Alfvén Waves In Multi-Component Plasmas

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In the linear cylindrical plasma device VINETA [REF: C. Franck, Phys. Plasmas, 9(8), 3254 (2002)] Alfvén waves (AWs) were measured in single- (Ar, He) and multi-ion species plasmas (Ar+O, He+Ar). The comparison of theoretically calculated dispersion relations for corresponding discharge parameters shows reasonable agreement of the dispersion behavior in theory and measurements below the ion cyclotron resonance frequency. The experimentally observed Alfvén speeds for each case are consistent with the theoretically expected ones.

Keywords: Alfvén wave, dispersion relation, multi-component plasma, magnetic field fluctuation

1. Introduction

Alfvén waves (AWs) [1], low frequency electromagnetic waves with \( \omega < \omega_{ci} \) the ion cyclotron resonance frequency, are basically responsible for transport processes of magnetic energy and heat in magnetized astrophysical [2,3] and laboratory plasmas [4–8]. The dispersion behavior of an AW is determined by the evolution of the parallel electron current and the perpendicular ion current. Hence the presence of multiple ion species strongly influence the dynamics of an AW, which is an important issue in investigations concerning space [9], fusion [10] and laboratory plasmas [11].

This paper discusses a simple collisionless theory for AW dispersion in single- and multi-ion species plasmas. This forms a basis for the comparison of experimental data with theoretical calculations. The experimental device VINETA and the diagnostic systems used are outlined as well.

2. Dispersion theory

A cold, current-free plasma with one or more ion species is considered. For simplicity collisional effects are neglected. The equation of motion and the total current density \( \mathbf{j} \) of a multi-ion plasma with density \( n_k \), mass \( m_k \), velocity \( \mathbf{v}_k \) and charge \( q_k = Z_k e \), where \( k \) indicates the number of ion species (for electrons \( Z_k = -1 \)), can be written as

\[
\frac{m_k}{\partial t} \mathbf{v}_k = q_k (\mathbf{E} + \mathbf{v}_k \times \mathbf{B}) \tag{1}
\]

\[
\mathbf{j} = \sum_k n_k q_k \mathbf{v}_k , \tag{2}
\]

with the electric and magnetic field \( \mathbf{E} \) and \( \mathbf{B} \), respectively.

Let \( \mathbf{B} = (0, 0, B_z) \), \( \omega_{ci,k} = q_k B/m_k \) (ion cyclotron frequency) and \( \omega_p^2 = q^2/(\epsilon_0 m) \) (plasma frequency).

We consider plane waves of the form

\[
\mathbf{E} = E_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \tag{3}
\]

with wave vector \( \mathbf{k} \), position vector \( \mathbf{r} \), time \( t \) and define the refractive index \( \mathbf{n} = c k/\omega \) having magnitude \( n = c/v \) (c: speed of light) where \( v = \omega/k \) is the phase velocity. Using Maxwell’s equation \( \nabla \times \mathbf{E} = i\omega \mathbf{B} \) and the Fourier transform \( \nabla \times (\nabla \times \mathbf{E}) = -\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) = (\omega^2/c^2) \mathbf{K} \cdot \mathbf{E} \) and

\[
\mathbf{n} \times (\mathbf{n} \times \mathbf{E}) = -\mathbf{K} \cdot \mathbf{E} . \tag{4}
\]

Having \( \mathbf{n} \) and \( \mathbf{k} \) lie in the \((x, z)\)-plane yields \( n_y = k_y = 0 \). The components of \( \mathbf{n} \times (\mathbf{n} \times \mathbf{E}) \) are

\[
\begin{bmatrix}
-n_z^2 & 0 & n_x n_z \\
0 & -(n_x^2 + n_z^2) & 0 \\
n_x n_z & 0 & -n_z^2
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix} \tag{5}
\]

and the product of the dielectric tensor \( \mathbf{K} \) and \( \mathbf{E} \) is

\[
\mathbf{K} \cdot \mathbf{E} =
\begin{bmatrix}
K_{xx} & K_{xy} & 0 \\
K_{yx} & K_{yy} & 0 \\
0 & 0 & K_{zz}
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix} . \tag{6}
\]

The solution of (4) gives the cold plasma dispersion relation [12] including finite electron mass, the Hall term in Ohm’s law, the vacuum displacement current and one or more ion species. As such, the dispersion relation describes all known cold plasma wave types, including the torsional and compressional AW.

