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Multi-scale Interaction between MHD, Turbulence and Transport in Tokamak Plasmas

Masatoshi Yagi and Sanae-I Itoh
Research Institute for Applied Mechanics, Kyushu University, Kasuga, Fukuoka 816-8580, Japan
Shinsuke Tokunaga and Seiya Nishimura
Interdisciplinary Graduate School of Engineering Sciences, Kyushu University, Kasuga, Fukuoka 816-8580, Japan
Kimitaka Itoh
National Institute for Fusion Science, Toki, Gifu 509-5292, Japan

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The recent achievement in the simulation study of multi-scale interaction among MHD, turbulence and transport is reported. Firstly, we discuss the formation and collapse of Internal Transport Barrier (ITB) in Tokamaks. The meso-scale structures are generated by three-wave interaction among drift wave turbulence. These structures interact each other, which generate the global mode around barrier location. This leads to the soft collapse of ITB. Secondly, we discuss multi-scale interaction among tearing mode and drift wave turbulence. The growth of tearing mode is accelerated by incoherent emission of drift wave turbulence. The possibility of sub-critical excitation and nonlinear sustainment of the magnetic island is discussed.

Keywords: multi-scale simulation, drift wave turbulence, MHD, meso-scale mode, weak turbulence theory.

1. Introduction

The multi-scale and multi-physics simulation study is a hot topic in fusion science. The advance in computer capability makes it possible to perform simulations with different time scale and special scales[1,2]. The multi-scale interaction plays an important role in not only plasma turbulence but also phase transition and critical phenomena in high temperature plasmas. In this paper, we report the recent achievement in the study of nonlinear simulations, putting an emphasis on multi-scale interactions among MHD, turbulence and transport.

Firstly, we discuss the formation and collapse of Internal Transport Barrier (ITB) in Tokamaks[3,4]. The meso-scale structures are generated by three-wave interaction among drift wave turbulence[5]. These structures interact each other, which generate the global mode around barrier location. This leads to the soft collapse of ITB. Secondly, we discuss multi-scale interaction among tearing mode and drift wave turbulence[6]. The growth of tearing mode is accelerated by incoherent emission of drift wave turbulence[7]. The possibility of sub-critical excitation and nonlinear sustainment of the magnetic island is also discussed.

2. ITB Formation and Collapse

A simple gyro-fluid model is used to investigate the ion temperature gradient driven drift wave (ITG) turbulence[8,9]. The model consists of the ion density, the ion parallel momentum and the ion temperature evolution equations. In this model, we normalize physical quantities by using the minor radius of the torus and Bohm diffusion time for the special and time scales, respectively. The heat source term is introduced into the ion temperature evolution equation. It is modeled by $S(r) = -8\times10^{-3}(2r^2-1)/(1-r_s^2)^2$ with $r_s = 0.6$. The equilibrium density is given by $n_0(r) = (1-r^2)/(1-r_s^2)$ which is fixed during the simulation to ensure no particle flux generation in the system. For the safety factor profile, the reversed magnetic shear profile is used which is given by $q(r) = q_{\min} + C_2(r^2-r_s^2)^2 + C_3(r^2-r_s^2)^3$ with $q_{\min} = 1.35$, $C_2 = 4.66$ and $C_3 = -0.987$ [10]. In this case, the minimum q value is 1.35 so that (4,3) mode is off-resonant mode.

The temporal evolution of fluctuating internal energy of each Fourier mode is investigated. In the early stages of evolution, the temperature gradient is small enough that the system is stable for the ITG modes and fluctuation amplitude remains at a noise level. When the temperature gradient exceeds the threshold value of the linear instability, they start to grow. Figure 1 shows the temporal evolution of fluctuating internal energy. It is found that the most unstable mode is (45,31) mode which is destabilized at $t\sim50$ and saturates at $t\sim58$. Other micro modes such as (29,20) and (30,20) modes start to grow at $t\sim62$ and damp at $t\sim65$. After saturation of these modes, meso-scale modes such as (16,11),(14,9),(11.7), (10,6) and (4,3) modes are excited. In this phase, ITB is formed in the vicinity of q minimum region. At $t\sim75$, the (4,3)

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author's e-mail: yagi@riam.kyushu-u.ac.jp

mode dominates the system and other meso-scale modes lose energies. It continues to grow and finally damps at $t \sim 81$. During this phase, the internal transport barrier begins to decay.

