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Analyses of Nonlinear Coupling for Turbulent Structural Formation in Magnetized Cylindrical Plasmas

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Turbulent plasmas form a variety of meso-scale structures such as a zonal flow and a streamer, which regulates micro-scale fluctuations. The mechanism of nonlinear structural formation has been studied with a three-field (density, potential and parallel velocity of electrons) reduced fluid model, which describes the resistive drift wave turbulence in magnetized cylindrical plasmas. The turbulent structure, streamer, is selectively formed by changing the ion-neutral collision frequency, which is the damping parameter of the zonal flow, in the target plasma. The turbulent structures are formed by nonlinear wave coupling of eigenmodes. The possible coupling can be controlled by restricting some modes in numerical simulations, and it is found that the same structural formation mechanism is dominant within several kinds of coupling paths.

Keywords: turbulence, structural formation, resistive drift wave, zonal flow, streamer

1. Introduction

Turbulent plasmas form a variety of meso-scale structures such as a zonal flow and a streamer [1,2]. A streamer, which is a localized vortex in the azimuthal direction, is generated by nonlinear wave coupling, and is sustained for a much longer duration than the oscillation period of micro instability. The turbulent structure affects the level of the anomalous transport in fusion plasmas by regulating micro-scale fluctuations, therefore, the formation and self-regulated mechanism of the turbulent structures should be taken into consideration to understand the transport processes.

Plasma experiments in a simple linear configuration have been carried out recently for quantitative understandings of the structural formation mechanism by turbulence [3-7]. A density gradient drives the drift wave turbulence in these plasmas. Two-dimensional measurements reveal the feature of turbulent structures [8-9] and their formation mechanisms by nonlinear mode coupling [10-11].

Numerical simulations of drift wave turbulence in linear devices have been carried out to understand the fundamental mechanism of structural formation by comparison with experiments [12-16]. We have developed a three-dimensional numerical simulation code, which simulates the resistive drift wave turbulence in magnetized cylindrical plasmas [12]. Selective formation of the turbulent structure can be identified by changing a damping parameter of the zonal flow [14]. There exist several paths for energy transfer by nonlinear couplings in plasmas. Each magnitude of the nonlinear coupling specifies the dominant three-wave coupling for the structural formation. To understand the self-organized selection mechanism, the number of modes included in numerical simulations is changed in this paper. The characteristic nonlinear energy transfer is compared to clarify the selection rule of the formed structure depending on mode coupling paths.

The paper is organized as follows. In Sec. 2, the set of model equations for the analyses are described. Simulations using this model are explained, and show formation of turbulent structures, a zonal flow and streamer in Sec. 3. The magnitudes of the nonlinear coupling are calculated to study the formation mechanism of the streamer, and calculation with additional modes is also carried out for comparison in Sec. 4. Then, we summarize our results in Sec. 5.

2. Model

We have been developing a three-dimensional numerical simulation code of the resistive drift wave turbulence in a linear device, called ‘Numerical Linear Device’ (NLD, details are described in [13]). The three-field (density, potential and parallel velocity of electrons) reduced fluid model is adopted. The plasma has a simple cylindrical shape, and the magnetic field has only the component in the axial direction with the uniform intensity. According to experiments, high density ($n_e > 1\times10^{19}$[m^{-3}]) and low temperature ($T_e < 5$ [eV]) plasmas in an argon discharge are analyzed. The density of neutral particles is high even in the plasma core region [17], so the effect of neutral particles is taken into consideration. The

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continuity equation, the vorticity equation and Ohm's law can be used to obtain the fluctuating density, potential and parallel velocity of electrons [18]:

$$\frac{dN}{dt} = -\nabla_{//}V - V\nabla_{//}N + \mu_N \nabla_\perp^2 N + S, \qquad (1)$$

$$\frac{d\nabla_\perp^2\phi}{dt} = \nabla N \cdot \left(-\nu_{in}\nabla_\perp\phi - \frac{d\nabla_\perp\phi}{dt}\right) - \nu_{in}\nabla_\perp^2\phi - \nabla_{//}V - V\nabla_{//}N + \mu_W \nabla_\perp^4\phi \quad, \qquad (2)$$

$$\frac{dV}{dt} = \frac{M}{m_e}\left(\nabla_{//}\phi - \nabla_{//}N\right) - \nu_e V + \mu_V \nabla_\perp^2 V, \qquad (3)$$

