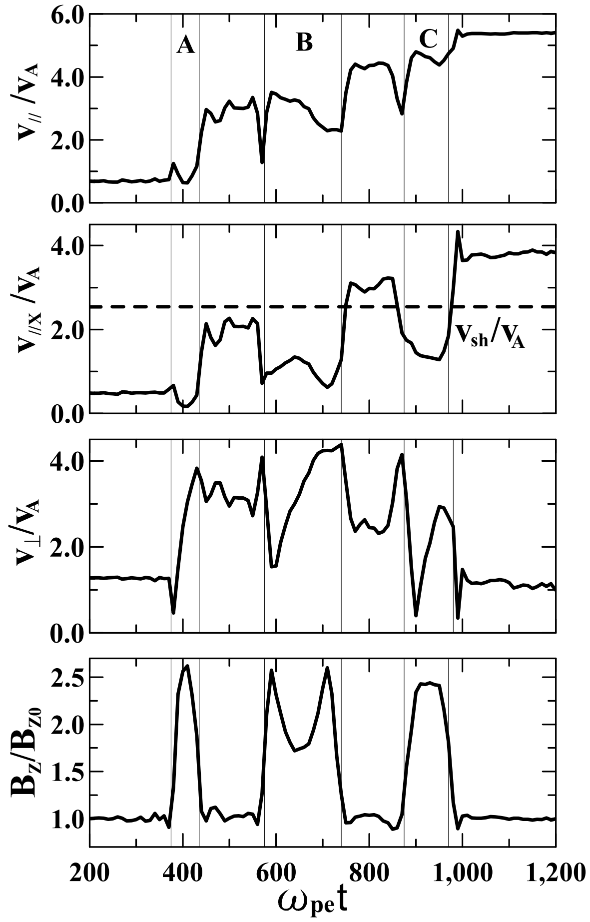

Fig. 3 Time variations of $v_\parallel$, $v_\perp$, and $B_z[x(t)]$. Here, $x(t)$ is the particle position.

shock region. The third and fourth panels show the work done by E_x and that by E_y, respectively. (Since the magnitude of E_z is much smaller than those of E_x and E_y, the work done by E_z is quite small and is not shown here.) The first energy gain A is from E_x, while the second and third ones (B and C) are mainly due to E_y. It has thus been shown that in a planar shock wave with $\theta = 45°$, some ions can suffer the repeated acceleration by E_y [2], after the reflection at the shock front.

In association with the repeated acceleration, the increase in $v_\parallel$ takes place owing to the change in the direction of the magnetic field $\boldsymbol{B}$. Figure 3 shows time variations of the parallel velocity $v_\parallel$, its x component $v_{\parallel x}$, perpendicular velocity $v_\perp$, and $B_z[(x(t)]$ at the particle position $x(t)$. Near the end of the first interaction with the shock wave (A), $v_\parallel$ increases, which is due to the decrease of B_z. The theory (3) gives a good estimate of $v_\parallel$ in the upstream region immediately after the reflection; the observed value of $v_\parallel$ is 75% of the theoretical value. Because $v_{\parallel x}$ is close to $v_{\rm sh}$, the particle can stay near the shock front, which leads to the second and third interactions in periods B and C. When the ion is in the shock region, $v_\perp$ increases owing to the transverse electric field. When the particle goes in and out the shock wave, $v_\parallel$ increases and $v_\perp$ decreases because of the change in $\boldsymbol{B}$. As a result of these processes, $v_{\parallel x}$ exceeds $v_{\rm sh}$, and the ion moves away to the upstream region.

3. Simulation for a cylindrical shock wave

By means of a two-dimensional (two space and three velocity components), electromagnetic particle code, we now study the ion motion in a cylindrical shock wave. Because of the curvature of the front, the angle θ between the wave normal and external magnetic field depends on the position on the shock front. We will show that the ions that are reflected at the front with $\theta \simeq 45°$ can suffer the repeated acceleration by the shock wave.

The system size of the simulation is $L_x \times L_y = 4096\Delta_g \times 4096\Delta_g$. The total number of simulation particles is $N \sim 1.0 \times 10^9$. The external magnetic field is taken to be $\boldsymbol{B}_0 = B_0(\cos 30°, 0, \sin 30°)$. The other parameters are the same as those in the one-dimensional simulation in Sec. 2.

Initially, we set the plasma density in the circular region with a radius, $r_0 = 250\Delta_g$, four times as high as that in the outer region. The ions and electrons in this high-density region have the shifted Maxwell velocity distribution functions; the local average velocity $\boldsymbol{v}_{\rm ps}(x, y)$ is in the radial direction of the circle. These particles act as a piston and initiate a cylindrical shock wave. Suppose that the front is given by

$$x^2 + y^2 = r^2 = v_{\rm sh}^2 t^2, \tag{4}$$

then θ is written as

$$\theta = \arccos\left[(1 - y^2/r^2)^{1/2} B_{x0}/B_0\right], \tag{5}$$

which indicates that θ is 30° at $y = 0$, increases with $|y|$, and is 90° at $y = \pm r$.

Figure 4 shows phase space plots (x, y, K) of the energetic ions with $K/(m_i v_{\rm A}^2) \geq 10$ at $\omega_{\rm pe}t = 720$ for a shock wave with $v_{\rm sh} \sim 2.5 v_{\rm A}$. The contour map of B in the (x, y) plane is also plotted. Energetic ions exist near the shock front. We note that the ions near $y = 0$ have particularly high energies. These ions have been repeatedly accelerated by the shock wave.