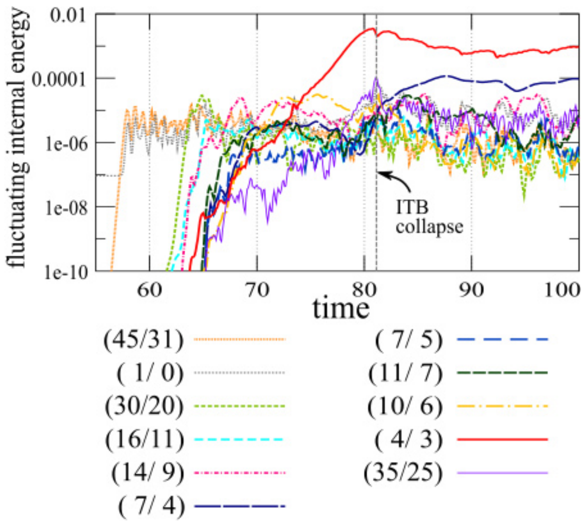

Fig.1 Time evolution of fluctuating internal energy.

Figure 2 shows time slices of the radial profile of turbulent heat diffusivity during the collapse phase. It is shown that the transport barrier gradually decays from inward boundary of the ITB region. The soft collapse (the relaxation) starts when the amplitude of (4,3) mode dominates the system. At $t \sim 81$, the energy inside of ITB bursts out abruptly. The trigger of the burst is the (35,25) mode which is observed as the spike of internal energy in Fig.1. The time scale is clearly different from the relaxation time scale.

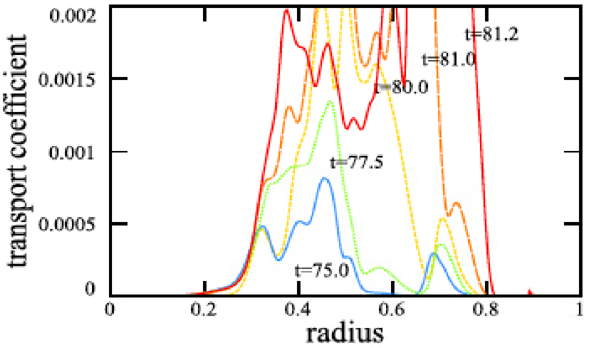

Fig.2 Time slices of turbulent heat diffusivity profile

The energy transfer to (4,3) mode is investigated at $t \sim 73.5$. It is found that the one from quasi-linear channel $(0,0)-(4,3)$ is comparable to those from micro channels: $(43,31)-(47,34) \sim (47,31)-(51,34)$ and $(48,34)-(52,37) \sim (51,34) \sim (55,37)$. The contribution from other meso-scale modes located on the core region: $(7,4)-(11,7)$ also exists, however, it is not as large as those from the micro modes. In the burst phase, the quasi-linear effect dominates the system and ITG modes are destabilized in the vicinity of $r = 0.7$, which lead to the burst. It is concluded that the amplitude of meso-scale modes and micro modes vary temporarily under the presence of the hear source and global relaxation.

3. Interaction between tearing mode and drift wave

The 4-field model equations including the neoclassical electron viscosity are derived by assuming cold ion and neglecting parallel ion momentum. The model consists of the vorticity equation, the Ohm's law, the continuity equation and the temperature evolution equation:

$$\frac{d}{dt}\nabla_\perp^2\phi = -\nabla_{//}\nabla_\perp^2 A + \mu_\perp \nabla_\perp^4 \phi$$

$$\frac{\partial}{\partial t}A = -\nabla_{//}(\phi - \delta p) + \alpha_T \delta \nabla_{//} T + \eta_{//}(1+\sqrt{\varepsilon})\nabla_\perp^2 A - \eta_{//}\sqrt{\varepsilon}\frac{q}{\varepsilon}\frac{\partial p}{\partial r}$$