where $N = \ln(n / n_0)$, $V = v_{//} / c_s$, $\phi = e\varphi / T_e$, n is the density, n_0 is the density at $r = 0$, $v_{//}$ is the electron velocity parallel to the magnetic field, c_s is the ion sound velocity, φ is the electrostatic potential, T_e is the electron temperature, $d / dt = \partial / \partial t + [\phi, \ ]$ is the convective derivative, S is a particle source term, M / m_e is mass ratio of ion and electron, ν_{in} is ion-neutral collision frequency, $\nu_e = \nu_{ei} + \nu_{en}$ is the sum of ion-electron and electron-neutral collision frequency, and μ_N, μ_V, μ_W are artificial viscosities. The ion cyclotron frequency Ω_{ci} and Larmor radius measured by the electron temperature ρ_s are used for the normalizations of the time and distance, respectively. The equations are solved in the cylindrical coordinate with spectral expansion in the azimuthal and axial directions assuming periodic boundary condition, where m and n are the azimuthal and axial mode number, respectively. The boundary condition in the radial direction are set to $f = 0$ at $r = 0$, a when $m \neq 0$, and $\partial f / \partial r = 0$ at $r = 0$, $f = 0$ at $r = a$ when $m = 0$, where f implies $\{N, \phi, V\}$, and $r = a$ gives an outer boundary of the plasma column.

3. Turbulent Structure

3.1. Simulation parameters

A nonlinear simulation has been performed to examine the saturation mechanism of the resistive drift wave turbulence. The following parameters are used: $B = 0.1$ [T], $T_e = 2$ [eV], $a = 10$ [cm], length of the device $\lambda = 1.7$ [m], $\mu_N = 1 \times 10^{-2}$, $\mu_V = \mu_W = 1 \times 10^{-4}$. Using these parameters, ν_e is estimated to be $\nu_e = 310$ [13]. The electron collisions (ν_{ei} and ν_{en}) destabilize and the ion-neutral collisions (ν_{in}) stabilize the resistive drift wave [13]. Therefore, the drift wave can be excited with large ν_e and small ν_{in}. There is ambiguity of the value of collision frequency ν_{in}, which depends on the neutral density. Therefore, ν_{in} is used as a parameter for controlling the instability in our simulations. The calculation with a fixed particle source has been carried out, where the time independent source profile is given by

$$S(r) = \frac{4S_0\mu_N}{L_N^2}\left[1 - \left(\frac{r}{L_N}\right)^2\right]\exp\left[-\left(\frac{r}{L_N}\right)^2\right], \qquad (4)$$

with $S_0 = 5.0$, $L_N = 5$ [cm]. This source profile gives a density profile proportional to $\exp[-(r/L_N)^2]$ in a linear phase, which is flattened for $r / a = 0.2$ - 0.8 in a nonlinear phase, as described in [13]. The density profile peaked at $r = 0$ destabilizes the resistive drift wave.

3.2. Nonlinear simulation

Linear analyses in the cylindrical geometry give linear growthrates and eigenfrequencies [13]. Only $n = 1$ modes can be unstable with these parameters. The dispersion relation of the linear eigenmodes with $n = 1$ shows weak dispersion in small m ($m < 4$), and $\partial\omega / \partial k_\theta \sim 0$ with $m = 4$ – 6. The initial condition is given to be $f = 0$ for $(m, n) = (0, 0)$ and $f = 1 \times 10^{-8} \sin(\pi r / a)$ for all the other modes, where f implies $\{N, \phi, V\}$. Simulations are performed with 256 grids in the radial direction. Fourier modes $(m, n) = (0, 0)$ and $m = \pm1$ - ±16, $n = \pm1$ - ±16 are taken ($m \times n = 16 \times 16$). Modes with $(m, n) = (3 – 6, 1)$ have the largest amplitudes in nonlinear phases, so this number of modes must be taken in calculations at least. The time evolution of each mode is calculated with Eqs. (1) – (3). Figure 1 shows an energy spectrum in a nonlinear saturation state, where $E_\phi(m) = \int dr^3 (\nabla_\perp \phi_m)^2 / 2$, and the dependency of the linear growthrate of $n = 1$ mode on m. The growthrates are calculated with the instantaneous density profile. The energy input comes from unstable modes with $m = 3 – 6$, where the instantaneous growthrate is positive, and is transferred to the higher m region by nonlinear mode coupling to be dissipated, and to the lower m region to be collisionally damped. The saturation state is sustained with these energy balances.