The trajectory of a repeatedly accelerated ion is depicted in the upper panel of Fig. 5, where the dashed lines represent the wave front at several different times from $\omega_{\rm pe}t = 30$ to 630. The circles and squares show the positions of the ion at these times and, respectively, indicate that the ion is in the shock and upstream regions. The lower panel of Fig. 5 shows its kinetic energy as a function of the particle position x. Even though the shock front has a curvature, the ion penetrates the shock region three

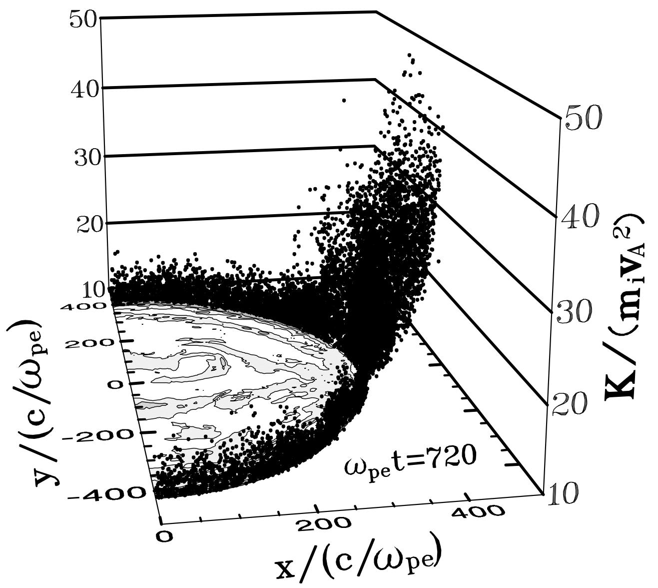

Fig. 4 Phase space plot (x, y, K) of energetic ions and contour map of B in the (x, y) plane for a shock wave with a cylindrical front.

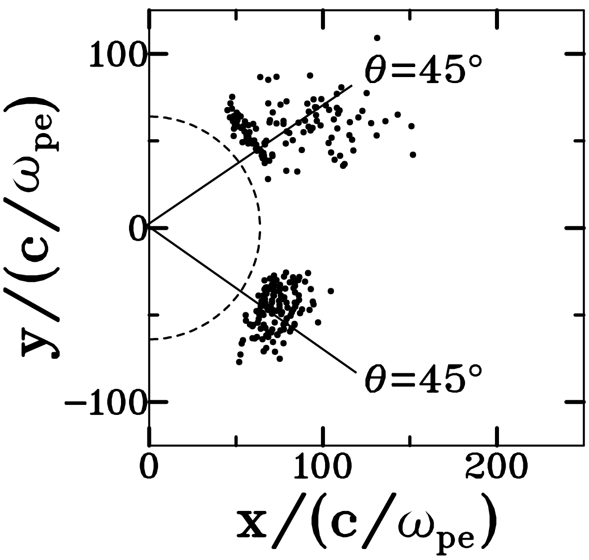

Fig. 6 Distribution of repeatedly accelerated ions. The positions at which the repeatedly accelerated ions had encountered the shock wave for the first time are plotted.

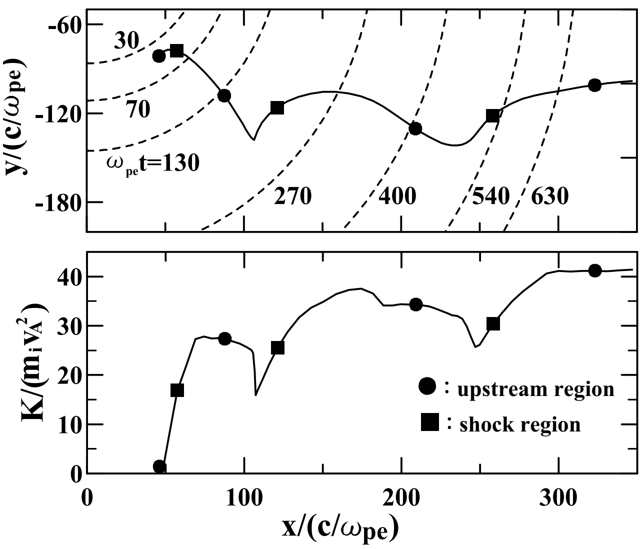

Fig. 5 Trajectory of a repeatedly accelerated ion (upper panel) and the dependence of K on x (lower panel).

times and is accelerated three times. The ion encountered the shock wave for the first time at the position $(x, y) \simeq (50c/\omega_{\rm pe}, -90c/\omega_{\rm pe})$, at which the value of θ was 45°.

Figure 6 shows the positions at which the ions that have been accelerated three times by $\omega_{\rm pe}t = 720$ had encountered the shock wave for the first time. These points are near the two solid lines of $\theta = 45°$. It is thus shown that the ions that are reflected at the shock front with $\theta \simeq 45°$ can be repeatedly energized by the shock wave.

4. Summary

We have studied the repeated acceleration of thermal ions by an oblique shock wave. We have been theoretically shown that if $\theta \simeq 45°$, thermal ions can suffer the repeated acceleration. We have then verified this prediction with one-dimensional, fully kinetic, electromagnetic particle simulations. In a planar shock wave with $\theta = 45°$, some of the thermal ions were energized three times; the first acceleration is caused by the longitudinal electric field and the second and third acceleration processes are caused by the transverse electric field. The parallel velocities of ions increase owing to the change in the direction of the magnetic field in the shock transition region, and the ions move away ahead of the shock wave. Furthermore, we have simulated a cylindrical shock wave by means of a two-dimensional code. Because of the curvature of the front, the value of θ varies as one moves along the front. The simulation has shown that some ions that encounter the shock wave for the first time at the positions with $\theta \simeq 45°$ can be repeatedly accelerated by the shock wave.

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