$$\frac{d}{dt}n + \beta\frac{d}{dt}p = -\beta\delta\nabla_{//}\nabla_\perp^2 A + \eta_\perp \beta \nabla_\perp^2 p - \beta\eta_{//}\sqrt{\varepsilon}\frac{1}{r}\frac{\partial}{\partial r}r\nabla_\perp^2 A + \beta\eta_{//}\sqrt{\varepsilon}\frac{q}{\varepsilon}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial p}{\partial r}$$

$$\frac{3}{2}\frac{d}{dt}T - \frac{d}{dt}n = -\alpha_T \delta\beta\nabla_{//}\nabla_\perp^2 A + \chi_\perp \nabla_\perp^2 T$$

where $d/dt = \partial/\partial t + [\phi,]$, $[,]$ is the Poisson bracket, $\nabla_{//} = ik_{//} - [A,]$, $\alpha_T = 0.71$, β is the plasma beta, ε is the inverse aspect ratio, $\delta \equiv (c/\omega_{pi})/a$ is the normalized ion skin depth, where c is the speed of light, ω_{pi} is the ion plasma frequency and a is the minor radius. The parameter δ indicates the strength of the drift wave coupling. The normalization is the same as those given in Ref.[6]. The neoclassical electron flow is simplified by assuming the neoclassical ion flow is zero. The rotation damping is neglected in the vorticity equation. Two types of q profile are used for simulations: $\Delta'_{2/1} = 11.1$ (standard q profile) and $\Delta'_{2/1} = -0.244$ (optimized q profile) shown in figure 3.

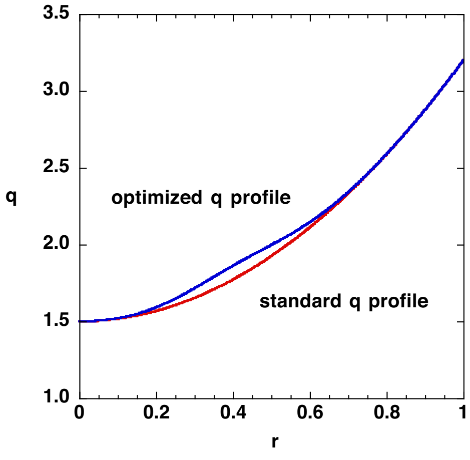

Fig.3 q profile

Simulation parameters are given by $\beta = 10^{-2}$,

$\varepsilon = 2\times10^{-1}$, $\mu_\perp = 10^{-4}$, $\eta_{//} = 10^{-5}$, $\eta_\perp = 10^{-4}$, $\chi_\perp = 10^{-4}$, $\delta = 10^{-3}$. Firstly, the linearized equations are solved as an initial value problem. Figure 4 shows the time evolution of electromagnetic energy of (2,1) modes. The cases with or without neoclassical viscosities are compared, i.e., 4th term in the Right Hand Side (RHS) of Ohm's law and 3rd and 4th terms in the RHS of the continuity equation. For $\Delta^{'} > 0$, the bootstrap current term enhances the growth rate of drift tearing mode. The neoclassical electron viscosity terms in the continuity equation have destabilizing and stabilizing effects, respectively and totally contribute to stabilize it, In addition, it is newly found that there is an instability even in the limit of $\Delta^{'} < 0$ if the bootstrap current term exists in the Ohm's law. The eigen-function shows the tearing-like mode. The transition between two eigen-state is observed in the time evolution of electromagnetic energy. It should be noted that this instability is only observed in the vicinity of $\Delta^{'} \approx 0$ and is stabilized by the drift effect and by collisional viscosities. The dependence of the growth rate on the drift parameter δ is also investigated for the optimized q profile. It is found that the stability threshold ($\delta \approx 8.5\times10^{-3}$) strongly depends on $\eta_\perp$ and $\chi_\perp$. If we set $\eta_\perp = \chi_\perp = 0$, no stability threshold is found and higher harmonics become more unstable than (2,1) mode.