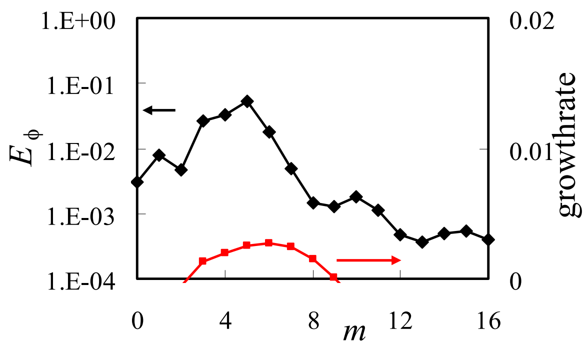

Fig. 1: Potential energy spectrum and instantaneous linear growthrate dependence on m. These are the case with $\nu_{in} = 0.1$ at $t = 7500$. Unstable modes with $m = 3$ - 6 drive the turbulence.

In the nonlinear saturation states, two kinds of turbulent structures have been obtained [14]; a zonal flow and a streamer. If the collision frequency is small, compared with the growth rate of unstable modes in saturated states, modulational coupling of unstable modes generates the (0, 0) mode, which is the zonal flow. If the collision frequency is large, the zonal flow remains stable, owing to strong collisional damping, and parametric coupling with modes, which have neighboring m and the frequency close to each other, forms a streamer. Snapshots of the contours of the potential are shown in Fig. 2. Typical structure with the streamer is represented in (c), compared with that with the zonal flow in (a), which shows the contour of the (0, 0) mode. The perturbation structure in the zonal flow case is a mixture of some modes, as shown in Fig. 2 (b).

Selective formation of the turbulent structure can be identified by changing ν_{in}, which represents the strength of the damping force of the zonal flow [14]. The zonal flow amplitude is small and the streamer modes are excited when $\nu_{in} > 0.05$. As ν_{in} is decreased, the zonal flow begins to be excited. The inflection point is given to be $\nu_{inc} \sim 0.052$ with the parameters in Subsec. 3.1, which is the clear indication to be nonlinear phenomena.

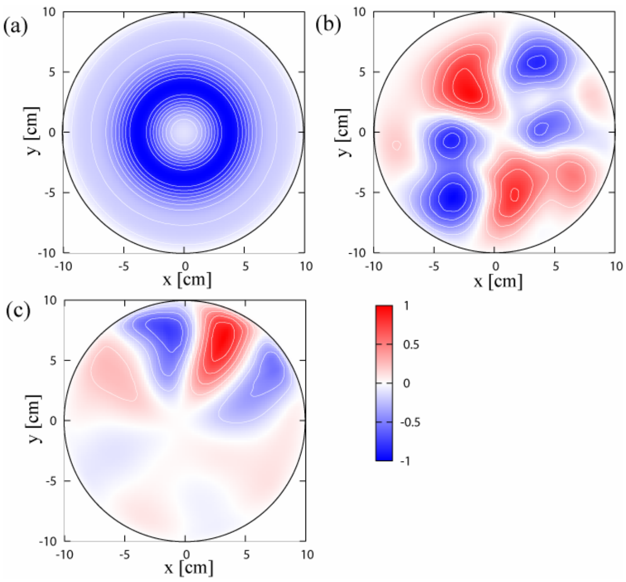

Fig. 2: Snapshots of the contours of the electrostatic potential, which is normalized by the maximum value at that time. (a) The (0, 0) component and (b) other component in the case of zonal flow formation with $\nu_{in} = 0.02$, and (c) that of streamer formation with $\nu_{in} = 0.1$ are shown.

4. Energy Exchange Paths

4.1. Competition of mode coupling

In the cases discussed in Sec. 3, there are two dominant energy exchange paths from $m \neq 0$ mode by mode coupling. One is that to (0, 0) mode to form the zonal flow, and the other is that to the mediator mode ((1, 2) in this case) to form the streamer. These two kinds of structural formation mechanisms are involved, but only one of the structures can appear in stationary states from their competitive nature. When the zonal flow is formed, the E × B shearing of the zonal flow breaks the phase locking of the modes, so the streamer is not formed, even though amplitudes of the modes are large.