The case of two ion species is described as follows, according to R. Cross [12]. The approximations \( \omega \ll \omega_{ce} \) and \( v_A \ll c \) are made, with the electron cyclotron frequency \( \omega_{ce} \) and the Alfvén speed \( v_A \). Let \( n_1 \) and \( n_2 \) be the number densities of the two ion species, \( m_1, m_2 \) their masses and \( Z_1, Z_2 \) their charge numbers. Then \( n_e = Z_1 n_1 + Z_2 n_2 \), \( x = (Z_2 n_2)/(Z_1 n_1) \), \( \Omega_1 = \omega/\omega_{ci1} \), \( \Omega_2 = \omega/\omega_{ci2} \), \( \Omega_e = -\omega/\omega_{ce} \) with \( \omega_{ci1} = Z_1 e B/m_1 \), \( \omega_{ci2} = Z_2 e B/m_2 \) and

\[
v_A^2 = \frac{B^2}{\mu_0 n_1 m_1} . \tag{7}
\]
Introducing the following abbreviations

\[
P = -\frac{(1 + x)e^2}{\Omega_1 \Omega_2 v_A^2},
\]

\[
R = \frac{e^2}{\Omega_1 v_A^2} \left[ (1 + x) - \frac{1}{1 + \Omega_1} \frac{x}{1 + \Omega_2} \right],
\]

\[
L = \frac{e^2}{\Omega_1 v_A^2} \left[ -(1 + x) + \frac{1}{1 - \Omega_1} \frac{x}{1 - \Omega_2} \right],
\]

\[
S = \frac{R + L}{2},
\]

\[
D = \frac{R - L}{2},
\]

the solution of (4) reduces to

\[
k_\parallel^2 = \frac{F^2 - G^2}{F^2},
\]

where \(F = A - k_\perp^2, A = \omega^2 S/c^2\) and \(G = -\omega^2 D/c^2\) [12]. Eq. (13) is quadratic in \(k_\parallel^2\) with solutions

\[
k_\parallel^2 = A - k_\perp^2/2 \pm [G^2 + k_\perp^4/4]^{1/2}.
\]

The \(\pm\) signs give torsional and compressional wave modes, respectively. Setting \(n_1 = 0\) or \(n_2 = 0\) the known solution for a single ion species plasma is recovered

\[
k_\parallel^2 = \frac{\omega^2}{v_A^2(1 - \Omega_1)}.
\]

These modes are still present in a two-ion species plasma. However, the torsional wave can have two resonances \(k_\parallel = \infty\), one at the lower cyclotron frequency and one at the higher cyclotron frequency. Assuming that the propagation of an AW is exactly parallel or perpendicular to \(\mathbf{B}\), two special cases can be discussed. If \(k_\perp = 0\) Eq. (13) yields \(F = \pm G\) such that \(k_\parallel^2 = A \pm G\). If \(k_\parallel = 0\) Eq. (13) yields \(k_\parallel^2 = (A + G)/(A - G)/A\). Thus for the torsional wave with propagation parallel to \(\mathbf{B}\) follows

\[
k_\parallel^2 = A + G = \frac{\omega^2}{v_A^2} L.
\]

With (10) this dispersion relation can be written as

\[
k_\parallel^2 = \frac{\omega^2}{v_A^2 \text{mult}} \frac{(a - b \Omega_2)}{a(1 - \Omega_1)(1 - \Omega_2)},
\]

where \(a = 1 + x \omega_{ci1}/\omega_{ci2}, b = 1 + x\) and

\[
v_A^2 \text{mult} = \frac{B^2}{\mu_0 (m_1 n_1 + m_2 n_2)}.
\]

3. Experimental setup

The measurements were conducted in the linear cylindrical plasma device VINETA. The vacuum vessel (total length 4.5m, \(\phi = 0.4\text{m}\)) is immersed in a set of 36 coils to generate a homogeneous magnetic field up to 100mT. The low-temperature plasmas are driven by a standard helicon antenna [13,14].