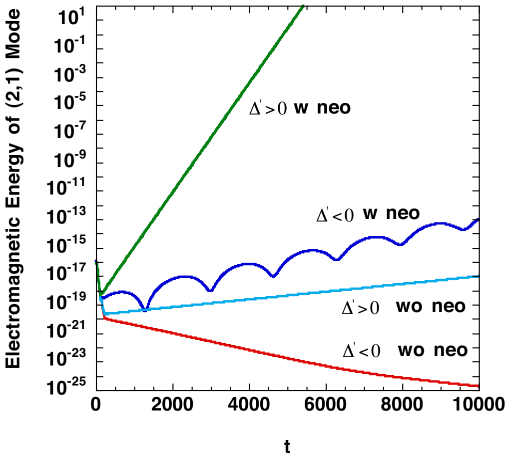

Fig.4 Time evolution of electromagnetic energy of (2,1) mode.

Using eigen-functions obtained by the simulation with standard q profile, we restart the run with the optimized q profile and investigate the dynamics of the neoclassical tearing mode (NTM). Figures 5 shows the time evolution of electromagnetic energy of (2,1) mode in case with $\delta = 10^{-2}$, where mode is linearly stable. It is found that the nonlinear sustainment does not occur but the magnetic island simply dumps. This result is different from that expected from the Rutherford model, where the magnetic island grows if $\Delta^{'} < 0$ and the seed island crosses the some threshold value.

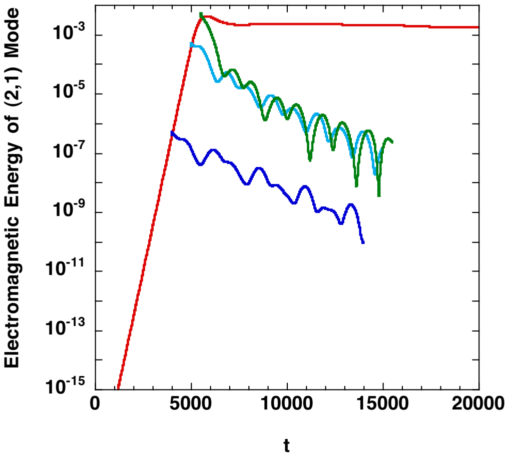

Fig.5 Time evolution of electromagnetic energy of (2,1) mode

To draw the decisive conclusion on sub-critical excitation of NTM in the drift tearing regime, it is necessary to investigate other parameter dependence on it, such as q profile and the corresponding δ, which gives the marginal growth rate. It is left for a future work.

4. Summary

The mechanism of ITB formation and collapse is investigated by using the multi-scale transport simulation code based on the gyro-fluid model. It is shown that many meso-scale modes are excited via three-wave interaction among ITG modes and quasi-linear effect. The amplitude of meso-scale and micro modes vary temporarily under the presence of the heat source and the global relaxation. The development of the turbulence structure is sensitively depends on the source profile. In this simulation, the collapse of the ITB starts without trigger. It can be regarded as a kind of the global relaxation. However, the final burst is triggered by unstable ITG modes in the vicinity of $r \approx 0.7$, which is destabilized by the quasi-linear effect. The results show the necessity of the global simulation to explore the mechanism of ITB formation and collapse.

The 4-field model equations including the neoclassical electron viscosity are derived by assuming cold ion and neglecting parallel ion momentum. Using this model, the drift tearing mode with the neoclassical effects is investigated. It is found that there is an instability even in the limit of $\Delta^{'} < 0$ if the bootstrap current term exists in the Ohm's law. This instability is only observed in the vicinity of $\Delta^{'} \approx 0$ and is stabilized by the drift effect and by collisional viscosities. We also investigate the dynamics of the NTM using the optimized q profile. So far, the sub-critical excitation of NTM has not been observed. Instead, the island simply dumps. To draw the decisive conclusion on sub-critical excitation of NTM in the drift tearing regime, it is necessary to investigate parameter dependence in detail. It is left for a future work.

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