4.2. Energy exchange for streamer formation

The detailed analysis of the energy exchange for streamer formation is carried out. Figure 3 (a) shows the time evolution of the fluctuation energy of the potential in the case with $\nu_{in} = 0.1$. Modes with $(m, n) = (\pm 4, \pm 1)$ and $(\pm 5, \pm 1)$ are dominant. A vortex structure localized in the θ direction, shown in Fig. 2 (c), are formed by these modes, and is sustained for a much longer duration than the drift wave oscillation period (more than 3000 Ω_{ci}). This mode matching comes from nonlinear mode coupling. Although the phase velocities of the linear eigenmodes of (4, 1) and (5, 1) are different from each other, nonlinear frequencies of (4, 1) and (5, 1) modes are downward shifted and become close [14].

To clarify the mechanism, the rate of the energy transfer of each mode $dE(m,n)/dt$ is calculated. Equations (1) – (3) can be divided into two parts: the part proportional to the own mode energy, and the other nonlinear coupling part. Linear (LT) and nonlinear (NT) contributions in $dE_\phi(m,n)/dt$ are calculated. The magnitude of NT is comparable to LT, and LT drives the mode variation, which is suppressed by nonlinear coupling term NT [14]. To clarify which mode couplings are dominant in nonlinear energy transfer, NT is decomposed into components, corresponding to each three-wave coupling (Fig. 3 (b)). Couplings with neighbouring modes, such as (4, 1) ← (3, -1) + (1, 2) or (5, -1) + (-1, 2), are dominant. Although the amplitude of (3, -1) + (1, 2) coupling is largest, the sign of the term NT is mainly dictated by (5, -1) + (-1, 2) coupling, which is most important for streamer formation in this case. If the mediate mode (1, 2) is artificially removed after the saturation, the streamer is not sustained and only a single mode becomes dominant [14]. These results suggests the

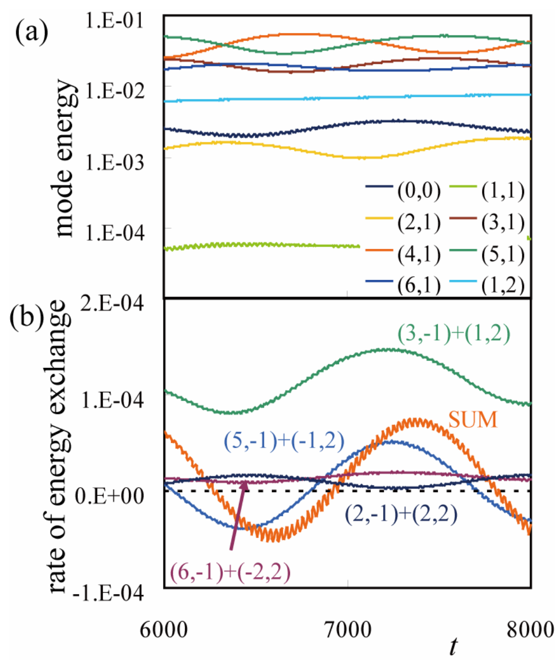

Fig. 3: Time evolution of (a) Fourier modes of the fluctuation energy, and (b) rates of the nonlinear energy transfer decomposed into each three-wave coupling in the nonlinear steady state, when $\nu_{in} = 0.1$. This is the case when $n = 0$ modes are not included in the calculation.

important role of the (1, 2) mode for the streamer formation. The mediator mode number is $(m, n) = (1, 2)$, because most unstable modes with neighboring m and $n = 1$ form the streamer. The axial mode number of the mediator does not change with a different device length, because $n = 1$ modes are still most unstable with parameters for experimental devices [13]. Note that mode coupling in the k_r space also affects on nonlinear saturation, which suggests three-dimensional mode coupling is essential for preservation of the streamer [19].