\(f = 13.56\text{MHz}, P = 4\text{kW pulsed}\) at the one end of the device. Swept Langmuir probes are used to measure densities \(n\) and electron temperatures \(T_e\) of the centrally peaked plasma (\(\phi = 0.1\text{m},\) working gas pressure 0.1Pa, ionization degree 35\%). Typical values are \(n = 5 \cdot 10^{19}\text{m}^{-3}, T_e = 2\text{eV}\). The ions are cold \(T_i = 0.2\text{eV}\) [15]. The ion cyclotron resonance frequencies are \(f_{ci}^{He} = 38.8\text{kHz}\) for Ar and \(f_{ci}^{He} = 388\text{kHz}\) for He. The relatively low temperatures lead to ion-neutral and ion-ion collisionalities in the range of \(\nu_{in} = 2 \cdot 10^4\text{Hz}\) and \(\nu_{ii} = 1 \cdot 10^7\text{Hz}\), respectively. Collisional effects have been shown to effect the AW dispersion behavior, especially when the AW frequency approaches \(f_{ci}\) [16]. Since collisional effects are not included in the present framework the investigations of the AW dispersion are restricted to frequencies smaller than \(f_{ci}\).

Electromagnetic waves are excited with an insulated Helmholtz coil pair in the plasma center. Each loop has 4 windings (\(\phi = 3.5\text{cm},\) distance of the loops 3.5cm). With a capacitive matching unit, typical alternating currents up to 50A are driven, which is equivalent to a magnetic perturbation of \(B_y \approx 5\text{mT}\) compared to \(B_z = 100\text{mT}\). Highly sensitive magnetic inductive probes are used to measure the magnetic field fluctuations of a wave. The \(x,y,z\)-loops (1000 windings each) of the probe were calibrated in a Helmholtz test field and have a lowest detection level of \(B_{low} \leq 10\text{nT}\), which is sufficiently below the expected magnetic fluctuations in the \(\mu\text{T}\) range. To minimize the influence on the discharge the whole probe body has been miniaturized to a length of 18mm and a height of 10mm, only. Great care was taken to reduce electrostatic pickup by shielding the probe.
Fig. 2 Three normalized AW dispersion relations (a). For [Ar1]: $B = 77\text{ mT}, n = 2 \cdot 10^{17}\text{ m}^{-3}$, [Ar2]: $B = 102\text{ mT}, n = 5 \cdot 10^{18}\text{ m}^{-3}$ and [Ar3]: $B = 77\text{ mT}, n = 4 \cdot 10^{18}\text{ m}^{-3}$. Also shown are the linear best fits of the dispersion relations. For changing magnetic field or density the expected behavior is found.

4. Single-component plasmas

Fig. 1 shows a measured AW dispersion in a pure He discharge with $B = 102\text{ mT}$ and $n = 1 \cdot 10^{18}\text{ m}^{-3}$ normalized to $v_A$ and $\omega_{ci}$ of He. The calculated dispersion relation (15) (solid line) and a linear best fit to the measurement (dashed line) are depicted as well. The slope of the fit yields the Alfvén speed $v_A^{fit}$ as listed in Tab. 1 and compared to the calculated values $v_A^{calc}$ (7). The Alfvén speeds agree quite well, but there is a systematic deviation of $Q_{He}^A = v_A^{fit}/v_A^{calc} \approx 0.77$. In the low frequency regime fluid plasma instabilities occur [17], which strongly influence the dynamics of the AW for $kv_A/\omega_{ci} < 0.25$ and lead to fluctuations of the AW phase velocity.