4.3. Streamer formation with $n = 0$ modes

Calculations including modes with $m \neq 0$ and $n = 0$ are also carried out. These axially homogeneous modes contributes to the formation of the turbulent structure, so inclusion of the modes is important for clarify the structural formation mechanism in the experimental device. In our model, stabilizing effect of modes with $m = 0$ and $n \neq 0$ is not included. This is because $(0, n)$ modes show bursty increase in the nonlinear phase, if these modes are added in the calculation. Therefore, we only discuss the role of $(m, 0)$ modes here. These modes are investigated in terms of geodestic acoustic mode (GAM) oscillation in tokamak plasmas [1].

Figure 4 (a) shows the time evolution of the fluctuation energy of the potential in the case with $\nu_{in} = 0.15$. Inclusion of $(m, 0)$ makes the system more unstable, and the zonal flow can be excited with larger ν_{in} than the case without $(m, 0)$ modes. Modes with $(m, n) = (\pm3, \pm1)$ and $(\pm4, \pm1)$ are dominant in this case, which form a streamer. The structure sustains for much longer duration than the drift wave oscillation period, but shorter than that in the case without $(m, 0)$ modes.

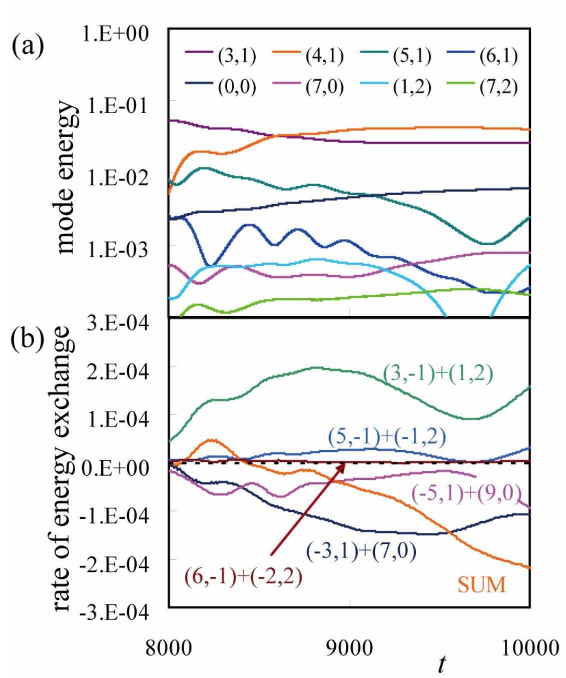

Fig. 4: Time evolution of (a) Fourier modes of the fluctuation energy, and (b) rates of the nonlinear energy transfer decomposed into each three-wave coupling in the nonlinear steady state, when $\nu_{in} = 0.15$. This is the case when $n = 0$ modes are included in the calculation.

To confirm which mode couplings are dominant in nonlinear energy transfer, the rate of energy exchange for (4, 1) mode is calculated, which is decomposed into components, corresponding to each three-wave coupling (Fig. 4 (b)). Among the couplings with $m \neq \pm4$ modes, couplings with neighbouring modes, such as $(4, 1) \leftarrow (3, -1) + (1, 2)$ or $(5, -1) + (-1, 2)$, are dominant for nonlinear mode excitation, as is the same in the case without $(m, 0)$ modes. Couplings mediated by $(m, 0)$ modes, such as $(4, 1) \leftarrow (-3, 1) + (7, 0)$ or $(-5, 1) + (9, 0)$, give other paths for energy transfer, which are main contributors of the difference between the sum of the rate of energy exchange and that from dominant coupling $(4, 1) \leftarrow (3, -1) + (1, 2)$. The couplings mediated by $(m, 0)$ modes affect as damping of the mode, whose rate of energy exchange has the negative sign. It is found that the couplings mediated by (1, 2) mode is still most important for streamer formation in this case. The quasi-linear effect of the (0, 0) mode is also important for mode saturation. The formation mechanism of the (0, 0) mode in the case with $(m, 0)$ modes will be studied in future works.

5. Summary

We have carried out the nonlinear simulation of the resistive drift wave in cylindrical plasmas. Turbulence with a zonal flow or a streamer was obtained in the nonlinear steady states. Detailed analyses show that the energy transfer between modes, which have neighboring m by means of the mediator mode (1, 2), is important for the streamer formation. The streamer is sustained with balance of some nonlinear mode couplings, and is selectively formed with ν_{in} larger than the critical value, in spite of the other possible energy exchange paths. In this way, our minimal model for analyzing the turbulent structural formation mechanism by mode coupling represents the selection rule of the structure.

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