Fig. 2 shows a set of measured AW dispersion relations in three different Ar discharges, where the magnetic field and the density are varied as follows: In [Ar1]: $B = 77\text{ mT}, n = 2 \cdot 10^{17}\text{ m}^{-3}$, in [Ar2]: $B = 102\text{ mT}, n = 5 \cdot 10^{18}\text{ m}^{-3}$ and in [Ar3]: $B = 77\text{ mT}, n = 4 \cdot 10^{18}\text{ m}^{-3}$. All dispersion relations are normalized to $v_A,Ar_2$ and $\omega_{ci,Ar_2}$ of discharge [Ar2]. The Alfvén speed as obtained from the linear best fit (solid for [Ar1], dashed for [Ar2], dash-dotted for [Ar3]) can be found in Tab. 1. Here the averaged ratio is $Q_A^{Ar} \approx 1.3$.

It is concluded that the AW dispersion in single ion discharges qualitatively shows the expected behavior and reasonable agreement between the calculation and the measured Alfvén speeds is found.

The presence of an additional ion species significantly influences the propagation AWs. The investigations of AW dispersion in two-ion species plasmas follows in the next section.

5. Multi-component plasmas

Fig. 3 shows AW dispersion relations in a set of different mixtures of Ar and O assuming that the corresponding ratio of the partial gas pressures reflects

<table>
<thead>
<tr>
<th>discharge</th>
<th>$v_A^{calc} [10^5 \text{ m/s}]$</th>
<th>$v_A^{fit} [10^5 \text{ m/s}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar (100%)</td>
<td>5.96</td>
<td>7.22</td>
</tr>
<tr>
<td>Ar (100%)</td>
<td>1.58</td>
<td>2.29</td>
</tr>
<tr>
<td>Ar (100%)</td>
<td>1.33</td>
<td>1.67</td>
</tr>
<tr>
<td>He (100%)</td>
<td>11</td>
<td>8.48</td>
</tr>
<tr>
<td>Ar (90%) + O (10%)</td>
<td>1.64</td>
<td>2.7</td>
</tr>
<tr>
<td>Ar (55%) + O (45%)</td>
<td>1.85</td>
<td>3.33</td>
</tr>
<tr>
<td>Ar (40%) + O (60%)</td>
<td>1.95</td>
<td>3.52</td>
</tr>
</tbody>
</table>

Tab. 1 Alfvén speed in different discharges (comparison of calculation and fit results). The average ratio $Q_A^{all} = v_A^{fit}/v_A^{calc}$ is 1.4.
the ion density ratio of Ar$^+$ and O$^-$ [18]: [ArO1] (small dash): Ar (90%), O (10%), [ArO2] (dash): Ar (55%), O (45%), [ArO3] (dash-dot): Ar (40%), O (60%) and pure Ar [Ar] (solid). Eq. (17) predicts an increase of the slope of the dispersion relation for increased concentration of oxygen as the negative ion species (Fig 3 (a)). The calculated Alfvén speeds (18) for each case are compiled in Tab. 1. The measurements (Fig 3 (b)) follow the expected trend. The corresponding Alfvén speeds observed in the experiment agree quite well with theory (Tab. 1, $Q_{mix}^A \approx 1.75$).

For a mixture of two positive ion species (He 90%, Ar 10%) the AW dispersion relation is shown in Fig 4. The axes are normalized by the Alfvén speed and ion cyclotron resonance frequency of the major component He. In this case (17) yields a cutoff at 0.19 $\omega_{ci,He}$ and a resonance at 0.1 $\omega_{ci,He}$, which is exactly the cyclotron resonance frequency of the minor component Ar. This resonance is also clearly seen in the experimental data (Fig 4). Above the resonance the slope of the measured dispersion relation is slightly steeper.

6. Summary

The measured AW dispersions in single- and multi-ion species helicon plasmas satisfy over a wide range of parameters (varying density, magnetic field, ion mass and mixing ratio of positive and negative ion species) the behavior of theoretically calculated dispersion relations. Except for a relatively small factor of $Q_{mix}^A = v_A^{fit}/v_A^{calc} \approx 1.4$ the experimentally observed Alfvén speeds show reasonable agreement with the prediction (see Tab. 1), although the plasma collisionality is not considered.

Future studies shall focus on frequency ranges close to $\omega_{ci}$. Detailed investigations concerning the influence of high collision frequencies and kinetic effects on the ion cyclotron resonance AWs are under way.

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